Geographical diversification in annuity portfolios

Clemente De Rosa[*], Elisa Luciano[†], Luca Regis[‡]

October 14, 2016

Abstract

This paper studies the problem of an insurance company that has to decide whether to expand her portfolio of policies selling contracts written on a foreign population. We propose a parsimonious continuous-time model for longevity risk, that captures the dependence across different ages in two populations and evaluate the diversification gains due to the international expansion. We present a calibrated example, based on annuity portfolios of UK and Italian males aged 65-75. The results of our application show that diversification gains, evaluated as the reduction in the portfolio risk margin following the international expansion, can be non-negligible, in particular when interest rates are low.

[*]Collegio Carlo Alberto, clemente.derosa@carloalberto.org, Via Real Collegio 30, 10024 Moncalieri, Italy, + 390116705228.

[†]University of Torino, Collegio Carlo Alberto and Netspar; ESOMAS Department, Corso Unione Sovietica 218/bis, 10134, Torino, Italy; e-mail: elisa.luciano@unito.it; phone number: + 390116705230 (5200).

[‡]IMT Institute for Advanced Studies Lucca and Collegio Carlo Alberto; AXES Research Unit – Piazza San Francesco 19, 55100 Lucca, Italy; e-mail: luca.regis@imtlucca.it.; phone number: +3905834326721.

1 Introduction

In the last twenty years, as a consequence of the deregulation trends in the financial industries, insurance companies have more and more expanded internationally, creating operating subsidiaries in different countries or via cross-border mergers and acquisitions.[1] The activities of reinsurance companies, by their own nature, have always been more geographically diversified than those of insurers (Cummins and Xie (2008)), because the degree of internationalization of their portfolios can be more easily disconnected with the geographical localization of their branches. Outreville (2008) indeed documents the high degree of internationalization of the largest insurers and reinsurers that in most cases concentrate the bulk of their activities in foreign countries. However, the extent to which this internationalization trend resulted in higher industry profitability has been the subject of some recent studies. Outreville (2012) finds that, in the reinsurance industry, the level of international diversification affects performance, but the relationship is non-linear. Empirical evidence on the life insurance industry Biener et al. (2015) shows that costs of coordination and organization of complex international structures may offset the potential benefits from internationalization, resulting in a negative relationship between performance and internationalization. Nonetheless, Cummins et al. (1999) put forward that geographical diversification was a primary determinant of mergers and acquisitions in the US insurance industry, as geographically diversified (across U.S. states) firms were more likely to be the target of acquisitions.

In this paper, on top of these efficiency/performance considerations, we focus on a potential benefit of internationalization in life insurance, arising from the diversification gains stemming from longevity risk pooling across populations. We consider the situation of an insurer who is facing the choice of expanding her portfolio of liabilities (for simplicity, an annuity portfolio held by policyholders of different ages) either in her domestic market or in a foreign one. Analogously, our setting can represent the problem of a reinsurer whose outstanding contracts are all written on the same underlying population and that has the option to reinsure a portfolio of annuities on the same or on a different population.

Abstracting from operational expenses, pooling portfolios of policies written on the lives of different populations of policyholders allows to diversify longevity risks. Even if the increasing longevity trend is a widespread phenomenon across countries, the unexpected fluctuations in the mortality rates across countries may be substantially non-perfectly correlated when analysing general population data. This fact has been recently deeply investigated in the literature, that has been studying the modeling and hedging of the basis risk implied in standardized longevity transfer transactions (see for instance **?** or **?**). Up to our knowledge, such non-perfect correlation in the comovements of mortality rates has instead been neglected when considering the effects it can have on the valuation of internationally diversified liabilities portfolios of life insurers and on the portfolios of contracts of reinsurers.

Assessing whether these diversifications gains may be sizable is by no means a trivial task, though. To this end, we indeed first need to model the joint mortality dynamics of the policyholders of the domestic and of the foreign market. We consider

[1] In Europe, the creation of a common regulation framework in the middle of the Nineties gave rise to a wave of international expansions and M& A operations.

policyholders who are heterogeneous by cohort and select a parsimonious model, that extends the longevity basis risk model in **?**. The model fits well the observed mortality rates and is able to capture the imperfect correlations observed between the rates across ages and populations. We apply such model to estimate the joint dynamics of the mortality rates of UK and Italian males aged between 65 and 75. Based on the level of co-movement between the rates of the two populations, which is captured by a single parameter in our model, we define a similarity/diversification index between two annuity portfolios written on two different populations. This index measures the degree of diversification that is possibly reached and is increasing in the potential diversification gains obtained by merging the two portfolios. We then compare the values of the portfolio of the insurer/reinsurer in the two cases of domestic and international expansion. Reminiscent of the Solvency II regulation framework, we define such values as the sum of the actuarially fair values of the liabilities and a risk margin, computed as a value-at-risk (VaR)-based loading. The value difference between the two alternatives provides us with a measure of the diversification benefits. Our application, that considers the choice of a UK-based annuity provider, shows that the risk margin can reduce up to 2% as a proportion of the actuarial value, in the case of a foreign expansion to Italy. Also, longevity risk mitigation effects are shown to be more sizable when the interest rate is lower.

The paper unfolds as follows. Section 2 describes the problem of the insurer and the value of its portfolio in the alternative cases in which he expands domestically vs. internationally. Section 3 presents the mortality model and our diversification index measure. Section 4 provides a calibrated application, describing our model calibration procedure, which is based on RMSE minimization and Gaussian mapping, and the results of our analysis. Section 5 concludes.

2 The Insurer's Problem

We consider an Annuity Provider, or Life-Insurer, based in a generic Country (that we call *Domestic*), having a portfolio of deferred annuities written on different cohorts belonging to the Domestic Population. Let $\mathcal{X} = \{x_1, \dots, x_m\}$ be the set of annuitants' ages at time zero, and let n_i, for $i = 1, \dots, m$, be the number of annuities sold to people aged x_i. We consider a portfolio in which the oldest policyholders are 75:

$$x_i \leq 75 \quad \forall i = 1, \dots, m. \tag{1}$$

When an annuity is sold at time zero, the annuitant pays an initial premium $\Pi_i^0(0)$. After signing the contract, he will receive a series of fixed annual payments R, starting from the year-end of his 65-th birthday if $x_i < 65$, or immediately if $x_i \geq 65$, until his death, that may happen at most when he reaches a final age ω (for instance $\omega = 115$). In Europe, the life-insurance business is regulated by the Solvency II regulation, that requires insurers to value their liabilities at market value and set aside VaR-based risk margins with respect to both financial and longevity risk. To reproduce such situation, we assume that the value $\Pi^0(t)$ of the portfolio at time t is the sum of two components: the Actuarial Value of the contracts of portfolio $AV_{\Pi^0}(t)$, which is the sum of the actuarial values of each individual contract $N_i(t)$, and the Risk Margin

$RM_{\Pi^0}(t)$ of the portfolio itself, i.e.

$$\Pi^0(t) = AV_{\Pi^0}(t) + RM_{\Pi^0}(t) = \sum_{i=1}^{m} n_i N_i(t) + RM_{\Pi^0}(t). \tag{2}$$

The portfolio risk margin $RM_{\Pi^0}(t)$ is defined as the Value-at-Risk, at a certain confidence interval $\alpha \in (0,1)$, of the unexpected portfolio's future actuarial value at a given time horizon T:

$$\begin{aligned} RM_{\Pi^0}(t) &= VaR_\alpha\big(AV_{\Pi^0}(t+T) - \mathbb{E}_t[AV_{\Pi^0}(t+T)]\big), &(3)\\ &= inf\{l \in \mathbb{R} : P(AV_{\Pi^0}(t+T) - \mathbb{E}_t[AV_{\Pi^0}(t+T)] > l) < 1-\alpha\}. &(4) \end{aligned}$$

Let us consider an annuity contract sold at time $t \geq 0$ to an individual aged x_i. The number of years before the individual reaches age 65 is $\tau = 65 - x_i$. If $\tau \geq 0$, then the contract is a deferred annuity and its actuarial value is

$$N_i(t) = D(t, t+\tau)S_i(t, t+\tau)\left[R\sum_{u=1}^{\omega-t-\tau} D(t+\tau, t+\tau+u)S_i(t+\tau, t+\tau+u)\right]. \tag{5}$$

If $\tau < 0$, then the individual is already aged at least 65, the contract is an immediate annuity and its actuarial value is:

$$N_i(t) = R\sum_{u=1}^{\omega-t} D(t, t+u)S_i(t, t+u). \tag{6}$$

Formula (5) can be used in both cases by using the convention $\tau = max(65 - x_i, 0)$.

2.1 Portfolio Expansion

We consider the case in which the Insurer wants to increase the size of her annuity portfolio and needs to choose between two possible strategies. The first one is just sell new contracts to her Domestic population. In this case, we denote with n'_i the number of new contracts sold to people aged x_i, with Π^D the portfolio composed of just these new annuities, and with Π^1 the total portfolio of old and new contracts. It is easy to see that the actuarial value of the new portfolio is

$$AV_{\Pi^D}(t) = \sum_{i=1}^{m} n'_i N_i(t), \tag{7}$$

and

$$AV_{\Pi^1}(t) = AV_{\Pi^0}(t) + AV_{\Pi^D}(t). \tag{8}$$

The value of the total portfolio Π is the sum of the actuarial value of the old portfolio, the actuarial value of the new portfolio and the risk margin of the total portfolio:

$$\Pi^1(t) = AV_{\Pi^1}(t) + RM_{\Pi^1}(t) = AV_{\Pi^0}(t) + AV_{\Pi^D}(t) + RM_{\Pi^1}(t). \tag{9}$$

The second possible strategy is to acquire a new portfolio of annuities Π^F, written on a foreign population. To compare the two strategies, we simply assume that for each age x_i the number of annuities written on people aged x_i in the foreign population is still n'_i. The actuarial value of this portfolio is

$$AV_{\Pi^F}(t) = \sum_{i=1}^{m} n'_i N_i^F(t), \tag{10}$$

and

$$AV_{\Pi^2}(t) = AV_{\Pi^0}(t) + AV_{\Pi^F}(t). \tag{11}$$

Moreover,

$$\Pi^2(t) = AV_{\Pi^2}(t) + RM_{\Pi^2}(t) = AV_{\Pi^0}(t) + AV_{\Pi^F}(t) + RM_{\Pi^2}(t). \tag{12}$$

Our objective is to compare these two strategies in order to study and quantify the effects of diversification in annuity portfolios.

3 Longevity Risk Modeling

We now turn to the description of the source of uncertainty that affects the value of the Insurer's portfolio: the risk of longevity, i.e. the risk that her policyholders live longer than expected. We set ourselves in the well-established continuous-time stochastic mortality setting initiated by Milevsky and Promislow (2001) that models the death of individuals as a Cox process. The time to death of an individual τ_{x_i} belonging to cohort x_i is the first jump time of a Poisson process with stochastic intensity. Let us consider two populations, each containing m different cohorts. The first population is called the *Domestic population* and the second one is called the *Foreign population*. A given cohort i, with $i = 1, \ldots, m$, belonging to one of the two populations, is identified by the (common) initial age x_i at time zero. The set $\mathcal{X}$ of initial ages is common to the two populations.

Domestic Population

The mortality intensity of each cohort x_i, for $i = 1, \ldots, m$, belonging to the Domestic population is denoted with $\lambda^d_{x_i}$, or simply λ^d_i, and follows a non-mean reverting CIR process:

$$d\lambda^d_{x_i}(t) = (a_i + b_i\lambda^d_{x_i}(t))dt + \sigma_i\sqrt{\lambda^d_{x_i}(t)}dW_i(t), \tag{13}$$

where $a_i, b_i, \sigma_i,\ \lambda^d_{x_i}(0) \in \mathbb{R}^{++}$ are strictly positive real constants and the W_i's are instantaneously correlated standard Brownian Motions, i.e. $dW_i(t)dW_j(t) = \rho_{ij}dt$ with $i, j \in \{1, \ldots, m\}$. This implies that the mortality intensities of two different cohorts belonging to the Domestic Population are instantaneously correlated.

Foreign Population

The mortality intensity of cohort x_i belonging to the Foreign population is denoted with $\lambda^f_{x_i}$, or simply with λ^f_i, and is given by the convex combination of the mortality intensity of the corresponding cohort belonging to the Domestic population $\lambda^d_{x_i}$ and an idiosyncratic component λ' which affects the Foreign population only and that depends on the initial age x_i in a deterministic way, i.e.

$$\lambda^f_{x_i}(t) = \delta_i \lambda^d_{x_i}(t) + (1 - \delta_i)\lambda'(t; x_i), \tag{14}$$

where

$$d\lambda'(t; x_i) = (a(x_i; a') + b(x_i; b')d\lambda'(t; x_i))dt + \sigma(x_i; \sigma', \gamma')\sqrt{d\lambda'(t; x_i)}dW'(t), \tag{15}$$

with $\delta_i \in [0, 1]$. The functions $a(x_i; a')$, $b(x_i; b')$, and $\sigma(x_i; \sigma', \gamma')$ are deterministic functions of the initial age x_i and, for every x_i with $i = 1, \dots, N$, they depend on the parameters $a' > 0$, $b' > 0$, $\sigma' > 0$ $\gamma' > 0$ respectively. W' is a standard Brownian Motion, that is assumed to be independent of W_i for each $i = 1, \dots, N$.
Intuitively, the idiosyncratic risk source W' is population specific in the sense that it is common to all the cohorts of the Foreign population. Two remarks are important.

- The idiosyncratic component $\lambda'(t; x_i)$ affects differently each cohort. The different impact on each cohort depends deterministically on their initial age x_i through the functional forms of $a(x_i; a')$, $b(x_i; b')$, and $\sigma(x_i; \sigma', \gamma')$;
- Each cohort x_i has, in general, a specific sensitivity to $\lambda'(t; x_i)$, which is given by the parameter δ_i.

Thus, the mortality intensities of two different cohorts of the Foreign population are correlated, because so are the corresponding cohorts of the Domestic population. In general, the correlation between $\lambda^f_{x_i}$ and $\lambda^f_{x_j}$ is different from the correlation between $\lambda^d_{x_i}$ and $\lambda^d_{x_j}$ because of the weights δ_i and δ_j. Moreover, we also model the non-perfect correlation between cohorts across the two populations, because of the presence of the idiosyncratic component λ' affecting the Foreign population.

The functional forms of $a(x_i; a')$, $b(x_i; b')$, and $\sigma(x_i; \sigma', \gamma')$ should be chosen in order to capture cohort effects in the Foreign Population. A possible choice is:

$$\begin{aligned} a(x_i; a') &= a' x_i, \\ b(x_i; b') &= b', \\ \sigma(x_i; \sigma') &= \sigma' e^{\gamma' x_i}, \end{aligned}$$

which means that the drift of $\lambda'(t; x_i)$ is linearly increasing with x_i and its diffusion coefficient is instead exponentially increasing with x_i. However, the simplest specification of the model, which we adopt in our application, is:

$$\begin{aligned} a(x_i; a') &= a', \\ b(x_i; b') &= b', \\ \sigma(x_i; \sigma') &= \sigma'. \end{aligned}$$

3.1 Similarity/Diversification index

Building up on the longevity model described in the previous section, we propose a synthetic measure to describe the similarity/dissimilarity between the annuity portfolios written on two populations, that we define as *Similarity* index. Let n^d_i be the number of annuities written on cohort x_i belonging to the domestic population, n_i^f the number of annuities written on cohort x_i belonging to the foreign population, $n_i = n_i^d + n_i^f$ and m^f the number of generation in the foreing portfolio with non-zero n_i^f. Then the Similarity Index (SI) is equal to:

$$SI = 1 - \frac{1}{m^f} \sum_{i=1}^{m} 1 - \frac{n_i^d + n_i^f \delta_i}{n_i}. \tag{16}$$

If $\delta_i = 1$ for every i, which means that the two portfolios are written on the same population, then, obviously, $SI = 1$. On the other hand, if $\delta_i = 0$ for every i, and $n_i^f \to \infty$ while n_i^d remains constant, we have that $SI \to 0$. We then define the Diversification index (DI) as the complement to 1 of the Similarity Index, interpreting it as a measure of the dissimilarity of the two portfolios. The higher the DI, the higher the diversification benefit that we should be expecting by coupling two portfolios.

4 Application

In this section, we try to quantify the diversification gains in an annuity portfolio in which UK is the Domestic country and Italy is the foreign one. We consider portfolios composed by 11 different cohorts: $x_i = 65, \ldots, 75$.

4.1 Mortality intensities estimation

We calibrate the parameters of the mortality model to the generations of UK and Italian males whose age, at 31/12/2012, is between 64 and 74, that is, the cohorts born between 1937 and 1947. We use the 1-year$\times$1-year cohort death rates data provided by the Human Mortality Database. The estimation of the parameters is carried out by minimizing the RMSE between the empirical and the theoretical survival probabilities using, for each cohort, the 20 observations from 1993 and 2012[2].

Table 1. Domestic Population (UK) calibration results.

Age	a	b	σ	λ_0	RMSE
65	$2.7878 \cdot 10^{-5}$	0.0723	0.0075	0.0116	0.00035
66	$6.5423 \cdot 10^{-5}$	0.0652	0.0059	0.0124	0.00028
67	$1.8424 \cdot 10^{-5}$	0.0740	0.0080	0.0135	0.00035
68	$5.3144 \cdot 10^{-5}$	0.0685	0.0084	0.0160	0.00043
69	$1.2500 \cdot 10^{-4}$	0.0589	0.0091	0.0164	0.00039
70	$8.4734 \cdot 10^{-5}$	0.0646	0.0108	0.0189	0.00056
71	$7.1323 \cdot 10^{-5}$	0.0667	0.0106	0.0212	0.00038
72	$4.1759 \cdot 10^{-5}$	0.0688	0.0073	0.0239	0.00040
73	$2.2984 \cdot 10^{-5}$	0.0689	0.0066	0.0262	0.00063
74	$9.6036 \cdot 10^{-5}$	0.0663	0.0131	0.0282	0.00040
75	$3.3898 \cdot 10^{-5}$	0.0684	0.0077	0.0316	0.00049

[2] These correspond to the last 20 observations for the italian males. However, since the UK dataset is updated until 31/12/2013, we have excluded the last available observation for the UK cohorts.

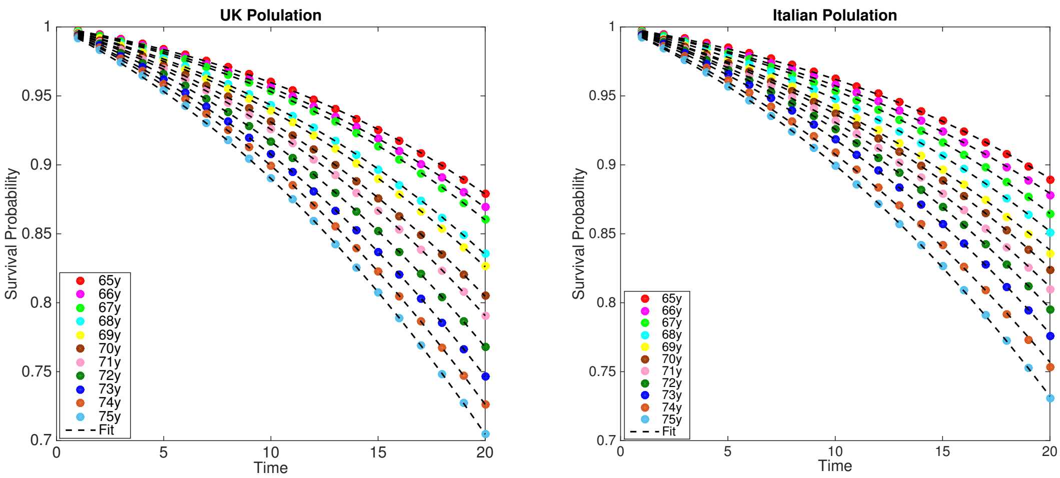

Figure 1. Observed and theoretical survival probabilities. The left panel shows the observed vs. fitted survival probabilities for the Foreign population, while the right reports the figures for the Domestic population.

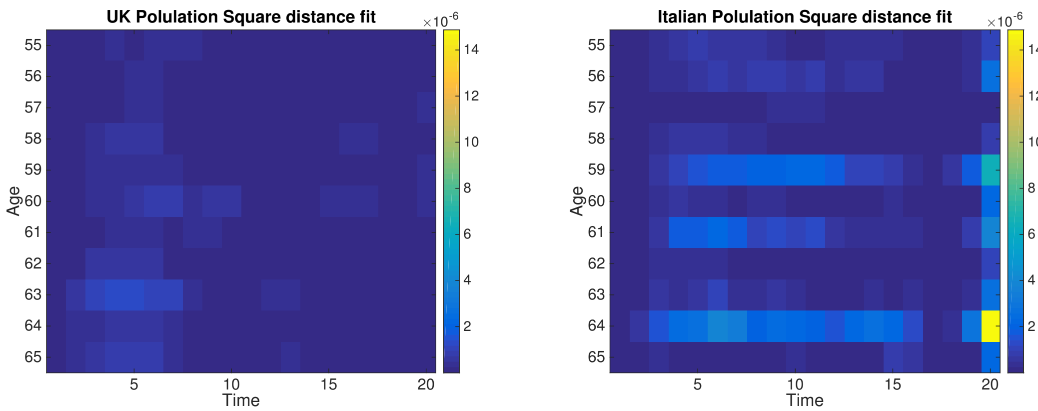

Figure 2. Calibration errors.

Table 2. Foreign Population (IT) calibration results.

Age	a'	b'	σ'	δ	RMSE	λ'_0
65				0.8071	0.00060	0.0075
66				0.8036	0.00073	0.0127
67				0.9348	0.00031	0.0190
68				0.8074	0.00045	0.0115
69				0.7893	0.00120	0.0163
70	$5.8458 \cdot 10^{-5}$	$4.2841 \cdot 10^{-11}$	$1.1464 \cdot 10^{-7}$	0.8119	0.00053	0.0141
71				0.7903	0.00099	0.0124
72				0.8006	0.00039	0.0092
73				0.8106	0.00064	0.0115
74				0.7622	0.00160	0.0209
75				0.8470	0.00053	0.0182

4.2 Correlation matrix estimation

To be able to estimate the instantaneous correlations implied by our longevity risk model, we apply the Gaussian Mapping technique, which we describe in the next section.

4.2.1 Gaussian Mapping

The Gaussian Mapping technique has been used in CDS pricing (see Brigo and Mercurio (2001)). It consists in mapping a CIR process into a Vasicek process that is as *close* as possible to the original one. While the meaning of *close* will be explained in what follows, we clarify that our objective is to use the mapped Vasicek processes to compute analytically the correlation between λ_i^d and λ_j^d, with $i, j = 1, \ldots, N$.
Starting from the CIR process (13) describing the mortality intensity of cohort x_i belonging to the domestic population, we consider a Vasicek process driven by the same Brownian Motion $W_i(t)$, having the same drift and the same initial point:

$$d\lambda_i^V(t) = (a_i + b_i\lambda_{x_i}^V(t))dt + \sigma_i^V dW_i(t), \quad \lambda_i^V(0) = \lambda_{x_i}^d(0). \tag{17}$$

The instantaneous volatility coefficient σ_i^V of (17) is then determined by making the two processes are as close as possible. Here, by close we mean that the two processes agree on the survival probability for a fixed maturity T:

$$S_i^d(t,T) = S_i^V(t,T;\sigma_i^V). \tag{18}$$

Then, we approximate the correlation between $\lambda_i^d(t)$ and $\lambda_j^d(t)$ by the correlation between $\lambda_i^V(t)$ and $\lambda_j^V(t)$:

$$Corr_0(\lambda_i^d(t), \lambda_j^d(t)) \approx Corr_0(\lambda_i^V(t), \lambda_j^V(t)), \tag{19}$$

since this last correlation can be computed analytically. A simple application of Itô's Lemma allows us to show that the solution to the SDE (17) is given by:

$$\lambda_i^V(t) = \lambda_i^V(0)e^{b_i t} + \frac{a_i}{b_i}\left(1 - e^{b_i t}\right) + \sigma_i^V \int_0^t e^{b_i(t-s)}\, dW_i(s). \tag{20}$$

Therefore, we have that:

$$\mathbb{E}_0\big[\lambda_i^V(t)\big] = \lambda_i^V(0)e^{b_i t} + \frac{a_i}{b_i}\big(1 - e^{b_i t}\big) \tag{21}$$

$$Var_0\big[\lambda_i^V(t)\big] = \frac{(\sigma_i^V)^2}{2b_i}\sqrt{e^{2b_i t} - 1}. \tag{22}$$

Since $\lambda_i^V(t) - \mathbb{E}_0\big[\lambda_i^V(t)\big] = \sigma_i^V \int_0^t e^{b_i(t-s)}\, dW_i(s)$, the covariance between $\lambda_i^V(t)$ and $\lambda_j^V(t)$ is:

$$\begin{aligned}
Cov_0(\lambda_i^V(t), \lambda_j^V(t)) &= \mathbb{E}_0\left[\sigma_i^V\sigma_j^V\Big(\int_0^t e^{b_i(t-s)}\, dW_i(s)\Big)\Big(\int_0^t e^{b_j(t-s)}\, dW_j(s)\Big)\right] \\
&= \mathbb{E}_0\left[\sigma_i^V\sigma_j^V\rho_{ij}\int_0^t e^{(b_i+b_j)(t-s)}\, ds\right] \\
&= \sigma_i^V\sigma_j^V\rho_{ij}\int_0^t e^{(b_i+b_j)(t-s)}\, ds \\
&= \frac{\sigma_i^V\sigma_j^V\rho_{ij}}{b_i+b_j}\Big(e^{(b_i+b_j)t} - 1\Big).
\end{aligned}$$

Finally, we have:

$$\begin{aligned}
Corr_0(\lambda_i^V(t), \lambda_j^V(t)) &= \frac{Cov_0(\lambda_i^V(t), \lambda_j^V(t))}{\sqrt{Var_0\big[\lambda_i^V(t)\big]Var_0\big[\lambda_j^V(t)\big]}} \\
&= \frac{2\rho_{ij}}{b_i+b_j}\cdot\frac{e^{(b_i+b_j)t} - 1}{\sqrt{\frac{(e^{2b_i t}-1)(e^{2b_j t}-1)}{b_i b_j}}}
\end{aligned} \tag{23}$$

4.2.2 Instantaneous correlation estimates

Using the central mortality rates data available in the UK life tables[3], we estimate the instantaneous correlation ρ_{ij} between $d\lambda_i$ and $d\lambda_j$ by inverting the approximated correlation expression (23). To compute the correlations between our 11 cohorts, we start from the central mortality rates in 1958 of the people aged between 1 and 11, and we follow the diagonal of the life table until we reach the central mortality rates of the people aged between 65 and 75 in 2012. The central mortality rates table constructed in this way has dimensions 55×11 and the estimated correlation coefficients are reported in table 3. Correlations are closet to 1, but they tend to decrease with the distance between the initial ages of the two considered cohorts.A graphical representation of the correlation matrix is also provided in Figure 3.

4.3 Evaluating the diversification gains

In this section, we describe our simulation assumptions and present the evaluation of the geographical diversification effect. We consider a simulation time horizon of 30 years, a constant interest rate of 2% and we disregard the modelling of interest rate

[3]Source: Human Mortality Database

Table 3. Correlation matrix UK Population.

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
65	1	0.9990	0.9983	0.9983	0.9973	0.9969	0.9972	0.9964	0.9962	0.9959	0.9957
66	0.9990	1	0.9992	0.9988	0.9985	0.9979	0.9977	0.9970	0.9970	0.9967	0.9960
67	0.9983	0.9992	1	0.9989	0.9988	0.9983	0.9983	0.9977	0.9976	0.9974	0.9964
68	0.9983	0.9988	0.9989	1	0.9993	0.9989	0.9987	0.9984	0.9985	0.9983	0.9974
69	0.9973	0.9985	0.9988	0.9993	1	0.9995	0.9988	0.9987	0.9988	0.9989	0.9978
70	0.9969	0.9979	0.9983	0.9989	0.9995	1	0.9986	0.9986	0.9989	0.9991	0.9981
71	0.9972	0.9977	0.9983	0.9987	0.9988	0.9986	1	0.9994	0.9992	0.9991	0.9990
72	0.9964	0.9970	0.9977	0.9984	0.9987	0.9986	0.9994	1	0.9997	0.9995	0.9996
73	0.9962	0.9970	0.9976	0.9985	0.9988	0.9989	0.9992	0.9997	1	0.9996	0.9994
74	0.9959	0.9967	0.9974	0.9983	0.9989	0.9991	0.9991	0.9995	0.9996	1	0.9995
75	0.9957	0.9960	0.9964	0.9974	0.9978	0.9981	0.9990	0.9996	0.9994	0.9995	1

risk or exchange rates to focus on the impact of longevity risk. This choice is made to isolate and capture any possible added benefit specifically due to the geographic diversification of an annuity portfolio. The time horizon at which the Risk Margin is computed is 15 years. Consistently with the Solvency II regulation, we select a 99.5% confidence interval when calculating the Risk Margin associated to the portfolio.

Initial Portfolio

We consider a UK Insurer with an initial portfolio Π^0, composed of contracts sold to males whose age, at 31/12/2012, is between 65 and 75. 100 contracts are sold to each generation and, therefore, the initial portfolio is composed of 1100 contracts. The initial Actuarial Value $AV_{\Pi^0}(0)$ of the portfolio is:

$$AV_{\Pi^0}(0) = 1.5288 \cdot 10^4, \tag{24}$$

while the Risk Margin computed at time 0 with a time horizon of 15 years and confidence interval $\alpha = 99.5\%$ is

$$RM_{\Pi^0}(0) = 1.3018 \cdot 10^3. \tag{25}$$

Hence, the initial portfolio value is

$$\Pi^0(0) = AV_{\Pi^0}(0) + RM_{\Pi^0}(0) = 1.6590 \cdot 10^4. \tag{26}$$

We observe that, in this case, the Risk Margin accounts for 8.52% of the initial portfolio Actuarial Value. The Risk Margin represents the amount of money that the Insurer needs to set aside in order to ensure its solvency (with a probability of 99.5%) in case of unexpected increase in longevity over a 15 years period.

Domestic Expansion

With a Domestic Expansion, the Insurer doubles the size of its annuity portfolio, selling additional policies to her domestic population, i.e. the UK population. The

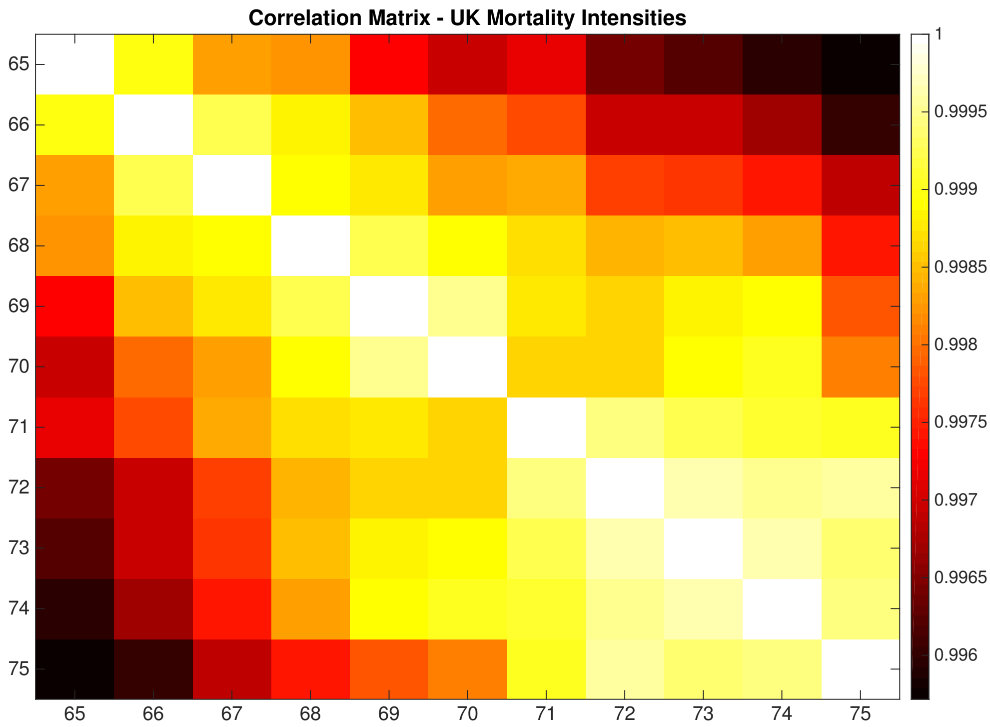

Figure 3. Estimated instantaneous correlation coefficients UK population.

new portfolio Π^1 is, therefore, composed of 2200 contracts, 200 for each generation. Hence,

$$AV_{\Pi^1}(0) = 3.0576 \cdot 10^4, \tag{27}$$

$$RM_{\Pi^1}(0) = 2.6036 \cdot 10^3, \tag{28}$$

$$\Pi^1(0) = 3.3179 \cdot 10^4. \tag{29}$$

The Risk Margin proportion relative to actuarial value is unaffected by the size of the portfolio, and still accounts for 8.52% of the Actuarial Value of the Domestically Expanded portfolio. In this case, we obviously have that the diversification index between Π^0 and $\Pi^1 - \Pi^0$ is 0, as no diversification gains can be obtained.

Foreign Expansion

In the case of a Foreign Expansion,the Insurer doubles the number of policies in its annuity portfolio by selling contracts written on policyholders belonging to the Foreign population, namely the Italian one. The new portfolio Π^2 is, therefore, composed of 1100 contracts sold to the UK population (100 for each generation) and of 1100 contracts sold to the Italian population (100 for each generation). Hence,

$$AV_{\Pi^2}(0) = 3.1252 \cdot 10^4, \tag{30}$$

$$RM_{\Pi^2}(0) = 2.4602 \cdot 10^3, \tag{31}$$

$$\Pi^2(0) = 3.3712 \cdot 10^4. \tag{32}$$

In this case, the Risk Margin accounts for 7.87% of the Actuarial Value of the Domestically Expanded portfolio. Now we have that the diversification index between Π^0 and $\Pi^2 - \Pi^0$ is 0.0925. Obviously, the size and the composition of the Foreign Portfolio affect the diversification gains. We then compute the values and risk margins of three additional portfolios, obtained by altering the number of policies sold to each different cohorts in the two populations. Table 4 summarizes the results, reporting the actuarial values, risk margins and total values of the portfolio. The column $\%RM$ represents the Risk Margin, expressed as a percentage of the Actuarial Value of the portfolio. Portfolio Π^3 is obtained fixing the number of policies sold to each generation of foreign policyholders as twice the number of policies in the initial domestic portfolio. Portfolio Π^F is an extreme case, exposed only to the foreign population useful for comparison. The portfolio Π^1_{opt} is obtained, like Π^1, selling additional policies to the Domestic population. However, it is obtained by optimizing the composition of the added contracts in order to minimize the risk margin, under the constraint that the number of the new contracts is to be 1100. It can then be considered as the maximally diversified portfolio, in the absence of geographical diversification. Its risk margin is 7.53% relative to value and is obtained by adding 1100 annuities sold to the UK 65 years old. Similarly, Π^2_{opt} is obtained by allowing for geographical diversification and optimizing the composition of the Foreign, Italian portfolio. The risk margin is 6.89%, obtained by selling the whole 1100 contracts to the Italian males aged 65. The DI of this last portfolio is the highest among the portfolios analyzed, 0.1768.

Table 4. Effects of Geographic Diversification ($r = 2\%$)

Portfolio	AV	RM	Π	$\%RM$	DI
Π^0	$1.5288 \cdot 10^4$	$1.3018 \cdot 10^3$	$1.6590 \cdot 10^4$	8.52%	-
Π^1	$3.0576 \cdot 10^4$	$2.6036 \cdot 10^3$	$3.3179 \cdot 10^4$	8.52%	0
Π^2	$3.1252 \cdot 10^4$	$2.4602 \cdot 10^3$	$3.3712 \cdot 10^4$	7.87%	0.0925
Π^3	$4.7217 \cdot 10^4$	$3.6186 \cdot 10^3$	$5.0835 \cdot 10^4$	7.66%	0.1233
Π^F	$3.1929 \cdot 10^4$	$2.3168 \cdot 10^3$	$3.4245 \cdot 10^4$	7.26%	-
Π^1_{opt}	$3.3002 \cdot 10^4$	$2.4842 \cdot 10^3$	$3.5486 \cdot 10^4$	7.53%	0
Π^2_{opt}	$3.3667 \cdot 10^4$	$2.3199 \cdot 10^3$	$3.5986 \cdot 10^4$	6.89%	0.1768

For the sake of completeness, Table 5 reports the results under the assumption of zero interest rate, i.e. $r = 0\%$.

Table 5. Effects of Geographic Diversification ($r = 0\%$)

Portfolio	AV	RM	Π	$\%RM$
Π^0	$1.9097 \cdot 10^4$	$2.1318 \cdot 10^3$	$2.1228 \cdot 10^4$	11.16%
Π^1	$3.8193 \cdot 10^4$	$4.2636 \cdot 10^3$	$4.2457 \cdot 10^4$	11.16%
Π^2	$3.9189 \cdot 10^4$	$4.0378 \cdot 10^3$	$4.3227 \cdot 10^4$	10.30%
Π^3	$5.9282 \cdot 10^4$	$5.9437 \cdot 10^3$	$6.5226 \cdot 10^4$	10.03%
Π^F	$4.0185 \cdot 10^4$	$3.8119 \cdot 10^3$	$4.3997 \cdot 10^4$	9.49%
Π^1_{opt}	$4.1683 \cdot 10^4$	$4.1488 \cdot 10^3$	$4.5831 \cdot 10^4$	9.95%
Π^2_{opt}	$4.2689 \cdot 10^4$	$3.8787 \cdot 10^3$	$4.6568 \cdot 10^4$	9.09%

Under this lower interest rate level, the magnitude of longevity risk is more severe, as expected: the percentage Risk Margins are higher for all portfolios, increasing in the best-case scenario to 9.09%, up from 6.89%.

5 Conclusions

The life insurance industry and the actuarial literature have recently focused on the basis risk arising in longevity transfers due to the non-perfect correlations between two populations. In this paper, we devoted our attention to the other side of the story, that is the possible benefits in geographically diversified portfolios, due to such non-perfect correlation between populations. Thus, we considered the problem of an insurer who has to decide whether to expand his liability portfolio in the country where it is based and runs its activity or in a foreign country. We pointed out that some diversification gains may be realized when expanding internationally, due to the mitigation of the exposure to longevity risk. To discuss whether these gains may be relevant in an annuity portfolio, we first proposed a longevity risk model that, while being parsimonious, is able to capture the non-perfect correlations among the different cohorts of a single generation and between the members of two different populations. We computed the risk margin coherently with the Solvency II internal modelling approach, as a loading on the actuarial value of the portfolio, computed as a 99.5% VaR of the whole portfolio value. Our application, based on an annuity portfolio written on the UK and the Italian populations, shows that a non-negligible reduction of the portfolio risk margins can be obtained by expanding internationally. Under a 0% interest rate assumption, we show that an optimally chosen foreign portfolio, on top of the domestic one, can lower the risk margin by more than 2% as a percentage of the actuarial value of the portfolio. This example can be considered as a conservative one, since the two male populations of UK and Italy present rather similar historical mortality dynamics. The diversification effect is shown to be more relevant, the lower the risk-free interest rate. Our paper contributes to the literature on the modeling of longevity risk, by proposing a multi-population model that captures the dependence structure within and across populations. It contributes to the understanding of longevity risk management, by pointing out that, although the same increasing longevity trend is common to the populations of almost every country, geographical diversification may lead to some benefit. Our model is able to quantify such effects. Finally, even though we framed our problem as representing the decision of an insurer, analogous consideration can be made in the case of a reinsurer willing to provide insurance to portfolios of policies written on different populations. This can have implications for securitization and for the structuring of longevity derivative products.

References

Biener, C., M. Eling, and R. Jia (2015). Globalization of the life insurance industry: Blessing or curse?

Brigo, D. and F. Mercurio (2001). *Interest rate models : theory and practice*. Springer finance. Berlin, Heidelberg, Paris: Springer.

Cummins, J. D., S. Tennyson, and M. A. Weiss (1999). Consolidation and efficiency in the us life insurance industry. *Journal of Banking & Finance 23*(2), 325–357.

Cummins, J. D. and X. Xie (2008). Mergers and acquisitions in the us property-liability insurance industry: Productivity and efficiency effects. *Journal of Banking & Finance 32*(1), 30–55.

Milevsky, M. A. and S. D. Promislow (2001). Mortality derivatives and the option to annuitise. *Insurance: Mathematics and Economics 29*(3), 299–318.

Outreville, J. F. (2008). Foreign affiliates of the largest insurance groups: Location-specific advantages. *Journal of Risk and Insurance 75*(2), 463–491.

Outreville, J. F. (2012). A note on geographical diversification and performance of the world's largest reinsurance groups. *Multinational Business Review 20*(4), 376–391.