Regress Under Stress

A Simple Least-Squares Method for Integrating Economic Scenarios with Risk Simulations

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1 Introduction

Scenarios are the language of *Risk*. The quality of a risk management analysis depends on our ability to generate relevant forward-looking scenarios that properly represent the future, and to understand clearly how these scenarios apply to our current situation, impact our specific portfolios and provide guidance on how to manage their risk effectively or take advantage of investment opportunities. While scenario analysis and stress testing have been an explicit part of risk management methodologies and systems for over two decades, the typical scenario and stress testing tools haven't evolved much and are still generally quite static and largely subjective. The future cannot be predicted, but the combination of expert economic analysis with advanced scenario and portfolio analytics can provide a strong basis for managing risk and making better, more informed investment decisions.

In this paper, we present a simple and powerful approach to create meaningful stress scenarios for risk management and investment analysis of multi-asset portfolios, which effectively combine economic forecasts and "expert" views with portfolio simulation methods. The intuition of the approach is straightforward. Expert scenarios are typically described in terms of a small number of key economic variables or market risk factors. However, when applied to a portfolio, they are incomplete – they generally do not describe what happens to all relevant *market risk factors* that affect the portfolio directly (or indirectly). We need to understand how these *market risk factors* behave, *conditional* on the outcome of the economic factors. For example, in a scenario with a large GDP growth, equity prices may rise to various degrees and CDS spreads contract. When the joint distribution of the economic and market factors is simple (e.g. Gaussian or Student t), conditional scenarios can be obtained analytically (see, e.g., Mood and Graybill (1963) for the Gaussian distribution, and Kotz and Nadarajah (2004) for the Student t distribution). The key insight to our approach is that the *conditional expectation*, or more generally the *full conditional distribution* of all the factors, and of the portfolio P&L, can be estimated directly from a pre-computed simulation using *Least Squares Regression (LSR)*. Specifically, we regress the market risk factors in the simulation, or directly the portfolio P&L values, on the economic factors (or some function of these), which define the economic scenario. All the conditional scenario analytics can be derived from the regression results.

The application of LSR on the cross-sectional information of a simulation to obtain conditional expectations is the key component of the popular Least-Squares Monte Carlo approach of Longstaff and Schwartz (2001), widely used to price American options. Here we show how similar ideas can also be effectively applied to portfolio risk management and stress testing. We refer to this approach as *Least Squares Stress Testing* (LSST).

LSST is a simulation-based method that offers many advantages over more traditional analytical methods, including:

- It can be easily applied to large portfolios and a large number of risk factors, including market and credit risks.
- Simulation allows the risk factors to follow completely general joint stochastic processes, with fat-tails, non-parametric and general codependence structures, autocorrelation, etc.
- Simulation techniques are simple, flexible, and provide very transparent results, which are auditable and easy to explain.
- LSST conditional scenarios analytics are easy to build on top of any existing scenario and portfolio simulation risk engine.
- LSST further decomposes the portfolio's performance in the simulation to produce explicit *risk factor P&L contributions*.
- The computational efficiency of post-simulation analytics allows users to run multiple scenarios and assumptions in real-time, thus providing multiple-portfolio views and an explicit assessment of model risk.

We focus this paper on conditional scenarios where several factor are given fixed values, i.e. *point scenarios*. Other recent work defines more general views for example on distribution parameters (e.g. moments or quantiles). See, for example, Meucci (2012, 2013), Ardia and Meucci (2015). We note that it is easy to extend the LSST methodology to more general views, including bounds on the factors, views on parameters of the factor distributions, etc.

The rest of the paper is organized as follows. The next section introduces briefly the problem and describes the basic setting for analyzing portfolios under conditional scenarios. Section 3 presents the LSST methodology. To illustrate the approach, Section 4 presents a real-life stress testing example where we analyze the performance of a typical multi-asset portfolio, dependent on a large number of risk factors, under recent economic research report and regulatory scenarios. Conclusions and further extensions are given in the last section.

2 Portfolio Analysis under Conditional Scenarios

The general problem can be formulated as follows. Scenarios typically coming from a research report or an analyst's views are naturally described in terms of a small number of key economic variables or risk factors. When applied to a given portfolio, they do not describe what happens to all relevant risk factors that affect the portfolio either directly or indirectly. In order to apply these economic scenarios to our portfolio, we have to find how all the relevant market factors behave as a given economic scenario unfolds. Implicitly or explicitly this relationship between all the factors must be assigned, and resolved when the scenario is applied to the portfolio. A simple approach commonly used by practitioners is to elicit subjective views or expert opinions on how some economic levels affect all the market factors in the portfolio. Alternatively, we focus on first applying statistical tools to model the joint behaviour of the factors, and use these models to create the "complete scenarios".

The general setting is as follows.

1. Portfolio and Market Risk Factors. Consider a multi-asset portfolio *P*, with positions for example in equities, bonds, CDSs and derivatives in multiple currencies. We are interested in the P&L distribution of the portfolio over a single horizon, *T,* say one month or one quarter. The discussion also applies to multiple horizons.

The portfolio P&L at the horizon is a function of a vector of *market risk factors, Y,*

$$\Delta V := \Delta V(Y) \tag{1}$$

Market risk factors include equity prices and indices, interest rate zero curves (Government & Swap curves), cash credit curves, CDS curves, FX rates, commodity prices etc. The number of risk factors can be large for a typical portfolio.

2. Market Scenarios and Portfolio Risk Metrics. Denote a *Market Scenario* by $Y = y$. This is a specific realization of the risk factors at *T.* Under the market scenario, the Portfolio P&L is $\Delta V(y)$.

Also, for risk analysis, we define a joint distribution of the risk factors at the horizon, $F_Y(y)$. This distribution is used with the P&L function (1) to obtain the portfolio's P&L distribution via either simulation or analytical methods, if tractable. Statistical risk measures for the portfolio, such as VaR and Expected Shortfall, are derived from this P&L distribution.

3. Economic Scenarios and Economic Risk Factors. Consider a vector of economic factors, *X*, and denote by $X = x_0$ an *Economic Scenario*, which may come from an economic report or an analyst's projections. Economic factors typically include macro-economic variables, such as GDP and unemployment, or other financial and market factors such as interest rates and market indices. We focus on scenarios as "point-wise views" of the form $X = x_0$, but the approach can be extended to more general views.

4. Joint Factor Evolution Model, and Joint Scenarios. A model of the joint evolution of the market and economic factors produces a joint distribution at the horizon *T*, $F_{X,Y}(x,y)$. The model and the resulting distribution *F* can be completely general: parametric or non-parametric, with fat-tailed marginals and general codependence structure).

5. Conditional Scenarios. Our objective is to understand the performance of the portfolio under the economic scenario, $X = x_0$. Thus we need to understand how the market risk factors behave, *conditional* on the defined outcome of the economic factors. The *expected conditional market scenario* is defined as

$$y_0 = E[Y \mid X = x_0] \tag{2}$$

More generally, we can obtain the full *conditional market factor distribution*

$$F_{Y|X}(y \mid x_0) = P[Y \leq y \mid X = x_0] \tag{3}$$

The case where the market and economic factors follow a joint multivariate Gaussian distribution,

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N(\mu, \Sigma)$$

results in simple analytical expressions for conditional scenarios and distributions. Denote the mean and covariance of the joint distribution by

$$\mu = \begin{pmatrix} \mu^{(1)} \\ \mu^{(2)} \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

Then, the conditional distribution of Y given $X = x_0$ is also multivariate normal with new mean *m* and covariance matrix *B* given by

$$\begin{aligned} m &= \mu^{(2)} + \Sigma_{21}\Sigma_{11}^{-1}(x_0 - \mu^{(1)}) \\ B &= \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12} \end{aligned}$$

Figure 1 provides a simple picture of the setting, for the 2-D case, with one market factor and one economic factor.

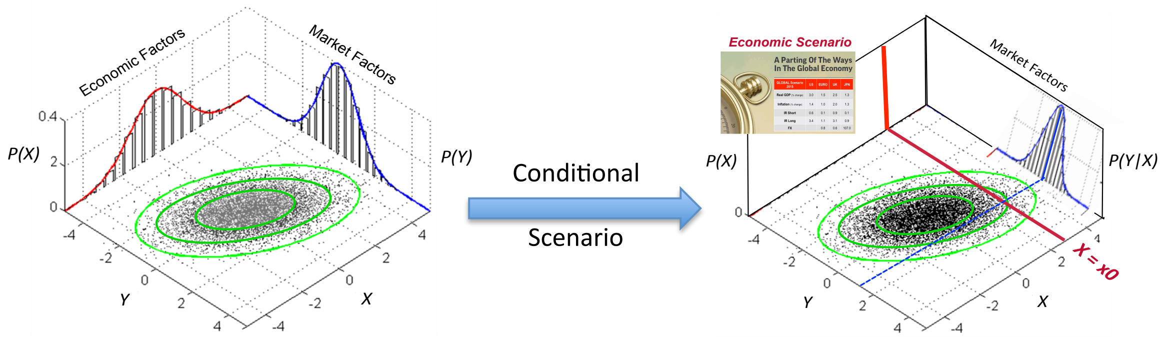

Figure 1: Conditional Scenario Setting.

6. Portfolio Stress Testing and Conditional Scenario Analytics. The impact of the economic scenario on the portfolio's performance is calculated by simulating the portfolio over the conditional scenarios.[3] As with expressions (2) and (3), we obtain the *conditional expected P&L scenario* and *the conditional distribution:*

$$\Delta V_0 = E[\Delta V \mid X = x_0] \quad \text{and} \quad F_{\Delta V|X}(v \mid x_0) = P[\Delta V \le v \mid X = x_0] \tag{4}$$

Note that in general, since V may be nonlinear, the expected conditional P&L scenario, ΔV_0, is not equal to the P&L in the expected conditional market scenario, $\Delta V(y_0)$.

All the risk metrics, including a conditional VaR or Expected Shortfall, are estimated from the conditional P&L distribution. In addition, one may want to attribute any metric to various positions or risk factors, and understand possible hedges.

[3] Alternatively, one might re-weight the original scenarios using the conditional distribution.

3 Conditional Scenarios and Factor Contributions using Post-Simulation LSST Analytics

We focus on a discrete version of the joint model, where we have a (large) set of scenarios $S = \{(x_j, y_j), \quad j = 1,...,m\}$ obtained from a simulation from the joint distribution $F_{X,Y}$ at the horizon T (as mentioned earlier, F can be completely general). The key insight for our approach is that the *conditional expected market scenario*, and more generally the *conditional market factor distribution* can be estimated directly from the matrix of simulated scenarios using *Least Squares Regression (LSR)*.

3.1 LSR for Market Risk Factors

Based on the set of scenarios S, we fit a general linear model of the form:

$$Y = BX + U \tag{5}$$

where Y is the vector of market factors; X is the vector of economic factors; B is the matrix of sensitivities of the market factors to the economic factors; and U is the vector of errors, with zero mean, and assumed to be independent of (or at least uncorrelated with) the vector of economic factors X. Often, the vector U is assumed to have a multivariate normal distribution, with covariance matrix Σ, but this is not necessary in our case.

This linear model (5) is not as restrictive as it may first appear. First, the market factors Y can be assumed to be non-linear functions of the linear combination of factors. More generally, also Y can depend on linear combinations of nonlinear functions of the economic factors, simply by adding these as new explanatory variables in the regression.[4]

Based on expression (5), the expected conditional market scenario is given by

$$y_0 = E[Y \mid X = x_0] = Bx_0 \tag{6}$$

Under the assumptions that the errors U are multivariate normal with mean zero and covariance matrix Σ, and independent of X, the conditional distribution of Y given $X{=}x_0$ is also multivariate normal with mean y_0 and covariance matrix Σ. When U is not normally distributed, but is still independent of X, then the conditional distribution of Y given $X{=}x_0$ can be calculated as:

[4] As an example consider the case where each factor is first transformed to a Normal variable (as in a Gaussian copula setting): $\tilde{Y}_i = \sum_{k=1}^{N} \beta_{ik} \tilde{X}_k + \tilde{U}_i$ with $\tilde{Y}_i = \Phi^{-1}\left(F(Y_i)\right),\ \tilde{X}_k = \Phi^{-1}\left(F(X_k)\right),\ k = 1,...,N$

$$
\begin{aligned}
P[Y \le y | X = x_0] &= P[BX + U \le y \mid X = x_0] \\
&= P[U \le y - BX \mid X = x_0] \\
&= P[U \le y - Bx_0]
\end{aligned} \tag{7}
$$

Expression (7) can be used to sample from the conditional distribution of *Y* given *X*, for example, when a fat-tailed distribution is used for the residuals *U*. Alternatively, the errors can be sampled directly using their empirical distribution (see below), thus avoiding any distributional assumptions on *U*.

The matrix of parameters *B* can be estimated in a number of ways, including Least Squares Regression (LSR) and Maximum Likelihood estimation. We find in practice it is straightforward to use Ordinary Least Squares Regression (OLSR) to estimate the parameters associated with each component of the vector *Y* separately.[5] That is, for each market factor, we consider the regression equation:

$$
Y_i = \sum_{k=1}^{N} \beta_{ik} X_k + U_i \tag{8}
$$

where β_{ik} is the (i,k)-th component of the matrix *B*.

We can write expression (8) in terms of the simulated scenarios $S = \{(x_j, y_j), \quad j = 1,\dots,m\}$

$$
y_{i,j} = \sum_{k=1}^{N} \beta_{ik} x_{k,j} + u_{i,j} \tag{9}
$$

where $y_{i,j}$ is the value of the *i*-th market factor under the *j*-th scenario, $x_{k,j}$ is the value of the *k*-th economic factor under the *j*-th scenario, and $u_{i,j}$ is the value of the *i*-th error under the *j*-th scenario.

The parameters β_{ik} in equation (9) are then estimated using OLSR on each factor separately. We denote the estimator of β_{ik} by $\hat{\beta}_{ik}$. Once the parameters have been estimated, we can construct the regression residuals:

$$
\hat{u}_{i,j} = y_{i,j} - \sum_{k=1}^{N} \hat{\beta}_{ik} x_{k,j} = y_{i,j} - \hat{y}_{i,0} \tag{10}
$$

where $\hat{y}_{i,0}$ is the estimated conditional mean:

[5] Alternative methods of estimation include using econometric techniques for simultaneous equations, for example see Ruud (2000) for details.

$$\hat{y}_{i,0} = \sum_{k=1}^{N} \hat{\beta}_{ik} x_{k,j} \tag{11}$$

The empirical estimator for the distribution of U is then given by [6]

$$P[U \le u] \approx m^{-1} \sum_{j=1}^{m} \mathbf{1}(\hat{u} \le u) \tag{12}$$

where $\mathbf{1}$(A) is equal to one if A is true and zero if A is false (inequalities involving vectors are interpreted component-wise). This is equivalent to assuming that

$$P[U = \hat{u}_j] = \frac{1}{m}, \quad j = 1, \ldots, m \tag{13}$$

From this, the resulting conditional distribution of the market scenarios is given by

$$P[X = \hat{u}_j + \hat{y}_0] = \frac{1}{m}, \quad j = 1, \ldots, m \tag{14}$$

3.2 LSR for Portfolio P&L

In the end, we are interested in computing the impact of the economic scenario on the portfolio's performance. As mentioned above, this can be calculated by first obtaining the expected conditional market scenario (or perhaps a full conditional market scenario distribution) and then simulating the portfolio over the conditional scenarios. A useful alternative in practice is to first simulate portfolio over the full set of scenarios *S,* and the regress directly the portfolio P&L directly against the economic factors:

$$\Delta V_j = \sum_{k=1}^{N} b_k x_{k,j} + \varepsilon_j, \quad j = 1, \ldots, m \tag{15}$$

with estimated residuals

$$\hat{\varepsilon}_j = \Delta V_j - \sum_{k=1}^{N} \hat{b}_k x_{k,j} \tag{16}$$

where the hat indicates the OLSR estimate. Based on the regression, we obtain the *conditional expected P&L scenario:*

---

[6] Note that this is only an approximation. If the true distribution of U is normal with mean 0 and covariance matrix σI, then $\hat{u}$ will be normal with mean 0 and covariance matrix $\sigma(I\text{-}H)$, where $H = X(X^T X)^{-1} X^T$.

$$\Delta V_0 = E[\Delta V \mid X = x_0] = \sum_{k=1}^{N} \hat{b}_k x_{k,0} \tag{17}$$

as well as the conditional distribution of the P&L:

$$P\left[\Delta V = \sum_{k=1}^{N} \hat{b}_{ik} x_{k,0} + \hat{\varepsilon}_j\right] = \frac{1}{m}, \quad j = 1,\ldots,m \tag{18}$$

There are several advantages of regressing the Portfolio P&L directly on the economic factors. First, for a given portfolio, it only requires a single regression, rather than multiple regressions (or a regression to fit a system of simultaneous equations). Although numerical algorithms for OLSR are now really efficient, this is much simpler and may also result in non-trivial reductions in computation time for some very large problems.

Second, and more importantly, by regressing the portfolio P&L directly on the economic factors, we can compute several conditional scenario portfolio analytics with a single portfolio simulation, which is typically the most expensive computational step. Also, a single simulation can be used to explore different sets of economic factors to fix in the conditioning. If we wish to constrain on a different set of economic factors, then the regression is rerun to generate a new set of residuals, over the same simulation, and thus new conditional means and distributions are obtained.

Finally, the portfolio P&L regression also has significant meaning in terms of (economic) *factor contributions* to a given scenario P&L, and more generally to portfolio risk measures such as VaR or Expected Shortfall (see Rosen and Saunders 2009, 2010, 2011, Meucci 2010). Given an economic scenario, X=x, the conditional portfolio P&L can be written as

$$\Delta V(x) = \sum_{k=1}^{N} C_k = \sum_{k=1}^{N} \hat{b}_k x_k \tag{19}$$

where C_k are the (linear) risk factor contributions. More generally, if the portfolio loss is written as the sum of its components

$$L = \sum_{k=1}^{n} L_k$$

then, under regularity conditions on the random variables involved (see, e.g. McNeil, Frey and Embrechts 2015), the contributions of the k-th component to Value-at-Risk and Expected Shortfall are given by

$$C_k^{\mathrm{VaR}} = E[L_k \mid L = \mathrm{VaR}], \qquad C_k^{\mathrm{ES}} = E[L_k \mid L \geq \mathrm{VaR}]. \tag{20}$$

Setting $L = -\Delta V$, we can estimate the risk contributions of the k-th economic risk factor using the regression results:

$$C_k^{\mathrm{VaR}} = \hat{b}_k x_{k,j^*}, \qquad C_k^{\mathrm{ES}} = \frac{1}{m'} \hat{b}_k \sum_{L_j \geq L_{j^*}} x_{k,j} \tag{21}$$

where j^* is the VaR scenario, and m' is the total number of scenarios in which losses exceed VaR.

3.3 Heteroskedasticity

An important additional assumption of the LSR model is that *the variance of the errors around the regression surface is everywhere the same*, i.e. $\mathrm{V}(U)=\mathrm{V}(Y \mid X) = \sigma^2$. In this case, we refer to the errors in the regression as *homoscedastic.* The situation where the variance of the errors depends on the level of the independent variables X is referred to as *heteroskedasticity.* Non-constant error variance does not cause biased estimates, but it does pose problems for efficiency, and the usual formulas for standard errors of the estimates are less accurate. Essentially, in this case, OLS estimates are *inefficient* because they give equal weight to all observations regardless of the fact that those with large residual errors contain less information about the regression. Standard econometric tests for heteroskedasticity include the White test and the Breusch-Pagan test (see, for example, Ruud 2000).

The presence of heteroskedasticity is in general not a big a problem to obtain conditional expected scenarios. However, it does have an impact when we are concerned with the full conditional scenario distributions.

There are several ways to correct for heteroskedastic errors in the regression. First, one can add more independent variables or transform both the dependent and independent variables with non-linear functions. Second, there are some techniques such as *Weighted LSR* to correct for this (e.g. Ruud 2000). Third, we may want to account for the fact that the conditional variance is not constant in the model, and perhaps adjust the variance of the error distribution conditional on the specific economic scenario (for example by looking at the variance surrounding that scenario, through the n closest points or the points within a given radius). Finally, particularly for the one-dimensional case of the Portfolio P&L, an alternative technique to OLSR when one is interested in obtaining specific conditional quantiles or the full conditional scenario distribution is the use of Quantile Regression Techniques (c.f. Koenker 2005). We discuss the application of these techniques in a follow-up paper.

4 Example

The Portfolio

Consider a typical USD-based multi-asset portfolio with positions in equities (EQ), rates (IR) and credit (CR) in four currencies: USD, EUR, GBP and JPY. Portfolio exposures are summarized in Figure 2. Its mark-to-market (MtM), as of April 28 2015, is $458 million USD, with $568M long and $110M short positions. Half of its MtM is in EQ with the rest equally split between IR and CR. In terms of currency exposure, almost 60% is USD, with 32% in EUR and just under 5% for each of GBP and JPY. The US EQ portfolio accounts for almost a third of the exposure, with diversified positions across sectors with the highest exposure in Energy (20%). The IR portfolio is long U.S. and German Government bonds, while the CDS portfolio has long-short strategies with high-yield and investment grade single name CDSs and indices.[7]

$ USD (Million)	NMV	Long	Short
Portfolio	457.5	567.8	110.3
EQ	221.4	225.5	4.1
IR	111.2	111.2	0.0
CR	124.9	231.1	106.2

NMV	USD	EUR	GBP	JPY	Total
EQ	32%	11%	4%	5%	52%
IR	10%	14%			24%
CR	18%	7%			24%
Total	59%	32%	4%	5%	100%

Portfolio	**458**
EQ	*221*
US EQ	136
EU EQ	48
GB EQ	16
JP EQ	22
IR	*111*
USD GOV	52
EUR SOV	59
CR	*125*
US IG	70
US HY	60
EU HY	32
US CDS	-38

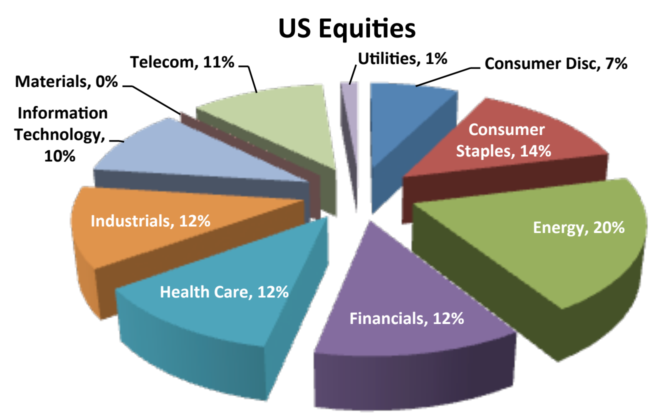

Figure 2: Portfolio Exposures.

[7] Portfolio Calculations performed by S&P Capital IQ Portfolio Risk. For Illustrative purposes only.

Figure 3 provides a summary of the risk of the portfolio, measured as 99% VaR over one month, and the risk contributions (stand-alone and marginal VaR contributions). The EQ portfolio accounts for half of the exposure but almost two thirds of the portfolio risk, with US EQ constituting more than half of this contribution (37%). In terms of currency, USD positions contribute slightly more than half of the risk, while EUR positions contribute about 40%, despite being only 32% of the portfolio exposure. The CR portfolio, although in principle more risky than the IR portfolio, seems reasonably well hedged, and only contributes to the total risk 3% more than IR. Looking deeper into US EQ, Energy contributes almost 30% of risk (with about 20% of exposure), and industrials are the second risk contributor at over 17% (with only 12% of exposure, about the same level as Financials, Healthcare and Telecom and below Consumer Staples).

$ USD (Million)	NMV	VaR	VaR (Marginal)
Portfolio	457.5	22.3	22.3
EQ	221.4	17.1	14.4
IR	111.2	6.3	3.6
CR	124.9	6.5	4.3

	USD	EUR	GBP	JPY	Total
EQ	37%	21%	4%	2%	65%
IR	3%	13%			16%
CR	13%	7%			19%
Total	53%	40%	4%	2%	100%

Portfolio	***458***	***22.3***	***22.3***
EQ	*221*	*17.1*	*14.4*
US EQ	136	11.5	8.3
EU EQ	48	6.2	4.6
UK EQ	16	1.5	1.0
JP EQ	22	2.7	0.6
IR	*111*	*6.3*	*3.6*
USD GOV	52	2.4	0.7
EUR SOV	59	4.6	2.9
CR	*125*	*6.5*	*4.3*
US IG	70	2.1	0.8
US HY	60	3.4	2.0
EU HY	32	2.1	1.5
US CDS	-38	1.9	0.0

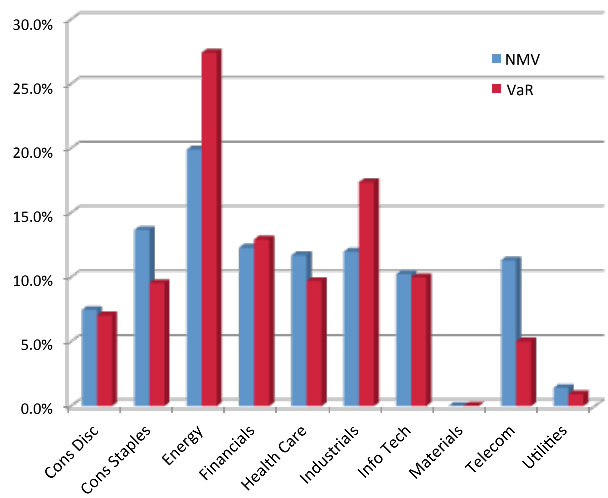

Figure 3: Portfolio Risk and Risk Contributions (VaR 99%, one month).

The Economic Report and Economic Scenarios

Suppose that the analysts just reviewed a new series of economic research reports[8] from which they draw a set of future scenarios on the global economy, in general, and the U.S.

[8] S&P economic research, Credit Week, April 22nd 2015.

economy specifically. We would like to understand how our portfolio will react if these conditions unfold over the next year. We focus on 4 economic scenarios:

- A Global Economic Scenario, and
- Three U.S. Outlook Scenarios.

Global Economic Outlook Scenario

The Global Scenario describes how the global economy, after being hit in 2008 by the worst financial crisis and recession since the Great Depression, had started to expand again by mid-2009, and has been expanding ever since. Economists expect the global economy to continue to grow, with about 3.5% real GDP growth in 2015 and 3.9% the following year. All of this despite a continuing slowdown in China, while the U.S., the Eurozone, Japan, and India all grow faster. The overall picture draws on the positives for global growth resulting from the decline in oil prices, the information technology revolution, the European Central Bank's quantitative easing program and the Fed's caution. normalization of monetary policy.

Out of the large number of economic variables in the forecast (about 120), we focus on a reasonable subset of 15 factors, which are relevant to our portfolio, as shown in Table 1. For example, Eurozone Interest Rates are forecasted to go down in 2015 from their 2014 levels, while the actions of the U.S. Fed result in the policy rate being raised, though the 10 year yield stays at 2.3%, below its 2014 level. GDP growth is expected to increase to 3% in the U.S. (from 2.4% in 2014), while the growth in the Eurozone and Japan is also expected to increase to 1.5% and 0.8%, respectively (from under 1% and negative 0.1% last year).

GLOBAL Scenario	US	EURO	UK	JPN
Real GDP (% change)	3	1.5	2.8	0.8
Inflation (% change)	-0.3	-0.3	0.1	0.4
IR Short	0.3	0.1	-	-
IR Long	2.3	0.3	-	-
FX	-	0.9	0.7	120.0

Table 1. Global Economic Scenario (One Year).

U.S. Economic Outlook Scenarios

According to the research report on the U.S. economy, its growth prospects remain favorable. Still, given a slightly softer-than-expected start to 2015, economists lowered their forecast for GDP growth from 3.3% to 3%. At the time of the report, it seemed plausible that the Federal Reserve would raise rates for the first time in almost 10 years. Based on this economic analysis, the report presents three scenarios (Base, Up, and Down) on over 30 economic variables. For this example, we construct scenarios based on a manageable set of 7 economic factors. In addition to GDP, Inflation and IRs, which are in the Global Scenario, we include Unemployment, a Market Index (S&P500), and Oil Prices. Table 2 presents these three economic scenarios. For example, while GDP grows at about 3% in the Base Scenario, increases by 3.4% in the Up Scenario, and is growth is merely 2.1% in the Down Scenario. The S&P500 is projected to grow between 7.6% and 10%. Also note that the Down Scenario has an 8.5% equity growth, which is higher than the Base Scenario. While perhaps counterintuitive, this is a consequence of the economists' model showing that the weaker than expected economic conditions in the Down Scenario cause the Fed Fund Rates to remain at zero well into 2017, and this drives the stock market levels higher than in the baseline case.

U.S. Scenario	UP	BASE	DOWN
Real GDP (% change)	3.4	3	2.1
Inflation (% change)	-0.3	-0.3	-0.4
Unemployment (%)	5.4	5.4	5.8
IR Short	0.4	0.3	0
IR Long	2.4	2.3	2.1
S&P 500 (%)	10.0%	7.6%	8.5%
Oil ($/bbl, WTI)	50.12	50	48.56

Table 2. U.S. Economic Scenarios (One Year).

The Joint Factor Simulation Model

We now construct a simulation model for the joint future evolution of the market factors affecting the portfolio P&L and the economic factors. This *statistical model* helps us translate the economic forecast scenarios into market scenarios, which we can use to simulate our investment portfolio and understand its risk.

The model is estimated using quarterly data for all the factors over about 20 years. It has a total of 18 economic factors (Tables 1 and 2) and 38 market factors, which include: 13 EQ factors (10 US, 1 Europe, 1 UK, and 1 Japan); IR curves in USD and EUR (8 points each), Bond spreads in US and EUR (IG and HY); CDS spreads in USD (HY and IG); and FX rates: EUR, GBP, JPY.

This joint factor model consists of two components: [9]

1. *Marginal Risk Factor Processes.* Based on each individual factor's time series, we construct a statistical model for its marginal processes. We use ARMA-GARCH models to filter the series and remove autocorrelations, etc. While perhaps more sophisticated models can be used in practice, this suffices to generate a rich enough model for this example, and highlights the main advantages of the conditional scenario methodology.

2. *Codependence structure of residuals.* After each factor's process is estimated, we model their codependence structure non-parametrically using directly the historical residuals. With 56 factors and 20 years of quarterly data, the factor codependence is described by a matrix of residuals consisting of 80 rows (80 quarters) and 56 columns.

We avoid the typical assumption of Gaussian residuals to generate more meaningful stress scenarios. The historical (non-parametric) residual codependence allows us to capture both non-normal marginal distributions with fat tails, as well as more complex tail dependence. Of course, when the number of market factors gets too large some dimensionality reduction may be necessary.

Figure 4 depicts the correlation matrix implied from the historical residuals. In general the correlations between the Economic Factors are not as strong, except perhaps for the EQ index and long rates, GDP and Oil, and a negative correlation between GDP and Unemployment, as expected. In contrast, the correlations between the Market factors are higher. The lower left side of the matrix gives the correlations between the market and the economic factors, which are generally not very strong. As expected, we can see for example that Long Rates are positively correlated to EQ and negatively to CDS spreads, and Oil Prices have strong correlation to Energy EQ returns.

[9] We use in this example a simple, but rich enough model to highlight the benefit of the LSST methodology.

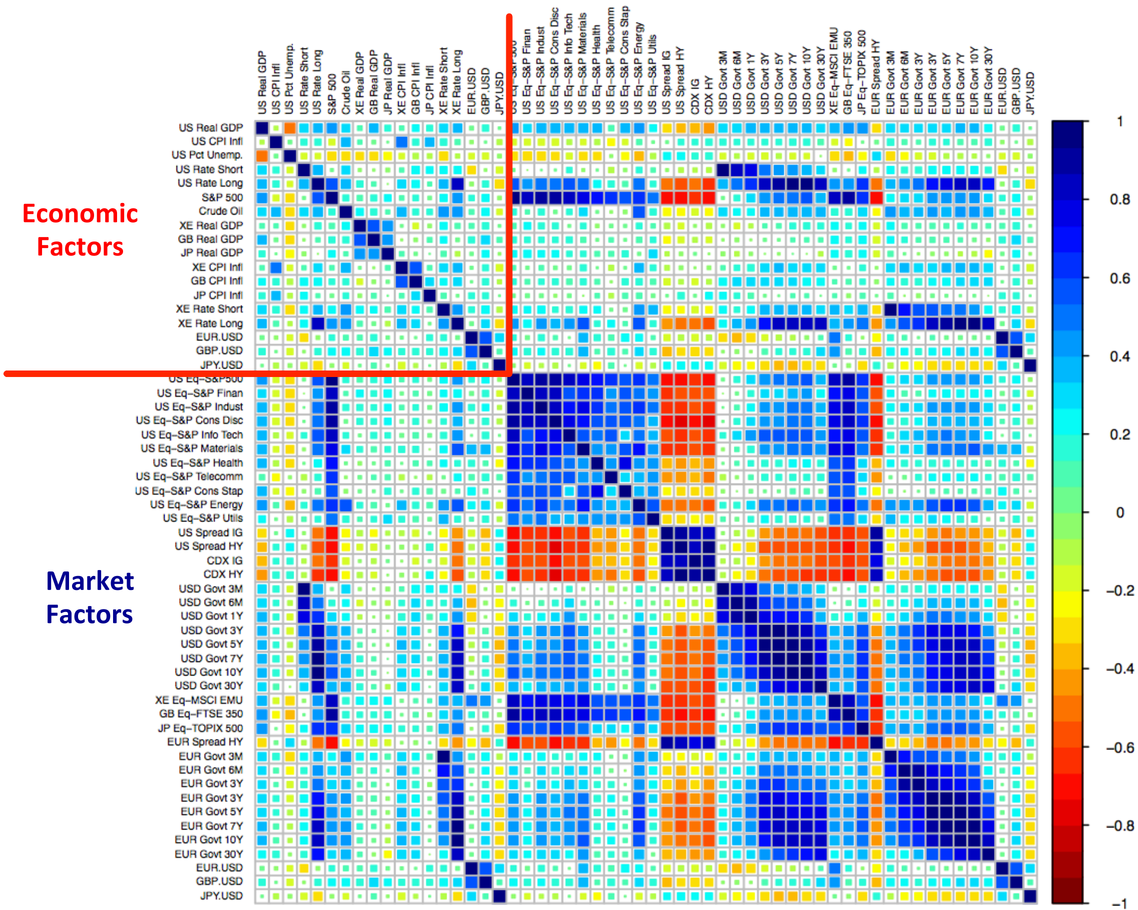

Figure 4. Correlation Matrix of Historical Factor Residuals.

Finally, with the calibrated model, we obtain a large number (say 1,000 to 10,000) joint scenarios for the economic and market factors, using Monte Carlo methods that simulate ARMA GARCH processes and randomly sample from the joint residuals. As specified by the Economic Scenarios, we run the simulation for a one-year horizon, by simulating four quarterly steps for each scenario. Although we have a small number of quarterly residuals (80), we can generate a large number of distinct yearly scenarios (almost 41 million). As an example, Figure 5 plots the joint quarterly residuals, as well as 1,000 simulated yearly scenarios, for 2-D projections of Euro vs. USD Long Rates, and for GDP vs. Unemployment. In general these scenarios are stored in a matrix of about 56 columns (the number of economic and market factors) and 1,000 rows (the number of scenarios).

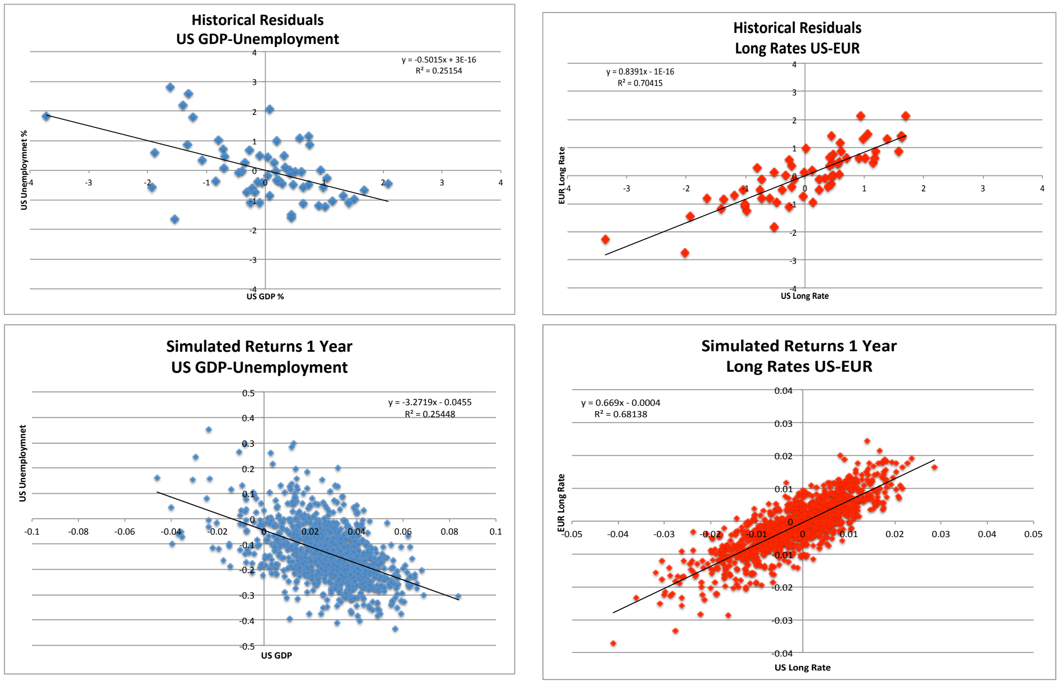

Figure 5. Joint Economic and Market Factor Scenarios (2-D Projections).

Conditional Scenarios

We cannot get analytical closed-form expressions for either the joint factor distribution, or for the conditional scenarios under this fairly rich (but still quite simple) factor model setting. However, as mentioned earlier, the LSST methodology can be applied in a straightforward way with the pre-computed joint scenarios on the economic and market factors, as simulated from the model. Figures 6 and 7 show the *Expected Conditional Market Scenarios* for the Global and the U.S. Economic Outlook, respectively. For simplicity, the scenarios are expressed in standardized form, i.e. as number of standard deviations from their expected value.

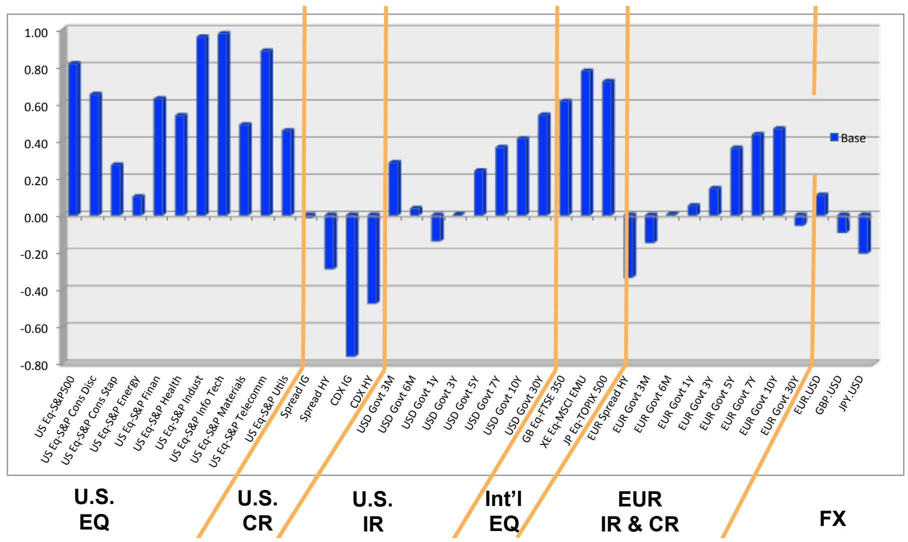

Figure 6. Global Economic Outlook – Expected Conditional Market Scenario.

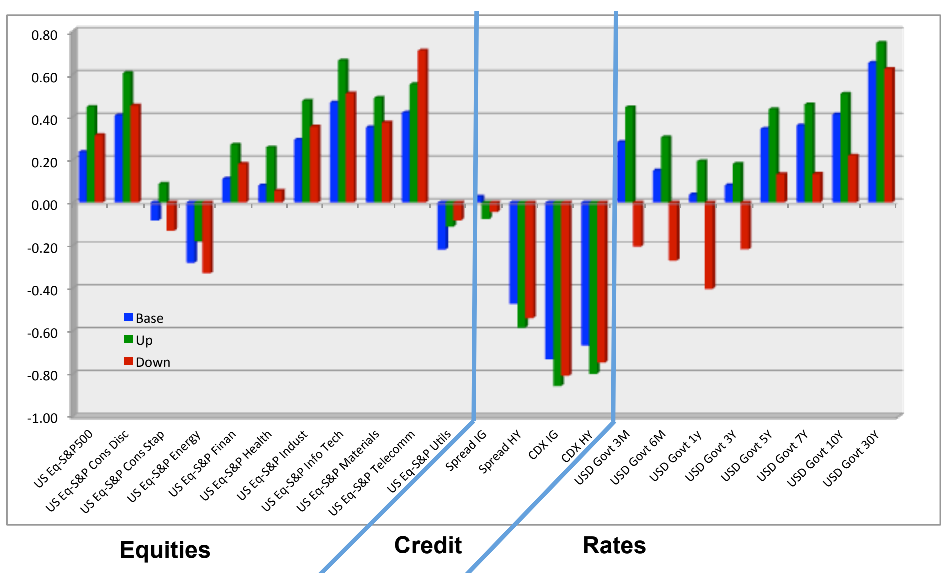

Figure 7. U.S. Outlook – Expected Conditional Market Scenarios.

In the Global Economic Expected Scenario (Figure 6) most US EQ sectors move up markedly (some one standard deviation) as a consequence of beneficial economic conditions. However, Energy increases only marginally, largely due to oil prices remaining quite low in the forecast. Credit spreads tend to tighten, but the impact is less clear for IRs in the figure, since the scenarios are expressed in standardized form (but essentially we know that the long end of the USD IR curve is at about 2.3% and short end at 0.3%).

In the U.S. Economic Outlook Scenarios, almost all equity sectors are up in the Base (from their expectation, which is generally positive) except for Energy, Utilities and Consumer Staples. Credit Spreads tend to decline while the IR term structure rises. The Up Scenario, as expected, essentially amplifies these effects, but not drastically. In contrast, the Down Scenario is a bit more complex. Equity returns for some sectors (e.g. Telcomm) are above their mean and for others (Energy) they are below. IR changes are generally less than expected for the short end of the curve, but they are positive and lower for the long-end of the curve. This is quite consistent with the earlier explanation of the scenario.

One can understand better these conditional scenarios by visualizing simple 2D projections of the joint scenarios, the regression and the resulting conditional distributions. For example, Figure 8 shows the projections of the conditional scenarios for a given EQ index (Industrials), against an economic factor (US Long Rate). The figure depicts both the Expected Conditional Scenario, as well as the full conditional distribution for each of the three U.S. Economic Outlook Scenarios.[10]

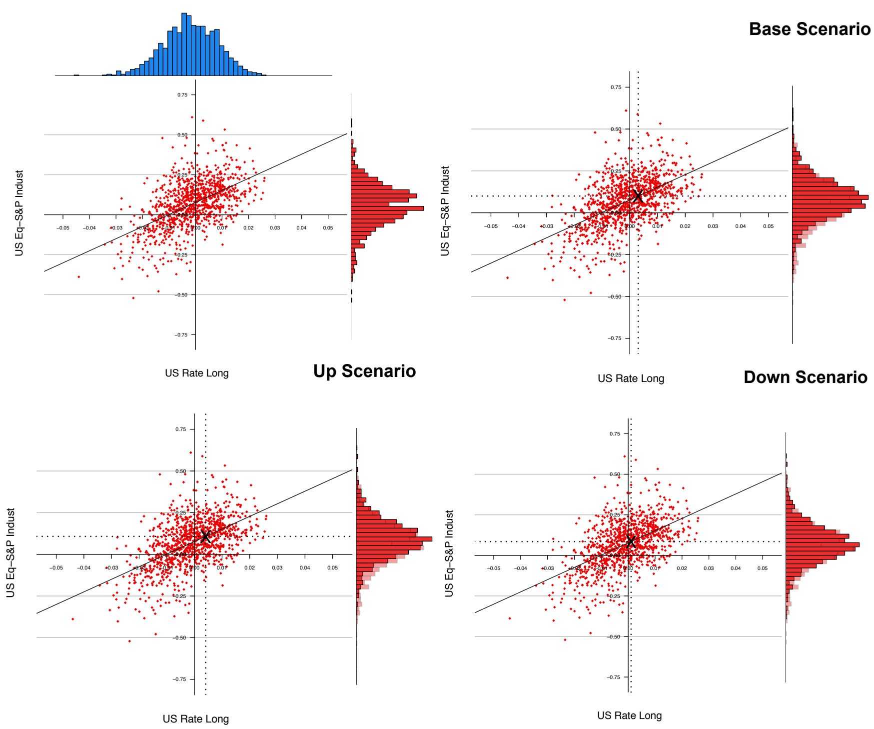

Figure 8. U.S. Outlook – Expected Conditional Scenarios (2D Projections).

[10] We point out that most of the factor regressions do not pass the homoskedasticity tests. While as mentioned earlier, the Expected Conditional Scenarios are not affected strongly by this, the conditional factor distributions are not expected to be accurate and are only shown here for illustration purposes.

Conditional scenarios can be very sensitive to the economic variables included in the forecast or view. The LSST methodology allows a quick assessment of this impact, without having to re-simulate the original unconditional scenarios. Take, for example, the Global Outlook Scenario, which originally included 15 economic variables (Table 1). We add first the price of Oil to the forecast and then the US EQ Index (as given in the U.S. Outlook Base Scenario in Table 2), and then re-run the LSR to obtain new conditional scenarios on the market factors. Figure 9 compares the Expected Conditional Scenarios for these three cases. Adding Oil Prices to the forecast does not affect the conditional scenario much (red bars). However, adding in the EQ Index (which is highly correlated to many of the market factors) affects the scenario quite drastically (green bars). The Expected Conditional Scenario becomes less bullish on equity prices, with correspondingly higher credit spreads, but does not affect IRs or FX much.

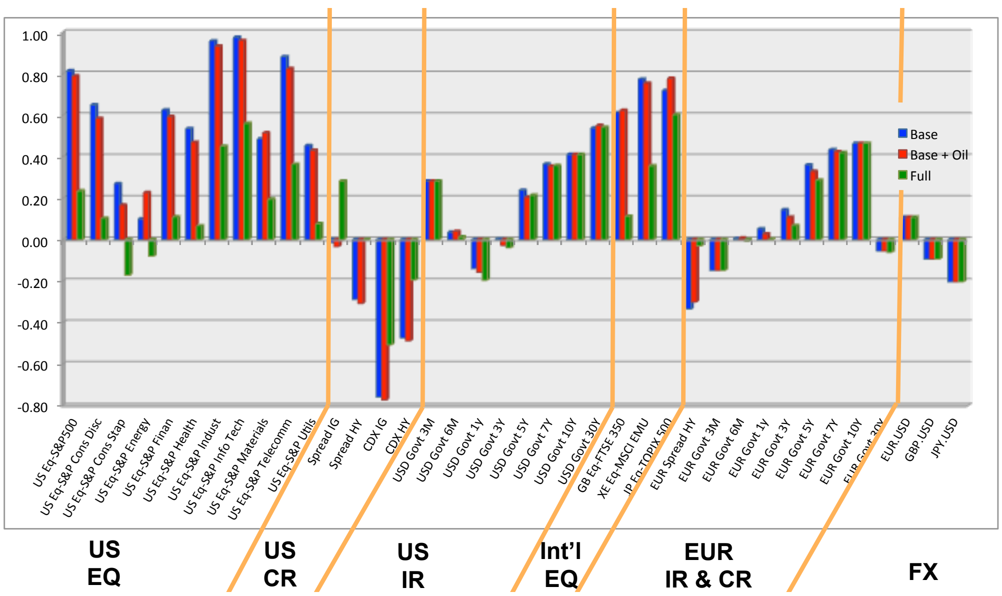

Figure 9. Effect of Scenario Variables on the Expected Conditional Market Scenario (Global Economic Outlook).

Portfolio Simulation and Analysis

Finally, we analyze the impact of the conditional scenarios on the portfolio. To recap, essentially the full process covers:

- Running a MC simulation with say 1,000 joint scenarios on the economic and market factors.

- Simulating the portfolio over these scenarios and calculating P&Ls for the portfolio (and all sub-portfolios) – this allows to get the “unconditional” P&L portfolio distribution over 1-year, which we can also use as a basis to understand the impact of the stress tests.

- Creating the *Expected Conditional Scenarios* for each of the four Economic scenarios in the report. This is done applying the LSST methodology on the generated scenario matrix.

- Simulating the portfolio directly over these scenarios, and analyzing the portfolio.

- Alternatively, also applying a LSR directly on the portfolio simulated P&L values (in the unconditional distribution), as explained earlier, to get the *Expected Conditional Portfolio P&L,* and also obtain full conditional portfolio P&L distributions.[11]

- Putting it all together to understand the impact on the portfolio.

To begin with, we look at the Global Outlook Scenario, which sets views on GDP, Inflation, short and long IRs and FX in the U.S., Europe, GB and Japan. Figure 10 gives the impact of the scenario on the portfolio P&L, and contrasts it to the portfolio’s mean P&L over the horizon, and its annual VaR (99%). The Global Outlook Scenario produces an excess return of 4.4% over the mean, with most of it coming from the EQ portfolio, and almost two thirds of this from the US EQ portfolio alone. Both the rates and credit portfolios have negative excess returns. The table on the right side further breaks down the scenario returns into the underlying sub-portfolio contributions.

[11] Note that the full conditional portfolio P&L distribution can also be obtained by first getting a new set of conditional scenarios from the factor LSR (using the full residuals from the regressions) and re-simulating the portfolio in each of these. This is computationally much more costly.

	NMV ($US M)	VaR (Annual)	P&L Mean	Global Scenario	Rel. Return
Portfolio	457.5	16.0%	3.7%	8.1%	4.4%
EQ	221.4	29.0%	6.7%	17.1%	10.4%
IR	111.2	14.4%	1.6%	-0.1%	-1.8%
CR	124.9	16.8%	0.4%	-0.5%	-0.8%

$US Million			NMV	P&L Mean	Global Scenario
Portfolio			**458**	**15.9**	**37.0**
	EQ		*221*	*14.5*	*37.8*
		US EQ	136	9.7	22.1
		EU EQ	48	2.6	9.8
		GB EQ	16	0.8	1.8
		JP EQ	22	1.3	4.1
	IR		*111*	*1.5*	*-0.1*
		USD GOV	52	0.3	-0.7
		EUR SOV	59	1.3	0.6
	CR		*125*	*-0.1*	*-0.6*
		US IG	70	-0.1	-1.0
		US HY	60	-0.6	0.2
		EU HY	32	0.4	1.1
		US CDS	-38	0.2	-1.0

Figure 10. Portfolio P&L – Expected Global Outlook Scenario

Figure 11 shows the impact on the Portfolio P&L as we add additional economic variables to the Global Outlook Scenario (Oil Prices and the EQ Index). As shown earlier in Figure 9, adding Oil to the forecast does not substantially alter the scenario itself, and hence has a small impact on the portfolio. However, adding the EQ Index results in much lower expected portfolio returns, with most of the return coming from the EQ portfolio, as expected. At first glance, the scenario does not significantly affect the positions in the Rates or the Credit portfolios. A more in-depth look at the sub-portfolios shows a more intricate story. The scenario has opposing effects on the USD and the EUR Rates portfolios, which cancel each other, since rates in the U.S. tend to increase and they are lowered for the Euro. For the CR portfolio, the opposing effects of the HY and IG positions hedge each other in the scenario.

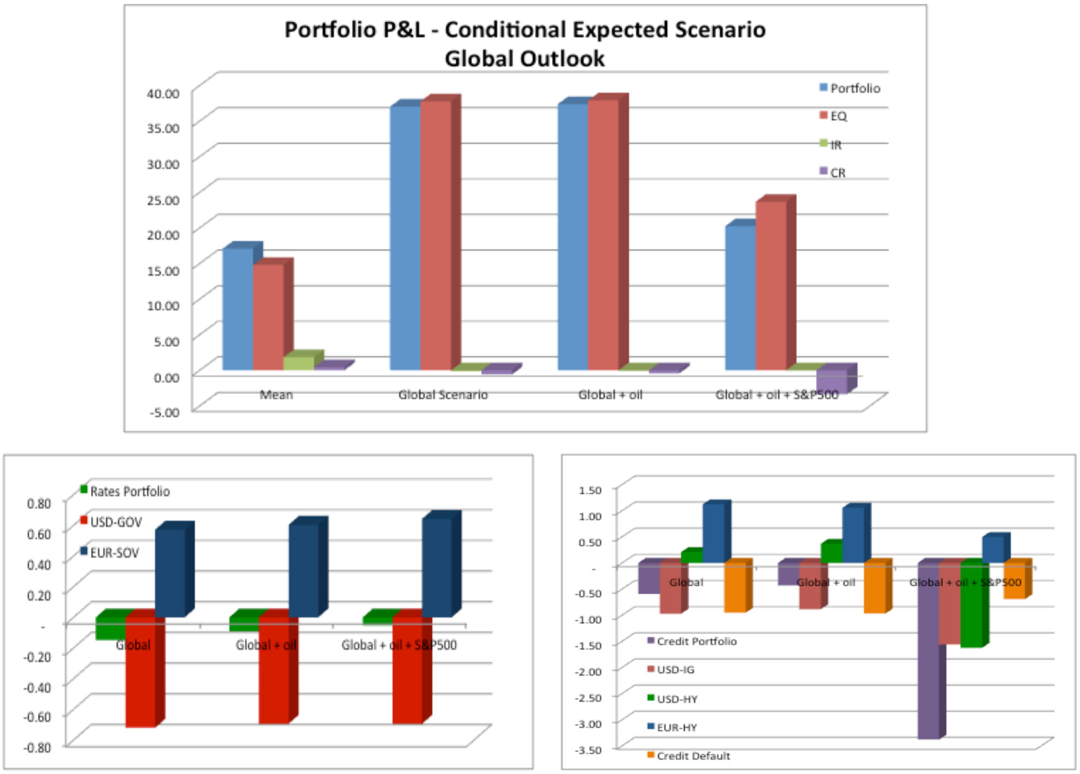

Figure 11. Effect of Scenario Variables on Expected Conditional Portfolio P&L – Global Outlook Scenario.

Figure 12 shows the U.S. Outlook Expected Conditional Scenarios for the US EQ portfolio, as well as the (unconditional) mean P&L. We focus now on this portfolio, as it dominates the results. The portfolio, with an NMV of $136M, has an (unconditional) mean P&L of almost $10M and VaR of $35M. The three U.S. Outlook Scenarios result in a P&L range of $12M-$15M, all above the mean P&L. The bottom graph in Figure 13 drills down to the sector P&L contributions (in the mean scenario as well as the three U.S. Outlook scenarios). While Energy has a large P&L contribution of almost $4M to the (unconditional) mean scenario, Industrials and Information Tech are essentially more dominant in the three U.S. Outlook Scenarios. This, of course, suggests some possible portfolio rebalancing, if we believe these forecasts and want to take some advantage of them.

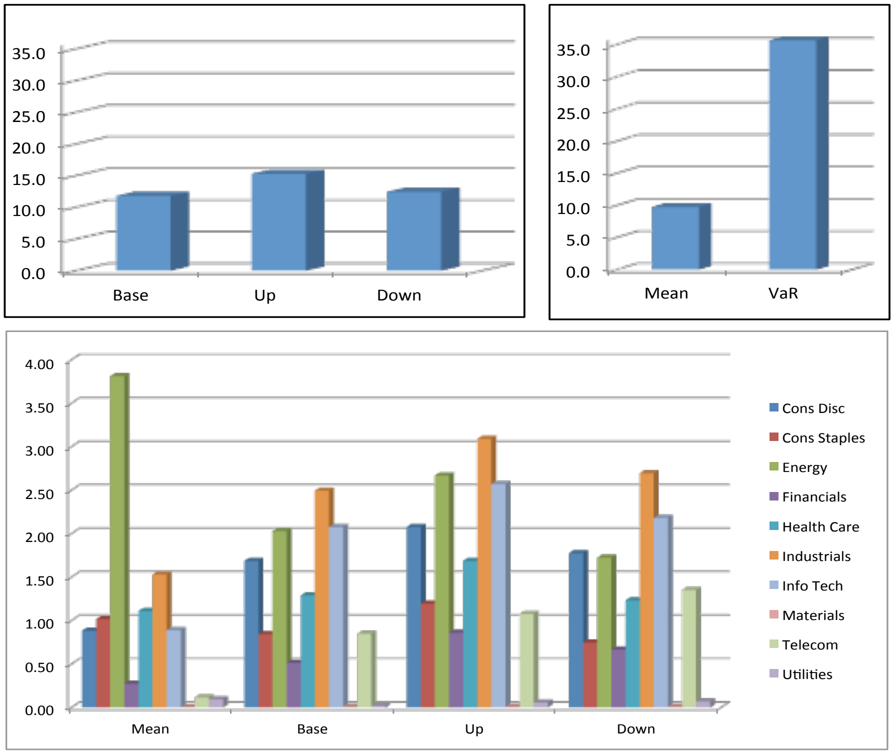

Figure 12. Expected Conditional US EQ Portfolio P&L – U.S. Outlook Scenarios.

In addition to the expected conditional scenario P&Ls, we can compute the full conditional scenario P&L distributions. Table 3 summarizes the Conditional VaR (99%) for the three U.S. Outlook Scenarios and compares them to the unconditional numbers. The Unexpected Loss (UL) defines the worst possible deviation from the mean at a 99% confidence level (the VaR minus the expected P&L). For comparability, we express the VaR as P&L, so that negative values correspond to losses. The UL of $11M is the same in all three Outlook Scenarios, and much lower than the unconditional one of nearly $45M. Under the LSR assumption of homoskedasticity, the distribution around the mean loss is the same under every scenario, once we define the fixed economic factors. The variance of this distribution also decreases as variables are added to the forecast. Hence, the large difference from $45M to $11M. Also note that, at the 99% level, all three Outlook Scenarios generate a profit!

Scenario	Mean P&L	Unexpected P&L	VaR
Unconditional	9.7	44.7	-35.0
Base	11.8	11.4	0.4
Up	15.3	11.4	3.9
Down	12.4	11.4	1.0

Table 3. Expected and Unexpected Conditional Portfolio P&L – U.S. Outlook Scenarios.

Figure 13 further shows the impact of the variables included in the Scenarios. We progressively remove the EQ Index and the Rates from the forecast. The bar chart shows the Expected Conditional Scenario for all cases under the Base, Up and Down Scenarios as well as the Conditional UL. It also contrasts these to the unconditional mean P&L and UL (the equivalent to not having any variables in the forecast). Removing the EQ Index from the conditioning variables results in substantial positive additional returns in the Base and Up Scenarios, but almost no impact on the Down Scenario. Removing both the S&P500 and Rates (thus leaving only GDP, Inflation, Unemployment and Oil Prices), leaves the Base and Up Scenarios almost unchanged, but reduces the P&L in the Down Scenario, since the forecasts essentially reflect slightly more bullish views. Finally, the conditional variance and UL increase when variables are removed from the forecast.[12] As we remove the EQ Index and then also Rates, the conditional UL goes from \$10M to \$30M to \$37M. At this point, the remaining economic factors have a smaller effect on the volatility of the market factors.

[12] Much as the R-squared of a regression always increases when more explanatory variables are added, Conditional UL always decreases as more economic variables are fixed (and these variables explain more of the portfolio returns). This observation depends on the assumption of homoskedasticity of the errors.

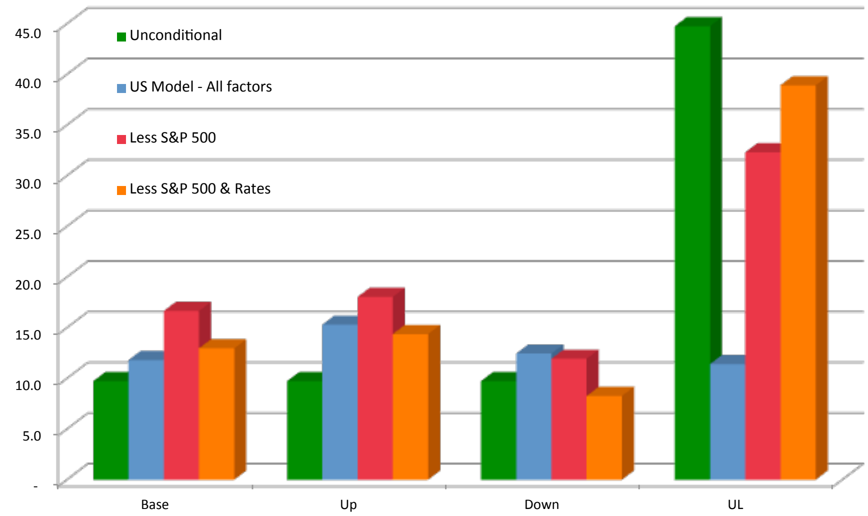

Figure 13. Effect of Scenario Variables on Expected and Unexpected Conditional US EQ Portfolio P&L – U.S. Outlook Scenarios.

The U.S. Outlook Scenarios look quite optimistic and do not produce any portfolio losses at the 99% level (including the Down Scenario). As part of a risk analysis, a fund manager may in addition want to look at downside economic scenarios coming, for example from regulators, who have dedicated substantial efforts to understand possible downfall scenarios. We test the portfolio against the CCAR 2015 adverse scenario, applied to the same economic variables.[13] Figure 14 summarizes the *Adverse CCAR* Economic Scenario, and contrasts the Expected Conditional Market Factor Scenarios, which result from fixing different sets of variables, with the U.S. Outlook scenario results. The CCAR Adverse Scenario causes portfolio losses of almost $30M (over 20% of NMV). Removing the EQ Index from the forecast reduces substantially the losses, but losses increase a bit again when Rates are also removed.

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[13] CCAR Scenarios produced by the US FED were originally designed for regulatory stress tests for bank holding companies with $50 billion or more of total consolidated assets. CCAR provides three Scenarios: Baseline, Adverse, and Severely Adverse, on 28 variables, including economic activity, unemployment, exchange rates, prices, incomes, and interest rates. The Baseline scenario is similar to average projections from surveys of economic forecasters (not the forecast of the Federal Reserve), while the Adverse & Severely Adverse scenarios are hypothetical events designed to assess the strength of organizations and resilience to adverse economic environments. In particular, the severely adverse scenario subjects the banks to a full year of global recession. As the Severely Adverse scenario is a bit extreme (e.g. a 60% drop in equity markets), we focus on the impact of the adverse scenario on our portfolio.

	CCAR Adverse
Real GDP (% change)	0.3
Inflation (% change)	4.0
Unemployment (%)	7.3
IR Short	2.6
IR Long	4.6
S&P 500 (% change)	– 20.5
Oil ($/bbl, WTI)	90.00

* Approximate

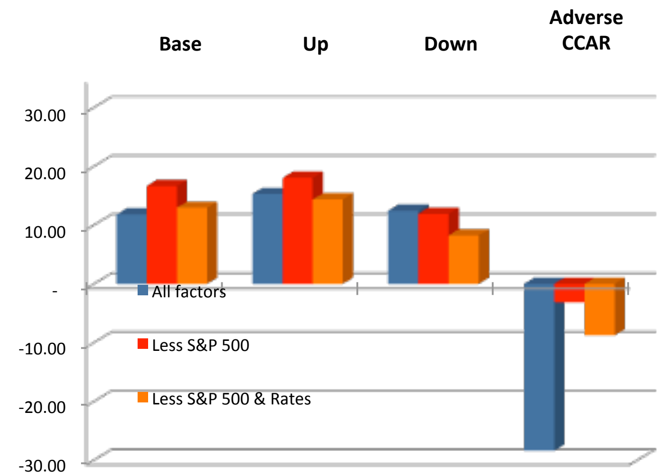

Figure 14. Conditional Expected Market Scenarios from CCAR Adverse Economic Scenario and U.S. Outlook Scenarios – US EQ Portfolio P&L.

Finally, as discussed in the methodology section, a significant advantage of the LSST framework is its ability to provide risk factor contributions. Essentially, on a conditional scenario, the P&L contribution of each economic risk factor is given by the product of the portfolio regression beta to the factor and the factor change in the scenario. Figure 15 presents the economic factor contributions (adding to a gross 100%) on the Expected Conditional CCAR Scenarios (for each of the three sets of variables). When included, the EQ Index dominates, accounting for almost 80% of the loss. Unemployment and Inflation become more important when the EQ Index is not present. Also, Oil has a large positive contribution once Rates are not included, since essentially the higher oil prices influence positively the energy positions. This simple ability to allocate factor P&L contributions provides a useful tool for improving the understanding of scenario outcomes, managing the risk of the portfolio, and constructing investment strategies.

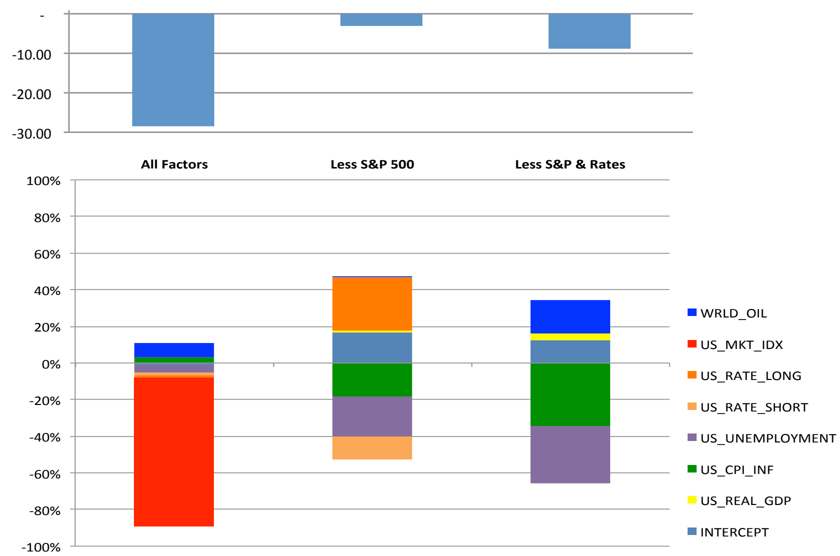

Figure 15. Factor Risk Contributions for CCAR Adverse Economic Scenarios – US EQ Portfolio P&L.

5 Concluding Remarks

LSST is a simple new approach to create meaningful stress scenarios for risk management and investment analysis of multi-asset portfolios, which effectively combine economic forecasts and "expert" views with portfolio simulation methods. While conditional market scenarios can be obtained for simple joint distributions such as Gaussian and Student t, LSST derives conditional scenarios from a pre-computed simulation, for completely general joint distributions and a large number of factors, using Least Squares Regression. The methodology is computationally efficient and can be built on top of any existing scenario and portfolio simulation risk engine, providing transparent results, which are auditable and easy to explain. It also defines a natural decomposition of a portfolio's performance into *risk factor P&L contributions*, allows users to run multiple scenarios and assumptions in real-time, and provides an explicit assessment of model risk.

While we focus this paper on *point scenarios,* conditional scenarios where factor are given fixed values, LSST allow one to define much more general views in terms of distributional parameters for the factors, etc. In particular one can also naturally define conditional scenarios where factors are prioritized, by running nested LSRs on a factor hierarchy, with progressive orthogonal incremental contributions, at each level (*Hierarchical LSRs*).

Finally, the focus of the paper has been largely on explaining the basic concepts and methodology, and showing its application to a real-life problem and portfolio. We have left some of the more advanced mathematical concepts, such as dealing with heteroskedasticity for the full conditional distributions, and methodology extensions to a follow-up paper.

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