Construction of Mortality Indexes

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DO'S AND DON'TS

The Life Market

The Life and Longevity Markets Association was set up in 2010 to promote the development of a liquid life market, where longevity-linked securities and liabilities are traded. It is hoped that via a deeper and more mature market, insurers and reinsurers can spread their longevity risk more readily amongst one another and those investors who want to diversify their portfolios with an uncorrelated market sector.

The Role of Mortality Indexes

Like many successful financial markets, it is indispensable to have transparent and widely adopted market indicators or indexes in place. With appropriate mortality indexes, one can construct standardized mortality-linked securities, such as mortality forwards and swaps, through which can be traded. Relative to custom-made hedging instruments, standardized securities are easier for investors to analyze and are more conducive to the development of liquidity. Attempts to create mortality indexes have been made by a number of investment banks, including Credit Suisse (an index based on the US life expectancy at birth), Goldman Sachs (the QxX Index), JP Morgan (the LifeMetrics Index), and Deutsche Börse (the Xpect Cohort Index).

What Makes a Good Mortality Index?

The mortality indexes mentioned earlier are all model- free. Although this may be seen as an advantage, a model-free index can only convey a limited amount of information. For example, an index that is based on the life expectancy at birth summarizes mortality improvement at the aggregate level, while indicates nothing about how the underlying mortality curve has actually evolved over time. Since shifts in a mortality curve are typically not uniform, an insurer may find it difficult to build a longevity hedge with such an index.

To represent the evolution of a mortality curve in a non-parametric way, there is a need to use a rather large number of indexes, such as those that are linked directly to death rates at different ages. However, if the number of indexes is large, tracking and concentrating liquidity would be difficult.

In contrast, a model-based construction method has the potential to improve the information content of mortality indexes. In recent years, several extrapolative stochastic mortality models have been developed. Generally, when they are fitted to historical data, one or more series of time-varying parameters are identified. By extrapolating these parameters into the future, a mortality forecast can be obtained. These parameters, which contain rich information about how the mortality of a population moves over time, can potentially be used as mortality indexes.

Not all stochastic mortality models are suitable for creating indexes. There are a number of criteria that should be satisfied. First, despite being small in dimension, the vector of indexes should represent the varying age-pattern of mortality improvement, not just the overall mortality level. Second, the model should possess what we refer to as the 'new-data- invariant' property. It means that when an additional year of mortality data becomes available and the model is updated accordingly, the index values in previous years will not be affected. This criterion is of crucial importance, as it would be impossible to track an index if its historical values are revised from time to time. Third, the indexes should be readily interpretable, so that they can be communicated to hedgers and investors at ease. Other criteria require that the indexes should be unambiguous, transparent, objective, measurable, timely, appropriate, popular, relevant, and stable.

The CBD Mortality Indexes

The Cairns-Blake-Dowd (CBD) model is a simple mortality model that fits the mentioned criteria. The CBD model is specified as:

$$\ln\left(\frac{q(x,t)}{1-q(x,t)}\right) = k(1,t) + k(2,t)\,(x-\bar{x})$$

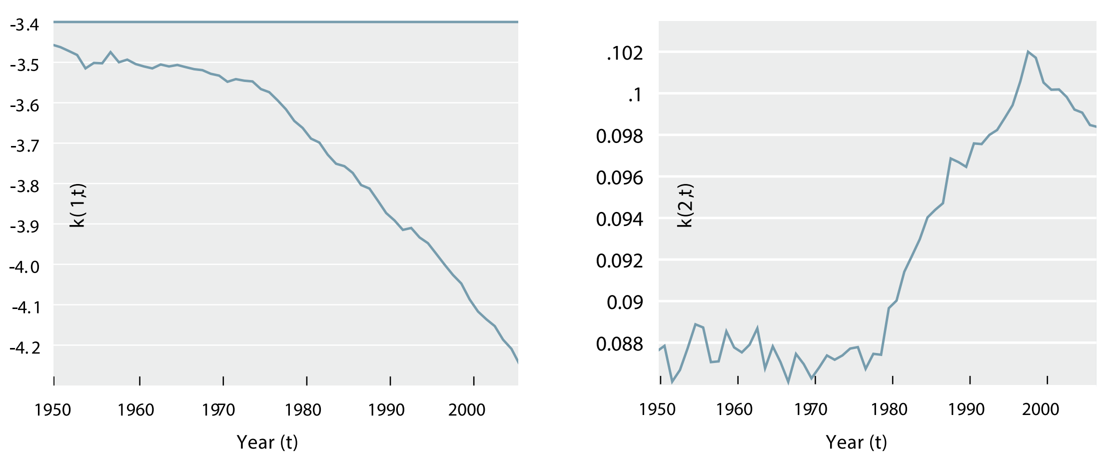
The term q(x,t) represents the (conditional) probability of death at age x in year t and is the average age over the sample age range. The model's two time-varying parameters, denoted by k(1,t) (intercept) and k(2, t) (slope), can be used jointly as mortality indexes.

Figure 1 plots the time-varying parameters that are calibrated using data from the Canadian male population (ages 40 to 90). It can be seen that k(1,t) tends to decrease but k(2,t) tends to increase over time. The former indicates the overall mortality improvement across time, and the latter suggests that mortality declines faster at younger ages than at older ages. The two mortality indexes, when considered jointly, can capture different patterns of shifts in the underlying mortality curve. A curvature term (i.e. a third index) may also be added to the model structure where necessary. In spite of all the advantages, the CBD model, like many other mathematical models, may not give a 'perfect' fit to the data. What makes it stand out is mainly its new-data-invariant property, which allows an investor or hedger to track the index values. Using the CBD model to create mortality indexes can be compared with applying the Black-Scholes option pricing model to construct implied volatility indexes. Both models may not be completely accurate, but they do produce well-defined indexes that are tractable by investors and other users.Interested readers are referred to the first paper in the list of references for further information concerning the CBD mortality indexes.

K-forwards

Longevity risk exposures may be transferred by trading standardized derivative securities that are linked to the two CBD mortality indexes. In particular, one can consider a security called K-forward, a zero- coupon swap that exchanges on the maturity date a pre-determined fixed amount for a random amount that is proportional to a CBD mortality index. For instance, to the fixed receiver, the payoff from a K-forward that is linked to the ith CBD index is \$(X (k(i,T)-k(i,T)), where X is the notional amount, T is the maturity date, and $\hat{k}$ (i,T) is the forward value. Payouts from a combination of properly calibrated K-forward contracts can potentially offset the undesired future outcomes of an annuity portfolio or a pension plan. The second paper in the list of references details how a K-forward longevity hedge can be calibrated and how effective such a hedge is.

Figure 1
Estimated CBD mortality indexes for Canadian males



References

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Tan, C.I., Li, J, Li, J.S.-H. and Balasooriya, U. (2014). Parametric Mortality Indexes: From Index Construction to Hedging Strategies. Insurance: Mathematics and Economics 59: 285-299.