INTEGRATING ECONOMIC SCENARIOS WITH MARKET AND CREDIT RISK SIMULATION ANALYTICS

Dan Rosen

CEO, ***d1g1t***

Director, Center for Financial Industries

Fields Institute

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Outline

- Introduction
- Case study: multi-asset investment portfolio and economic analysis
- Conditional Scenarios
- Least Squares Stress Testing Methodology (LSST)
  - Extensions to credit risk portfolios
- Concluding Remarks

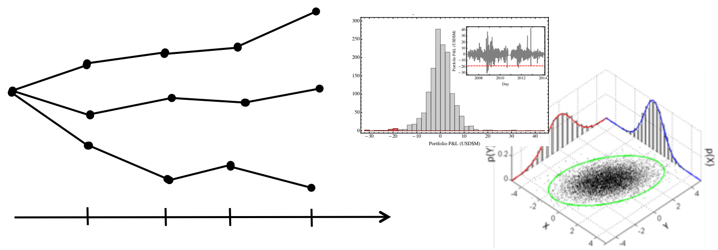
Rosen D. and Saunders D. (2016), Regress Under Stress: A Simple Least-Squares Method for Integrating Economic Scenarios with Risk Simulations, *Journal of Risk Management in Financial Institutions*, 9(4), Autumn/Fall 2016

Introduction – What Is A Scenario?

- A scenario is the basic descriptor of the *future* evolution of the *state-of-the-world* over time
  - Joint realization of

    all the relevant financial and economic risk factors

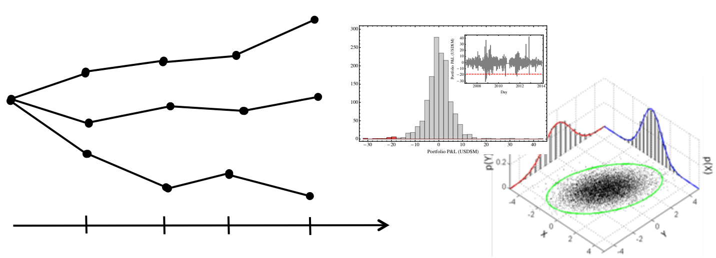
    at a point in time or, more generally, a discrete set of times ***in the future***



What Is A Scenario?

- A scenario is the basic descriptor of the *future* evolution of the *state-of-the-world* over time

- Scenario generation differs considerably from forecasting
  - A forecast is a prediction that a single scenario will occur, and its accuracy is therefore crucial
  - But… we really can't predict accurately specific financial events in the future



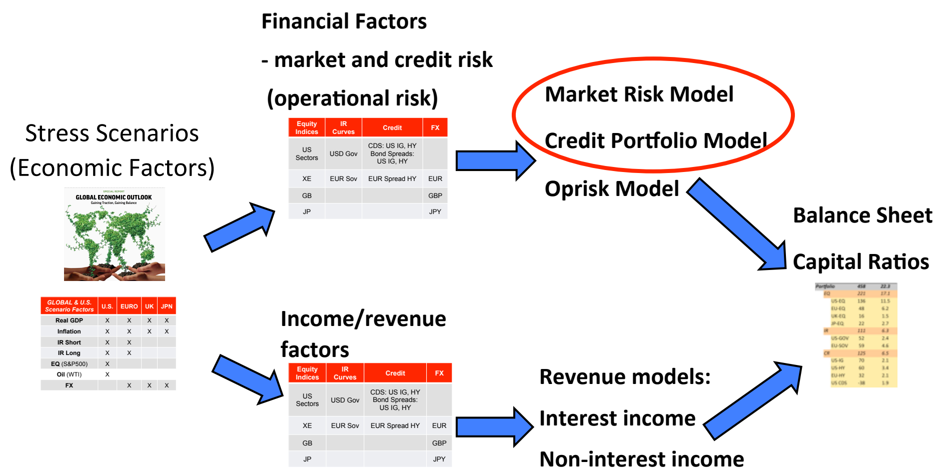
Scenarios and Risk Management

Conditional Scenarios

- Objective: to describe how *all the Risk Factors affecting a Portfolio* should behave as a given economic scenario unfolds
- General setting: economic or subjective scenarios are typically described in terms of a small number of key economic variables or risk factors
  - When applied to a portfolio, the scenario is incomplete – does not describe what happens to ***all relevant risk factors*** that affect the portfolio directly…or *indirectly*
- Implicitly or explicitly the relationship between all the factors must be accounted for when the scenario is applied to the portfolio
  - e.g., simple approach: use subjective or expert opinion
- Statistical or financial engineering tools can be used to model the the joint behaviour of the factors – and then create "complete scenarios"

Example: Regulatory Stress Testing

Macro scenario *→ Losses & revenues → Balance sheet → Capital adequacy*



Economic Outlook – Global Scenario

GLOBAL ECONOMIC OUTLOOK

Gaining Traction, Gaining Balance

GLOBAL Scenario 2015	US	EURO	UK	JPN
Real GDP (% change)	3	1.5	2.8	0.8
Inflation (% change)	-0.3	-0.3	0.1	0.4
IR Short	0.3	0.1	0.5	0.05
IR Long	2.3	0.3	2.0	-
FX		0.9	0.7	120.0

Source: Standard & Poor's Ratings Services economic research report dated April 22 2015. Indexes are unmanaged, statistical composites and it is not possible to invest directly in an index. These results are inherently limited because they do not represent the results of actual trading and were constructed with the benefit of hindsight. The returns shown do not reflect payment of any sales charges or fees an investor would pay to purchase the securities they represent. The imposition of these fees and charges would cause actual and back tested performance to be lower than the performance shown. Returns exclude dividends.

Economic Outlook – US Scenarios

U.S. Economic Forecast

The Emperor's New Groove

U.S. Scenario 2015	UPSIDE	BASE	DOWNSIDE
Real GDP (% change)	3.4	3	2.1
Inflation (% change)	-0.3	-0.3	-0.4
Unemployment (%)	5.4	5.4	5.8
IR Short	0.4	0.3	0
IR Long	2.4	2.3	2.1
S&P 500 (%)	10.0%	7.6%	8.5%
Oil ($/bbl, WTI)	50.12	50	48.56

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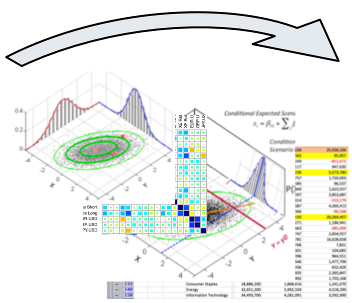
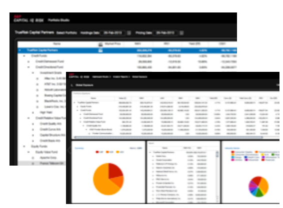
From Economic Research to Portfolio Scenario Analysis





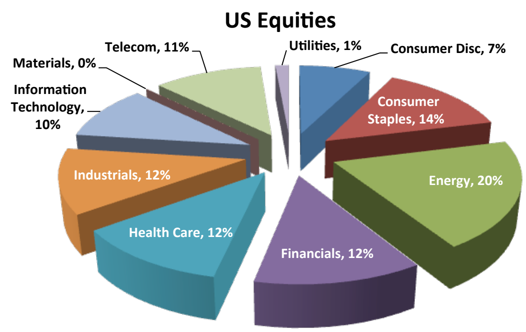
1. The Portfolio
2. Joint Factor Simulation Model – Economic and Risk (Market & Credit) Factors
3. Economic Scenarios – Model Mapping
4. Conditional Scenarios
5. Portfolio Simulation and Analysis

1. The Portfolio

$ USD (Million)	NMV	Long	Short
Portfolio	457.5	567.8	110.3
EQ	221.4	225.5	4.1
IR	111.2	111.2	0.0
CR	124.9	231.1	106.2

NMV	USD	EUR	GBP	JPY	Total
EQ	32%	11%	4%	5%	52%
IR	10%	14%			24%
CR	18%	7%			24%
Total	59%	32%	4%	5%	100%

Portfolio	**458**
EQ	*221*
USD-EQ	136
EUR-EQ	48
GBP-EQ	16
JPY-EQ	22
IR	*111*
USD-GOV	52
EUR-SOV	59
CR	*125*
USD-IG	70
USD-HY	60
EUR-HY	32
USD CDS	-38



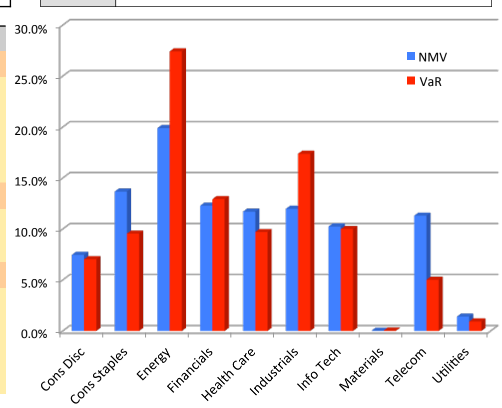
Source: S&P Capital IQ. For illustrative purposes only.

The Portfolio – VaR & Contributions (99% confidence, one month)

$ USD (Million)	NMV	VaR	VaR (Marginal)
Portfolio	457.5	22.3	22.3
EQ	221.4	17.1	14.4
IR	111.2	6.3	3.6
CR	124.9	6.5	4.3

	USD	EUR	GBP	JPY	Total
EQ	37%	21%	4%	2%	65%
IR	3%	13%			16%
CR	13%	7%			19%
Total	53%	40%	4%	2%	100%

Portfolio	***458***	***22.3***	***22.3***
EQ	*221*	*17.1*	*14.4*
US-EQ	136	11.5	8.3
EU-EQ	48	6.2	4.6
UK-EQ	16	1.5	1.0
JP-EQ	22	2.7	0.6
IR	*111*	*6.3*	*3.6*
US-GOV	52	2.4	0.7
EU-SOV	59	4.6	2.9
CR	*125*	*6.5*	*4.3*
US-IG	70	2.1	0.8
US-HY	60	3.4	2.0
EU-HY	32	2.1	1.5
US CDS	-38	1.9	0.0



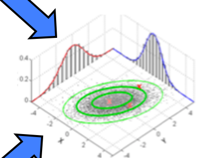
Source: S&P Capital IQ. For illustrative purposes only.

2. Joint Factor Simulation Model

Economic Research Scenarios

Economic Factors

GLOBAL & U.S. Scenario Factors	U.S.	EURO	UK	JPN
Real GDP	X	X	X	X
Inflation	X	X	X	X
Unemployment	X			
IR Short	X	X		
IR Long	X	X		
EQ (S&P500)	X			
Oil (WTI)	X			
FX		X	X	X



Portfolio

Portfolio	*458*	*22.3*
EQ	*221*	*17.1*
US-EQ	136	11.5
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CR	*125*	*6.5*
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US-HY	60	3.4
EU-HY	32	2.1
US CDS	-38	1.9

Market Factors

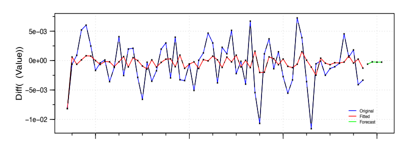
Equity Indices	IR Curves	Credit	FX
US Sectors	USD Gov	CDS: US IG, HY Bond Spreads: US IG, HY	
XE	EUR Sov	EUR Spread HY	EUR
GB			GBP
JP			JPY

Joint Factor Simulation Model

- Input: quarterly data for all the factors (20+ years)

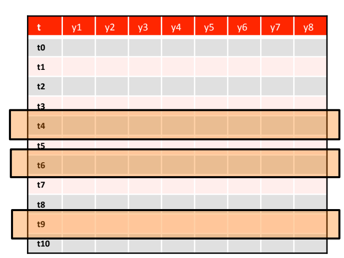
1. Marginal processes for each factor: ARMA GARCH model (filtering)

$$y_t = a_0 + a_1 y_{t-1} + b_1 \epsilon_{t-1} + \epsilon_t$$
$$\epsilon_t = \sigma \eta_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$



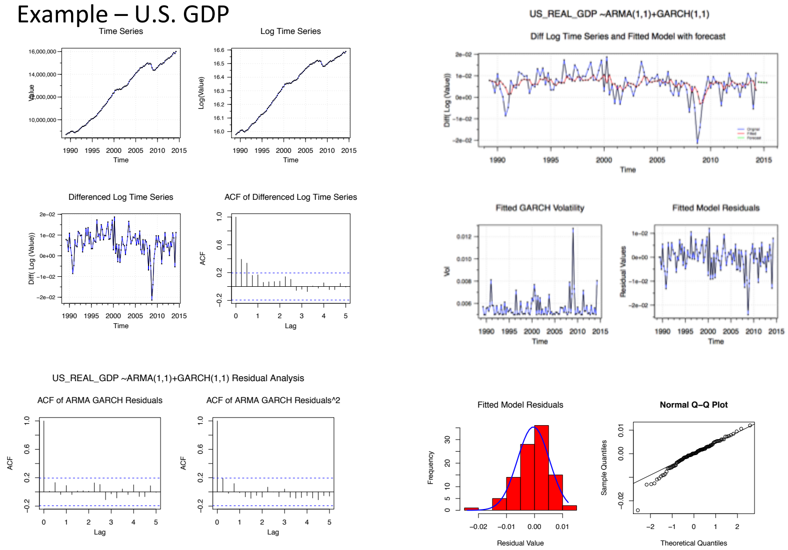
2. Historical codependence of residuals

(allows for non-Gaussian fat tails and tail dependence)



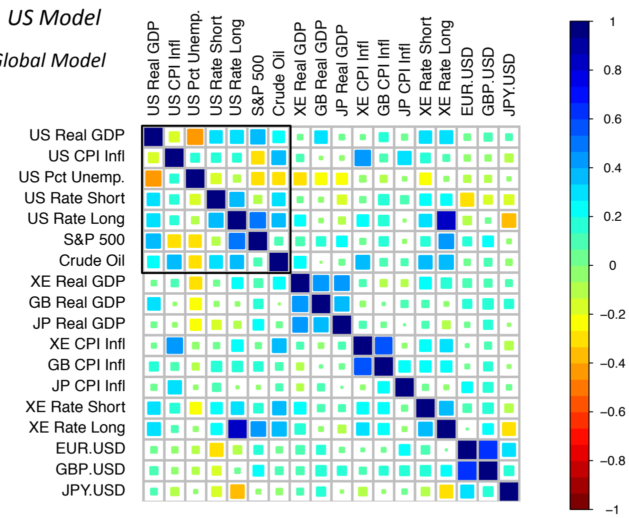
Joint Factor Simulation Model

Example – U.S. GDP



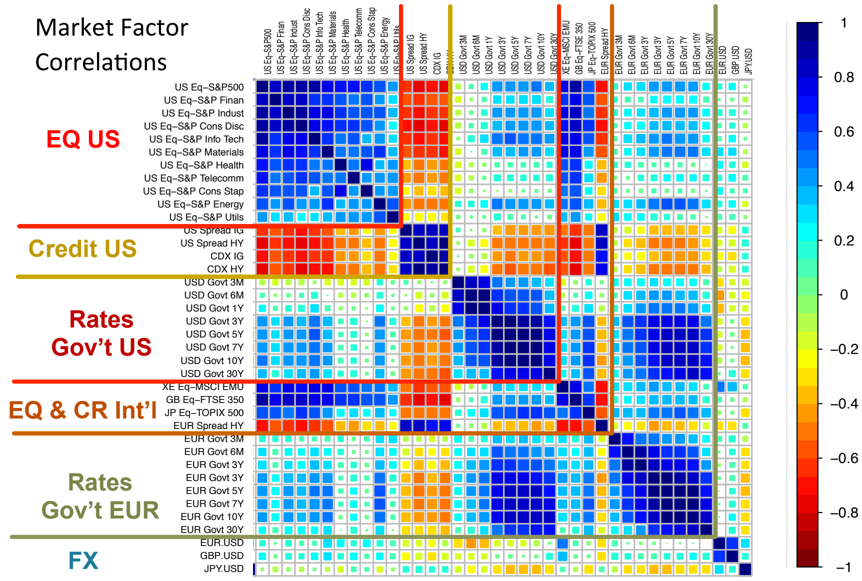
Joint Factor Simulation Model

- Economic Factor Residuals – Correlation Matrix
  - *US Model*
  - *Global Model*

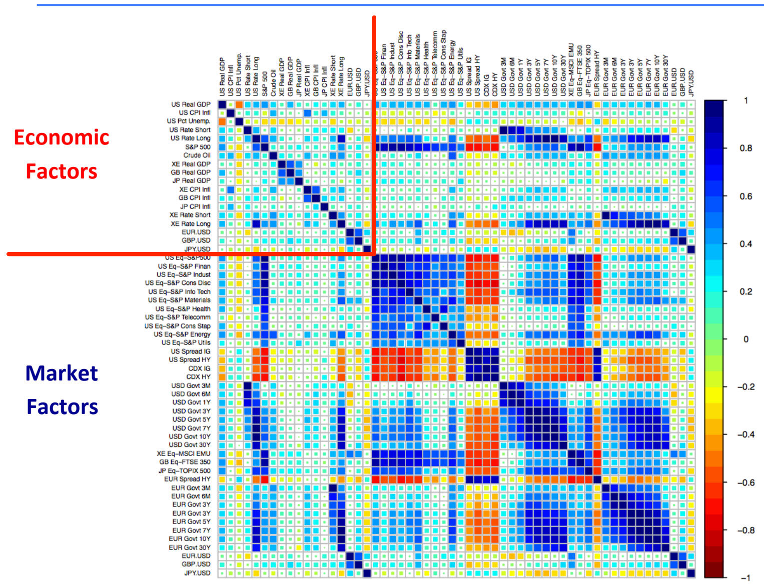


Joint Factor Simulation Model

Market Factor Correlations



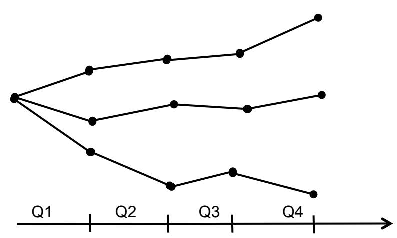
Joint Factor Simulation Model

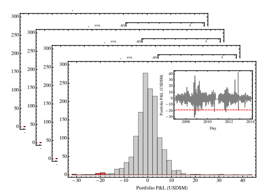


Joint Factor Simulation Model

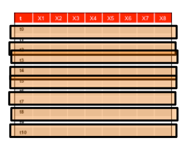
Empirical (simulated) factor terminal distribution (horizon = 1 year)

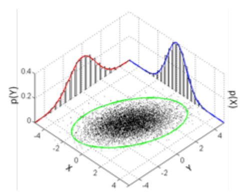
– Monte Carlo simulation of *N* scenarios of the joint factor processes over 4 quarterly steps (*N = 1K – 10K)*





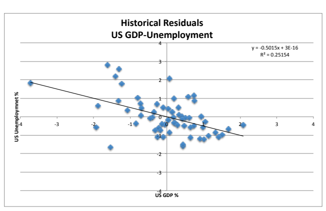
$$y_t = a_0 + a_1 y_{t-1} + b_1 \epsilon_{t-1} + \epsilon_t$$
$$\epsilon_t = \sigma \eta_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2,$$

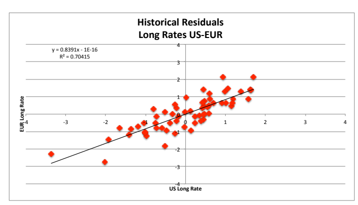


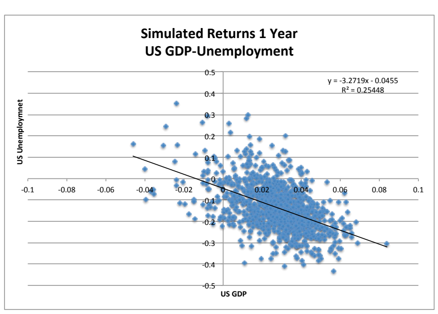


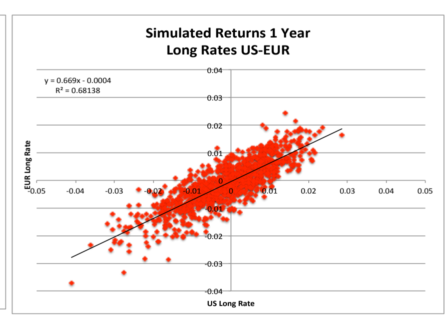
Joint Factor Simulation Model

Example – Empirical and simulated factor terminal distribution





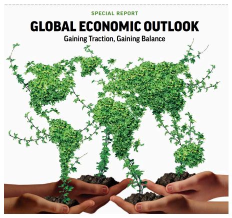


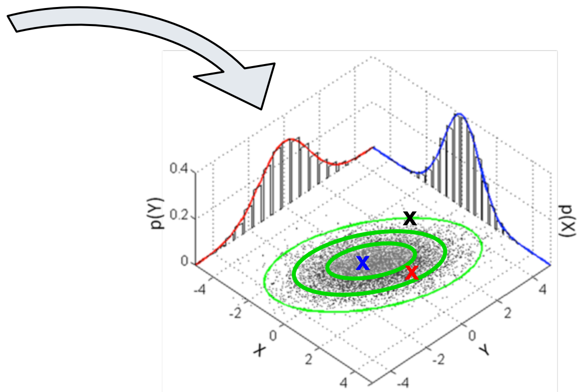


3. Economic Scenarios – Model Mapping

Objective: express forecasted scenarios in the context of joint factor simulated distribution

– Standardized economic scenarios (expressed as number of standard deviations of factors, and also in terms of likelihood within the model)





Economic Scenarios – Model Mapping

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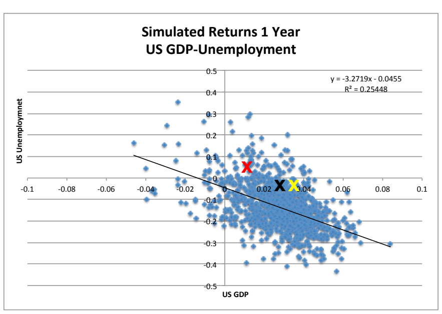

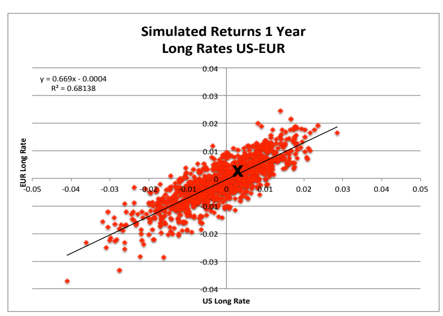

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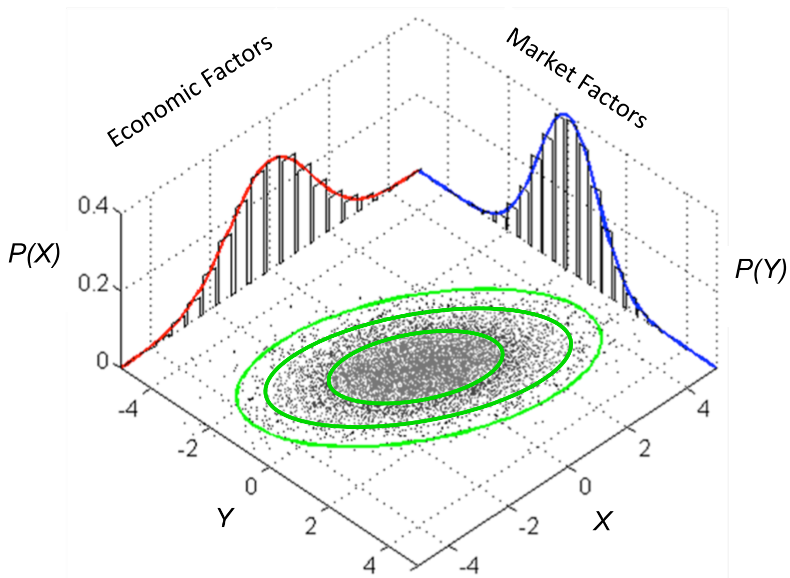
Standardized Mapped Scenarios (number of std. dev. from mean)

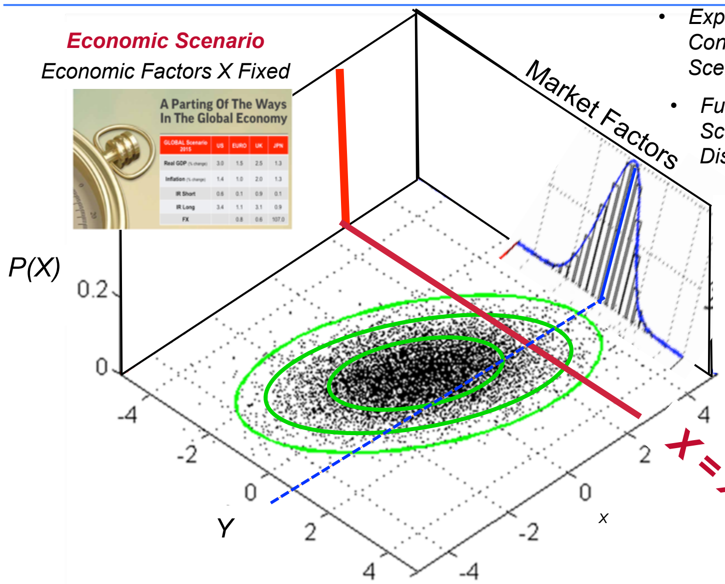
U.S. Scenario 2015	UPSIDE	BASE	DOWNSIDE
Real GDP (% change)	0.33	0.09	-0.44
Inflation (% change)	-2.29	-2.29	-2.39
Unemployment (%)	0.99	0.99	1.74
IR Short	0.44	0.28	-0.21
IR Long	0.51	0.41	0.22
S&P 500	0.45	0.24	0.31
Oil ($/bbl, WTI)	-0.37	-0.38	-0.49

GLOBAL Scenario 2015	U.S.	EURO	UK	JPN
Real GDP (% change)	0.1	0.1	-0.1	-0.2
Inflation (% change)	-2.3	-1.9	-1.5	1.2
IR Short	0.3	-0.1		
IR Long	0.4	0.5		
FX		0.1	-0.1	-0.2

4. Conditional Scenarios



4. Conditional Scenarios

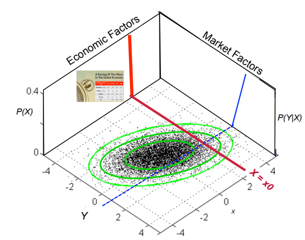


- *Expected Conditional Scenario of Y*
- *Full Conditional Scenario Distribution*

Conditional Scenarios – Definitions

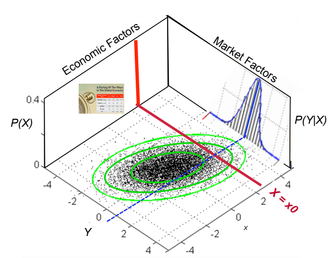
- Conditional Market Scenario
  - Expected conditional market scenario

$$y_0 = E[Y \mid X = x_0]$$



  - Full conditional market factor distribution

$$F_{Y|X}(y \mid x_0) = P[Y \le y \mid X = x_0]$$



Conditional Scenarios – Definitions

- Conditional Portfolio P&L
  - Portfolio P&L in the expected conditional market scenario

$$\Delta V(y_0) \qquad y_0 = E[Y \mid X = x_0]$$

  - Conditional expected Portfolio P&L scenario

$$\Delta V_0 = E[\Delta V \mid X = x_0]$$

  - Conditional distribution

$$F_{\Delta V|X}(v \mid x_0) = P[\Delta V \le v \mid X = x_0]$$

Conditional Scenarios: Analytical Methods

- Conditional factor distributions are available analytically for certain joint distributions

  e.g. Multi-variate Gaussian

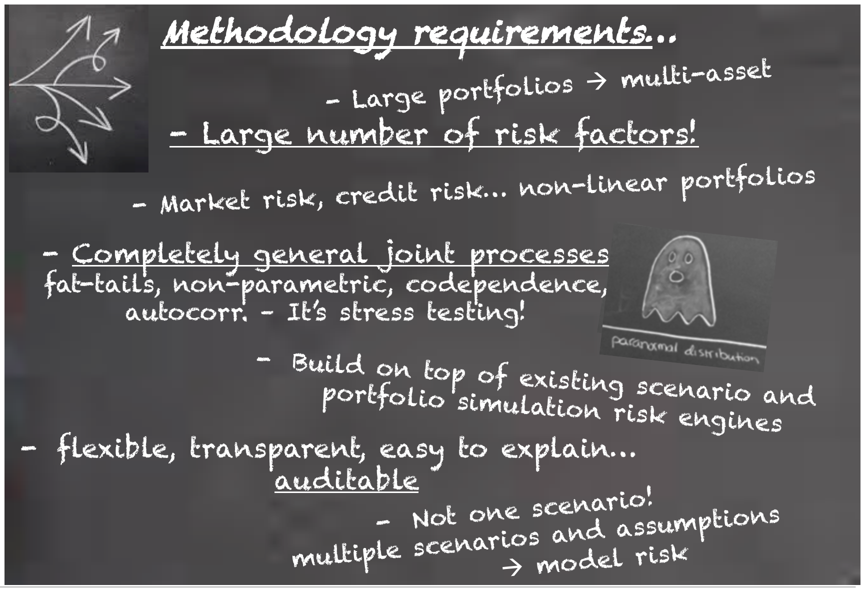
$$X = \begin{pmatrix} X^{(1)} \\ X^{(2)} \end{pmatrix} \sim N(\mu, \Sigma)$$

$$\mu = \begin{pmatrix} \mu^{(1)} \\ \mu^{(2)} \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

Conditional distribution of $X^{(2)}$ given $X^{(1)} = x^{(1)}$ is *multivariate normal* with mean *m* and covariance matrix *B*

$$m = \mu^{(2)} + \Sigma_{21}\Sigma_{11}^{-1}(x^{(1)} - \mu^{(1)})$$
$$B = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$$

Stress Testing and Conditional Scenarios

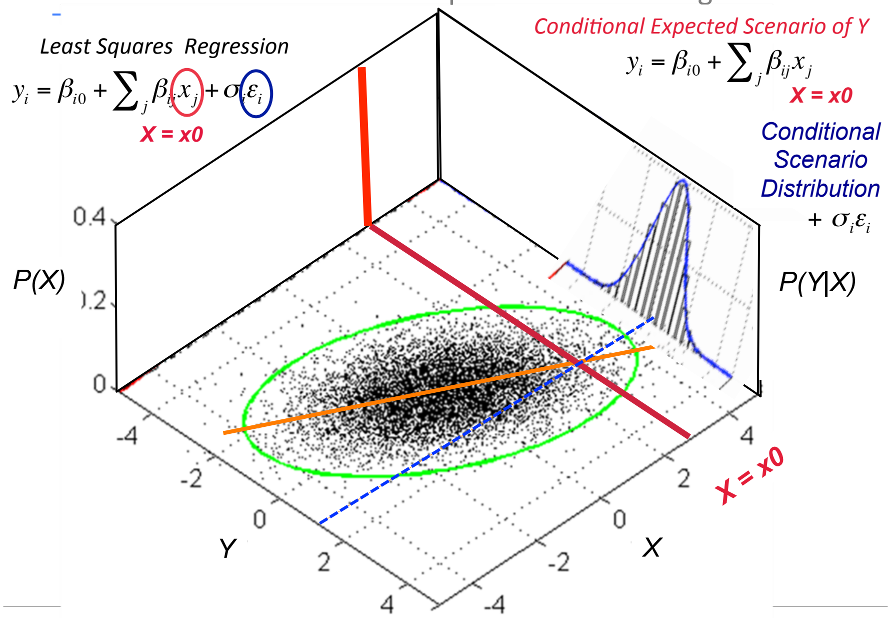


Conditional Scenarios: Least Squares Stress Testing (*LSST*)

- Key insight: conditional expectation of all the factors (and more generally the full conditional distribution) can be estimated directly from a pre-computed simulation using Least Squares Regression (LSR)
- Conditional scenario analytics, including risk factor contributions, can be derived from the regression results

- The application of LSR on the cross-sectional information of a simulation to obtain conditional expectations is the key component of LSM to price American options (Longstaff and Schwartz 2001)
  - Applied here to portfolio risk management and stress testing

Conditional Scenarios: Least Squares Stress Testing



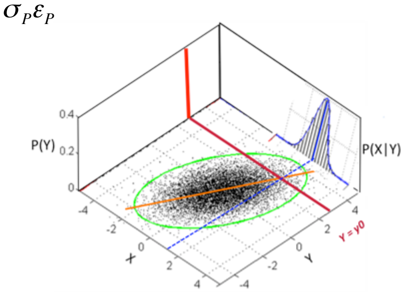
Conditional Scenarios: Least Squares Stress Testing

– ***LSR of Market Factors*** over a simulation of *N factor* scenarios

$$y_i = \beta_{i0} + \sum_j \beta_{ij} x_j + \sigma_i \varepsilon_i$$

– ***LSR of Portfolio P&L*** over a simulation of *N* factor scenarios

$$\Delta V = \beta_{P0} + \sum_j \beta_{Pj} x_j + \sigma_P \varepsilon_P$$



Least Squares Stress Testing on Portfolio P&L

Several advantages of regressing Portfolio P&L directly on economic factors

- Single regression – rather than multiple regressions for all the market factors
- Compute multiple conditional economic scenarios and portfolio analytics with a single portfolio simulation – typically the most expensive computational step
- Risk factor contributions: portfolio P&L regression also has meaning in terms of (economic) *factor contributions* to a given scenario P&L

$$\Delta V(x) = \sum_{k=1}^{N} C_k = \sum_{k=1}^{N} \hat{b}_k x_k$$

as wells as to risk measures such as VaR or Expected Shortfall

$$C_k^{\mathrm{VaR}} = \hat{b}_k x_{k,j^*} \quad , \qquad C_k^{\mathrm{ES}} = \frac{1}{m'} \hat{b}_k \sum_{L_j \geq L_{j^*}} x_{k,j}$$

where *j** is the VaR scenario, and *m'* is the number of scenarios in which losses exceed VaR

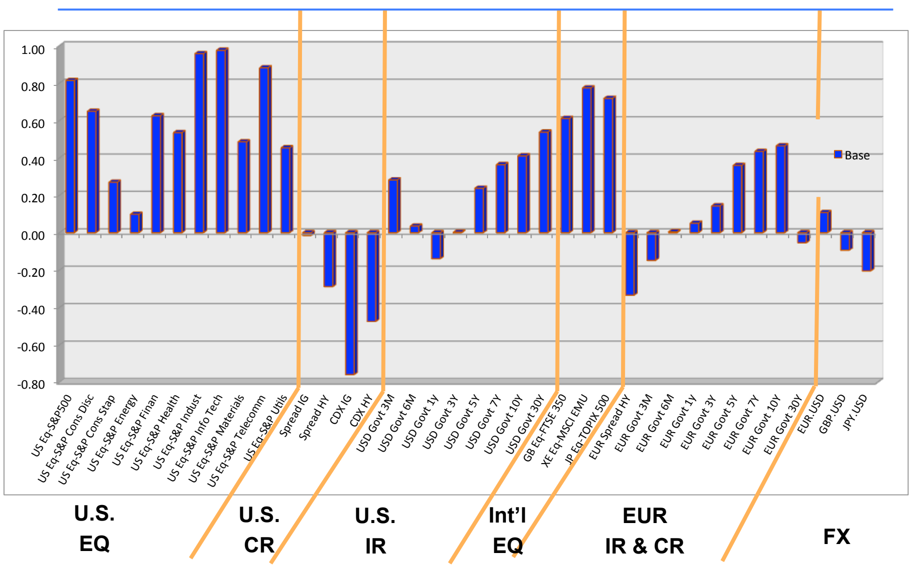
Conditional Scenarios: Least Squares Stress Testing

- LSST offers many advantages
  - Easily applied to large portfolios and large number of risk factors (market and credit risk)
  - General joint processes (fat-tails, non-parametric, codependence, autocorr.)
  - Simulation: simple, flexible, transparent results, auditable and easy to explain
  - Easy to build on top of any existing scenario and portfolio simulation risk engine
  - Explicit *Risk Factor P&L contributions*
  - Computational efficiency of post-simulation analytics: multiple scenarios and assumptions in real-time, providing multiple-portfolio views and an explicit assessment of model risk
- General conditional scenario framework
  - Examples here are for conditional scenarios where factor are given fixed values ("point scenarios" or views), but easy to extend to more general views, e.g. bounds on the variables, parameters of the factor distributions, etc.

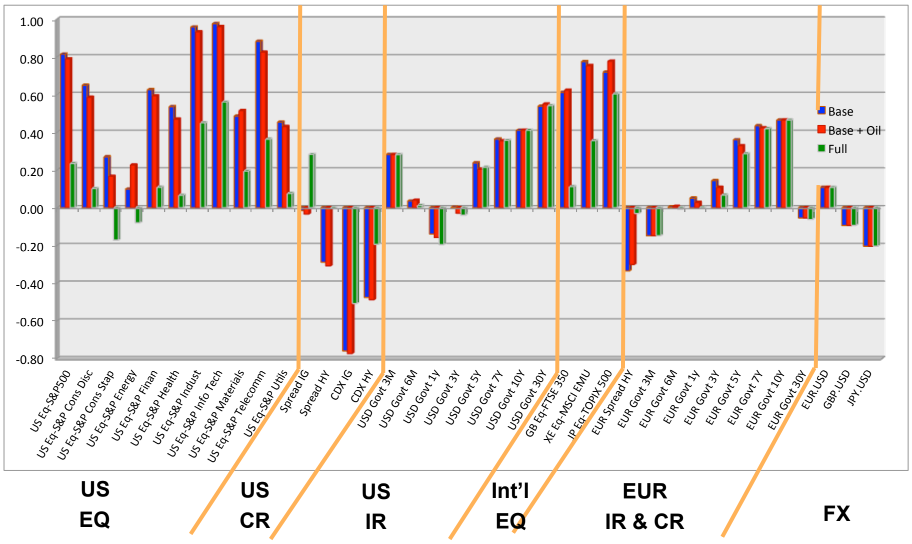
A Note on Regression and Heteroskedasticity

- Assumption of LSR is that *the variance of the errors around the regression surface is everywhere the same*, i.e. $V(U)=V(Y \mid X) = \sigma^2$ *(Homoskedastiticy)*
  - *Heteroskedasticity does* not cause biased estimates, but it does pose problems for efficiency – usual formulae for standard errors of the estimates are less accurate
  - Standard econometric tests for heteroskedasticity: White test, Breusch-Pagan test
- Heteroskedasticity is in general not a big problem for *conditional expected scenarios*
  - But important when we are concerned with full conditional scenario distributions
- Several ways to correct for heteroskedasticity in LSST
  - Transform both the dependent and independent variables with non-linear functions (or add polynomial terms or *basis functions*)
  - Techniques such as *Weighted LSR*
  - Adjust the variance of the error distribution conditional on the specific economic scenario
  - *Quantile Regression* Techniques: can be applied when one is interested in specific quantiles (e.g. VaR) or obtaining the full conditional scenario distribution
    - Useful specifically for the one-dimensional case of the Portfolio P&L

Global Outlook – Conditional Scenarios



Global Outlook – Conditional Scenarios



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US Outlook – Conditional Scenarios

Conditional Expected Scenarios on Market Factors

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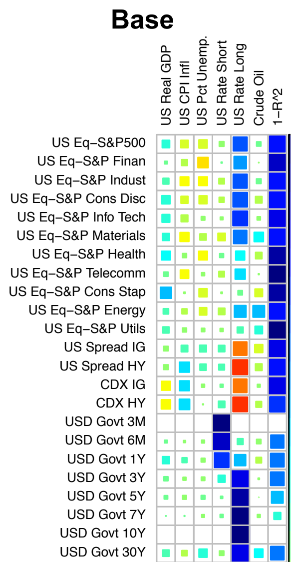
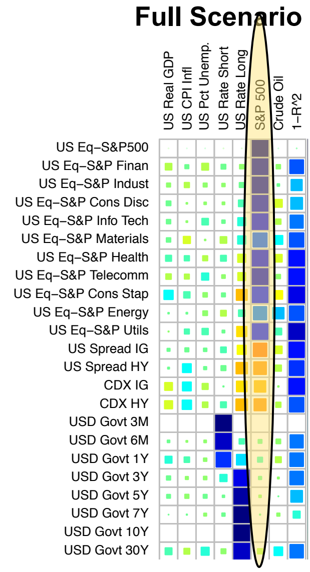
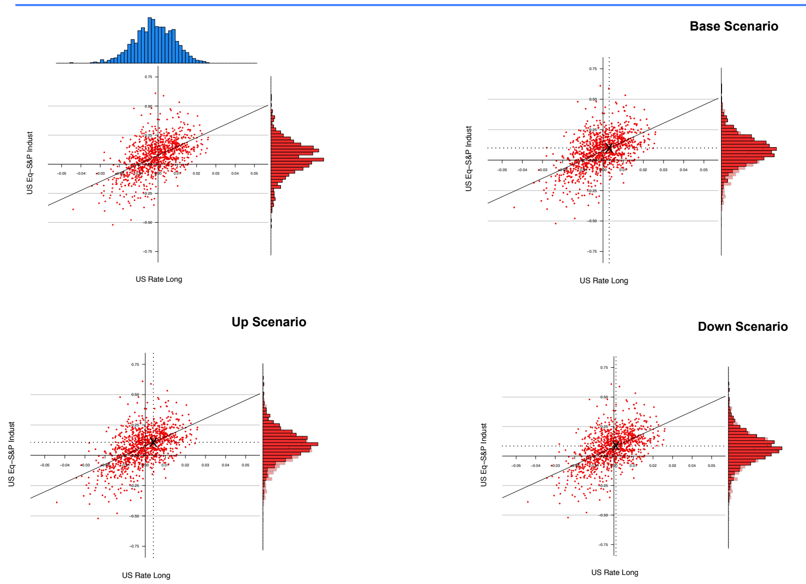
US Outlook – Factor Regressions





Conditional Scenarios – Example



5. Portfolio Simulation And Analysis – Global Scenario

	NMV	VaR (Annual)	P&L Mean	Global Scenario	Rel. Return
Portfolio	457.5	16.0%	3.7%	8.1%	4.4%
EQ	221.4	29.0%	6.7%	17.1%	10.4%
IR	111.2	14.4%	1.6%	-0.1%	-1.8%
CR	124.9	16.8%	0.4%	-0.5%	-0.8%

$US Million		NMV	P&L Mean	Global Scenario
Portfolio		**458**	**15.9**	**37.0**
EQ		*221*	*14.5*	*37.8*
	US EQ	136	9.7	22.1
	EU EQ	48	2.6	9.8
	GB EQ	16	0.8	1.8
	JP EQ	22	1.3	4.1
IR		*111*	*1.5*	*-0.1*
	US GOV	52	0.3	-0.7
	EU SOV	59	1.3	0.6
CR		*125*	*-0.1*	*-0.6*
	US IG	70	-0.1	-1.0
	US HY	60	-0.6	0.2
	EU HY	32	0.4	1.1
	US CDS	-38	0.2	-1.0

GLOBAL ECONOMIC OUTLOOK

Gaining Traction, Gaining Balance

GLOBAL Scenario 2015	US	EURO	UK	JPN
Real GDP (% change)	3	1.5	2.8	0.8
Inflation (% change)	-0.3	-0.3	0.1	0.4
IR Short	0.	0.1	0.5	0.1
IR Long	2.3	0.3	2.0	0.3
FX		0.9	0.7	120.0

Portfolio Simulation And Analysis – Global Scenario

	NMV	VaR (Annual)	P&L Mean	Global Scenario	Rel. Return
Portfolio	457.5	16.0%	3.7%	8.1%	4.4%
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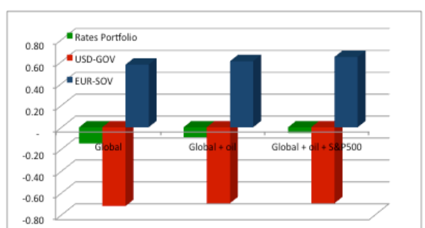
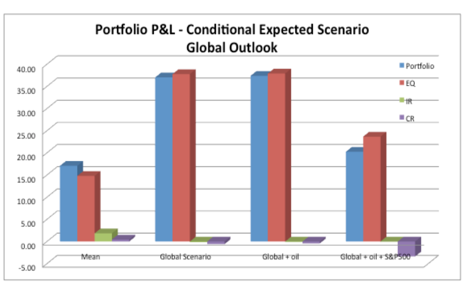
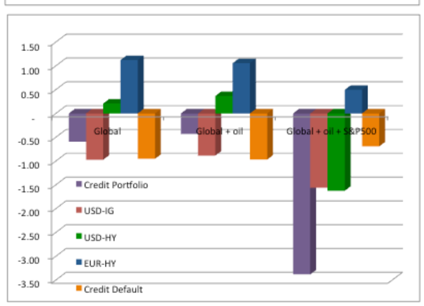
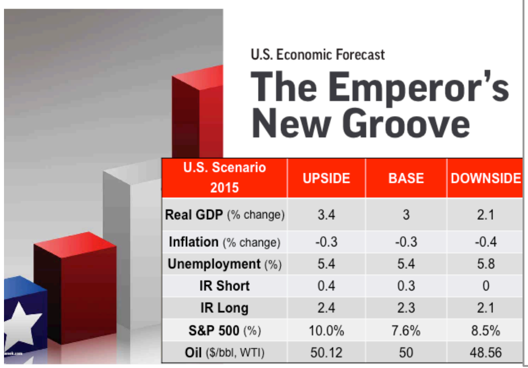




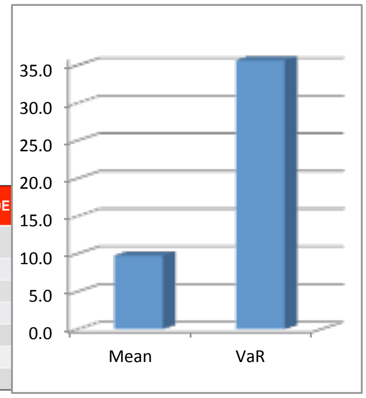


Portfolio Simulation and Analysis – US Outlook Scenarios

US Equity Portfolio
(NMV = US $136 M)



U.S. Scenario 2015	UPSIDE	BASE	DOWNSIDE
Real GDP (% change)	3.4	3	2.1
Inflation (% change)	-0.3	-0.3	-0.4
Unemployment (%)	5.4	5.4	5.8
IR Short	0.4	0.3	0
IR Long	2.4	2.3	2.1
S&P 500 (%)	10.0%	7.6%	8.5%
Oil ($/bbl, WTI)	50.12	50	48.56



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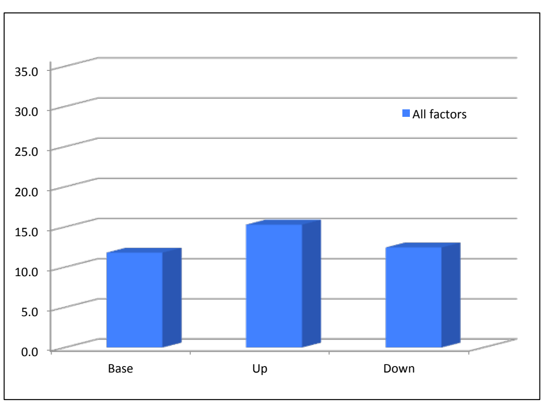
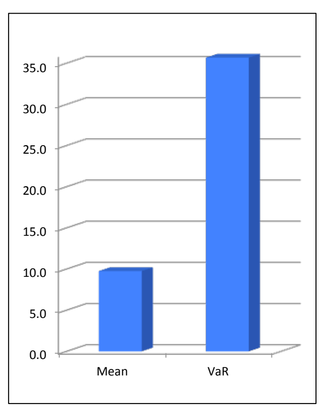
Portfolio Simulation and Analysis – US Outlook Scenarios

US Equity Portfolio – Expected Scenarios
(NMV = US $136 M)





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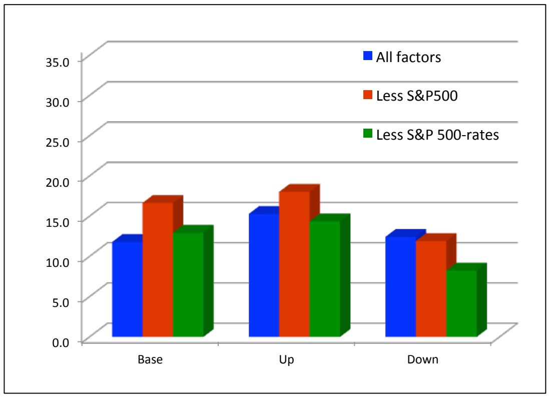
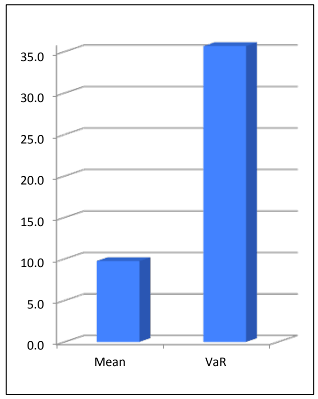
Portfolio Simulation and Analysis – US Outlook Scenarios

US Equity Portfolio – Impact of Variables entering Expected Scenarios





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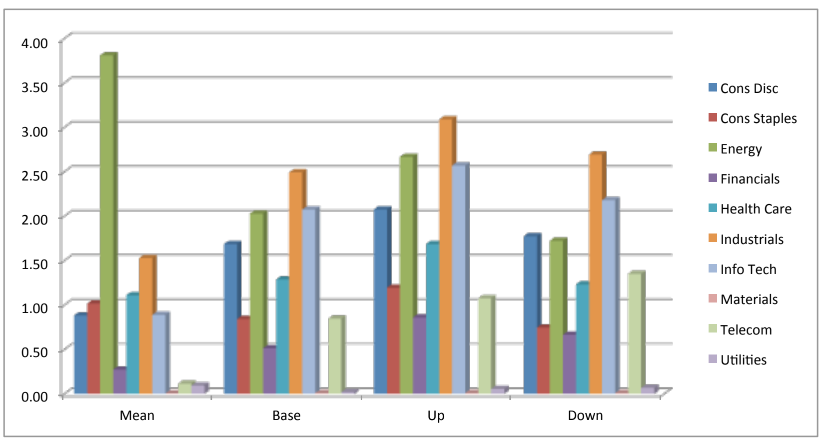
Portfolio Simulation and Analysis – US Outlook Scenarios

US Equity Portfolio – Sector Breakdowns in Expected Scenarios



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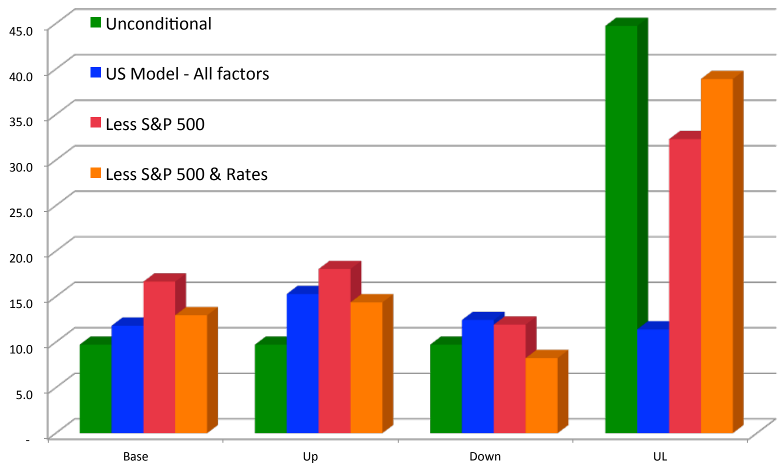
Portfolio Simulation and Analysis – US Outlook Scenarios

US Equity Portfolio – Expected Scenarios and Conditional Tail Losses

UL (unexpected loss) = VaR 99% + Expected P&L

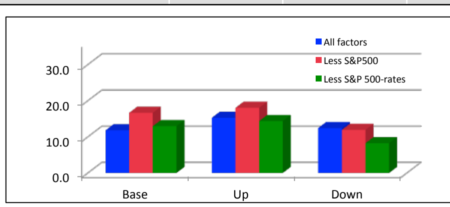


Portfolio Simulation and Analysis – Adverse Scenarios

CCAR Regulatory Scenarios 2015 *

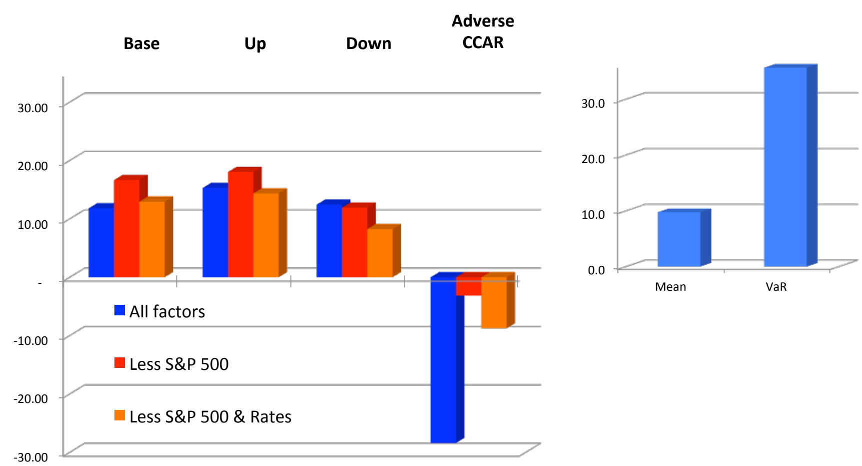
U.S. Scenario 2015	UPSIDE	BASE	DOWNSIDE	Adverse	Severely Adverse
Real GDP (% change)	3.4	3	2.1	0.3	– 1.5
Inflation (% change)	-0.3	-0.3	-0.4	4.0	1.1
Unemployment (%)	5.4	5.4	5.8	7.3	9.9
IR Short	0.4	0.3	0	2.6	0.1
IR Long	2.4	2.3	2.1	4.6	1.5
S&P 500 (% change)	10.0	7.6	8.5	– 20.5	– 57.9
Oil ($/bbl, WTI)	50.12	50	48.56	90.00	110.00

* Approximate



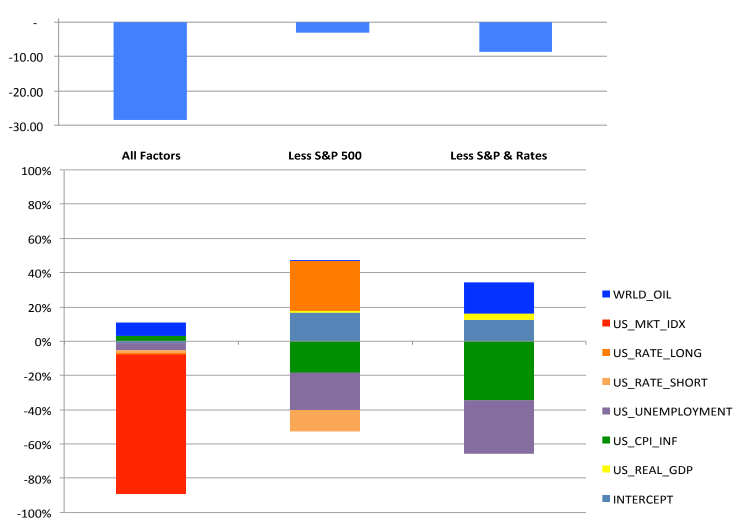
Portfolio Simulation and Analysis – Adverse Scenarios

US Equity Portfolio – Comparing against CCAR Adverse scenario (P&L in $ Million)

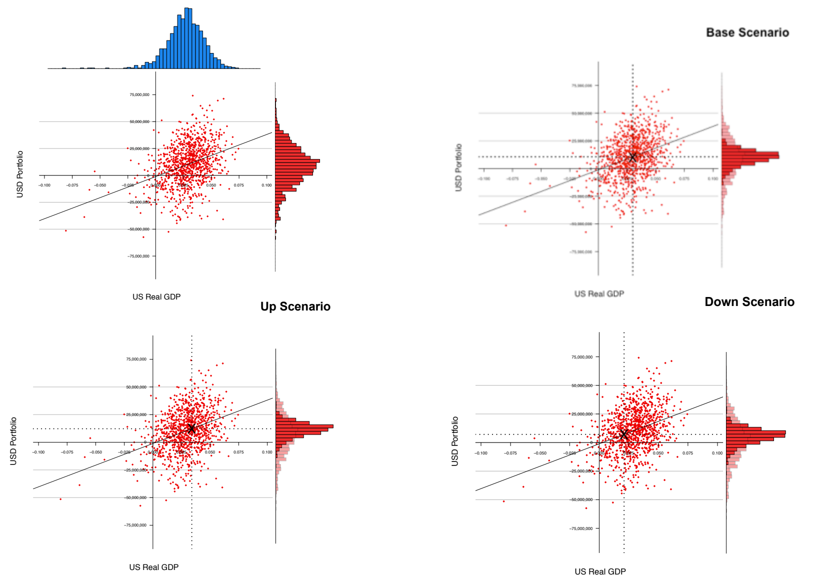


Portfolio Simulation and Analysis – Adverse Scenarios

US Equity Portfolio – Risk Factor Contributions – CCAR Adverse scenario



Portfolio Simulation And Analysis – Example



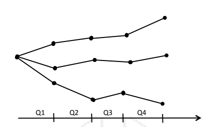
Integrated Market-Credit Simulation

Economic Scenarios

Economic Factors

GLOBAL & U.S. Scenario Factors	U.S.	EURO	UK	JPN
Real GDP	X	X	X	X
Inflation	X	X	X	X
IR Short	X	X		
IR Long	X	X		
EQ (S&P500)	X			
Oil (WTI)	X			
FX		X	X	X

Joint factor processes {**X, Y, Z**}
Required for aggregated market-credit losses and economic scenario stress testing



Portfolio

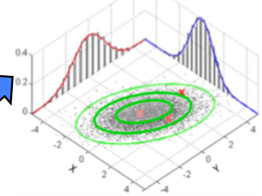
Portfolio	458	22.3
EQ	221	17.1
US-EQ	136	11.5
EU-EQ	48	6.2
UK-EQ	16	1.5
JP-EQ	22	2.7
IR	111	6.3
US-GOV	52	2.4
EU-SOV	59	4.6
CR	125	6.5
US-IG	70	2.1
US-HY	60	3.4
EU-HY	32	2.1
US CDS	-38	1.9

Market Factors

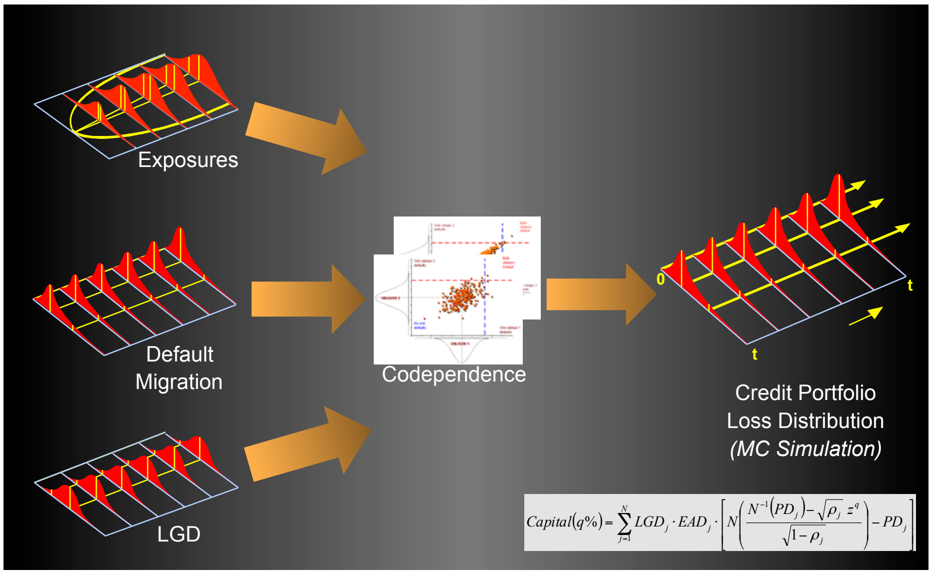
Equity Indices	IR Curves	Credit	FX
US Sectors	USD Gov	CDS: US IG, HY Bond Spreads: US IG, HY	
XE	EUR Sov	EUR Spread HY	EUR
GB			GBP
JP			JPY

Credit Factors

T	Z_1	Z_2	Z_k
1	z11	z21	zk1
2	z12	z22	zk2
3	z13	z23	zk3
.	.	.	.
.	.	.	.
.	.	.	.
n	z1n	z2n	zkn



Credit Portfolio Model Components



Applying Conditional Scenarios and LSST to Credit Portfolios

Key issues:

- Multi-step joint economic and risk factor model
  - Multi-step dynamic credit portfolio model
- Non-linear regression transformation for credit losses
- LSST and multi-step economic scenarios (defined “path” scenarios)
- Modelling systematic vs. idiosyncratic losses
- Computational efficiency
  - Exploiting the credit model’s conditional independence structure

General Credit Portfolio Model

Conditional PD $PD(Z = z)$

$$PD(z) = \mathrm{G}(a + b\,z) = G\left(\frac{H^{-1}(PD) - \sqrt{\rho}\,z}{\sqrt{1-\rho}}\right)$$

$$a = \frac{H^{-1}(PD)}{\sqrt{1-\rho}}\ , \qquad b = \sqrt{\frac{\rho}{1-\rho}}$$

– *G* = distribution of the idiosyncratic component; *H* = distribution of CWI

Models – examples

– Standard Probit (Gaussian copula/Vasicek/Merton type)

$$F(x) = G(x) = H(x) = N(x)$$

– Standard Logit

$$F(x) = N(x)\ , \quad G(x) = \frac{e^x}{1+e^x}\ , \quad H(x) \text{ implicit from F and G}$$

Multi-step Credit Portfolio Model – Extensions

Model 1: Static (Multi-Step) Copula

– Assume calibrated model for 1 period (e.g quarterly)

– Defined process for Z (which can be simulated say over 4 steps)

$$z_t = z_{t-1} + \Delta z_t, \quad t = 1, \ldots, n$$

– Define the cumulative PD to time *t* $PD_t, \quad t = 1, \ldots, n$

– Then, in the spirit of the "static" copula model:

$$PD_n(z) = \mathrm{G}(a_n + b\,z^*) = G\left(\frac{H^{-1}(PD_n) - \sqrt{\rho}\,z^*}{\sqrt{1-\rho}}\right)$$

with the *canonical* (or standardized) factor $z^*_n = z_n / \sigma_n$

– $PD_n(z)$ is the conditional probability given the entire path of the *z* process

– Essentially, the model is calibrated to the quarterly data and then the constant parameter adjusted for the yearly probability

Multi-step Credit Portfolio Model – Extensions

Model 2: Dynamic Conditional Probability Model

- Assume calibrated model for one period (e.g quarterly)
- Defined process for Z (which can be simulated say over 4 steps)

$$z_t = z_{t-1} + \Delta z_t, \quad t = 1, \ldots, n$$

- Assume that, at each step: the *PD* conditional on
  - Survival of the previous step, and
  - the path of the systematic factor, *z*

  is given by the single step model.

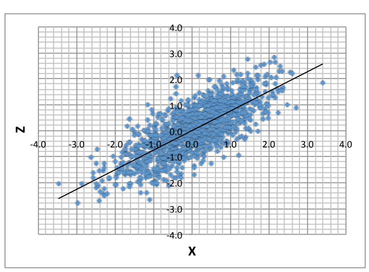
- Then, for example, for two periods, the cumulative PD is

$$PD_2(z) = PD_1(z) + (1 - PD_1(z))\ G(a + b\ z^*_2)$$

where, as before, *z** is the canonical (or standardized) factor (and assume *z* i.i.d. increments, otherwise some complications)

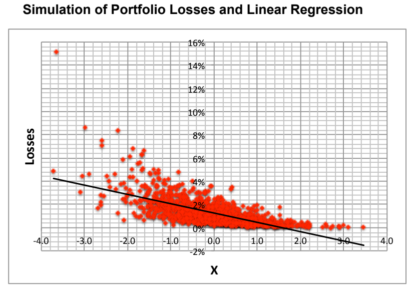
Non-Linear LSST for Credit Risk – Example

Correlation of Credit Factor to Market Factor = 77%

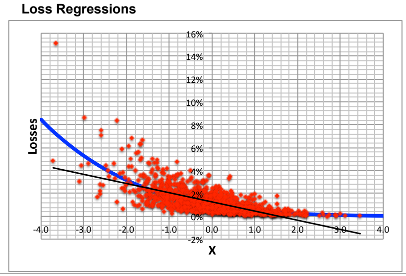


$$PD(z) = \mathrm{G}(a\ +\ b\ z)$$
$$= G\left(\frac{H^{-1}(PD) - \sqrt{\rho}\ z}{\sqrt{1-\rho}}\right)$$

Simulation of Portfolio Losses and Linear Regression



Loss Regressions



Scenario Methodologies – Some Extensions

- Hierarchical LSR
  - Orthogonal incremental contributions with prioritized factors
- Non-linear regression and generalized linear models
- Quantile regression methodologies
- Maximum Likelihood Methods
  - Provide bounds and distributional estimates
  - Conditional estimates in the tails
- Parametric or semi-parametric distribution approximations in the tails
- Kernel and entropy methods
  - Bandwidth optimization
  - Tail extrapolations

Concluding Remarks

Bio – Dan Rosen

Dr. Dan Rosen is a *FinTech* Entrepreneur and Quant.

He is currently the Chief Executive Officer of ***d1g1t Inc.,*** a new digital wealth management platform, powered by analytics, that offers advanced transparent portfolio management services to advisors and their individual investors. He is also the *Director* of the *Centre for Financial Industries* at the ***Fields Institute*** *for Research in Mathematical Sciences*, as well as an Adjunct Professor of Mathematical Finance at the ***University of Toronto***.

Dr. Rosen was the co-founder and *CEO* ***R^2 Financial Technologies,*** acquired by S&P Capital IQ in 2012, and where he was Managing Director for Risk and Analytics until 2015. Prior to starting R^2 in 2006, Dr. Rosen had a successful career over a decade at Algorithmics Inc., where he lead financial engineering and research, strategy, products and marketing.

In addition to working with numerous financial institutions around the world, he lectures extensively on financial engineering, portfolio management, enterprise risk and capital management, credit and market risk, valuation of derivatives and structured finance. He has authored numerous risk management and financial engineering publications, including two books, and several patents, and serves in the editorial board of various industrial and academic journals.

Dr. Rosen was inducted in 2010 a *Fellow of the Fields Institute* for his "outstanding contributions to the Fields Institute, its programs, and to the Canadian mathematical community". He currently serves in the Board of Directors of the Fields Institute, as well as in the Advisory Boards of the OSC on Fintech, Canada's Institute Innovation Platform (IIP), International Association of Quantitative Finance (IAQF), Global Risk Institute (GRI), Center for Advanced Financial Studies at the University of Waterloo, and the Institute for Leadership Education in Engineering (iLead) at the University of Toronto. He is one of the founders of the Professional Risk Management International Association (PRMIA) and of RiskLab, initiated at the University of Toronto.

He holds an M.A.Sc. and Ph.D. in Chemical Engineering from the University of Toronto, and was a Post-Doctoral fellow at the Centre for Management of Technology and Entrepreneurship. His B.A.Sc. is in Chemical Engineering from Universidad Autonoma Metropolitana, in Mexico City, where he was awarded in 2015 the recognition of *Distinguished Alumni*.

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