Discount Rate Regulation for Canadian Private Defined Benefit Pension Plans

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EXECUTIVE SUMMARY

In Canada, private DB plans follow the Canadian Institute of Chartered Accountants' (CICA) guidelines for calculating their accounting liabilities. According to regulatory guidance CICA 3461, private DB plans should use the yield of high quality fixed income securities that generate cash flows matching the expected amount and timing of the plan's benefit payments to discount pension liabilities.

This regulation is challenged by the incomplete long-term bond market. Since DB pension plans are often confronted with ultra-long-term commitments with maturities of more than 70 years, the valuation of DB pension liabilities rely on ultra-long term (30 years +) interest rates. The market for these ultra-long-term instruments is much less liquid and in Canada high-quality corporate bonds with maturities of more than 10 years virtually non-existent.

Due to the incompleteness of the long-term bond market, it is necessary to extrapolate the term structure of the corporate bond interest rates beyond the maturity of the longest dated market-available instrument. The current extrapolation method (according to a CIA Educational Note)[1] that extends the smoothed or regressed function beyond the LLP to complete the yield curve, can be very risky and chaotic. Under both interpolation and extrapolation, smoothing of the discontinuous bond prices is susceptible to both parameter and model risk.

This paper proposes to use an ultimate forward rate (UFR) method to extrapolate the corporate yield. This is a subjective method that extrapolates the liquid market interest rates such that they converge in the long run to an unconditional ultimate forward rate. Different from the UFR method adopted in Europe, the Canadian UFR includes additional default risk premium so as to adjust the spread between government and corporate long-term yields. Table below lists the five core components of the Canadian UFR. A survey study suggests that setting UFR to 4.7% is suitable to the Canadian market.

COMPONENT	RATE
Real expected short-term interest rate	2.2%
Long term expected inflation	2.0%
Long term nominal term premium	0.1%
Long term nominal convexity effect	-0.2%
Long term default risk premium	0.6%
Canadian Ultimate Forward Rate	**4.7%**

The adoption of UFR provides two possible benefits: First, it effectively shrinks the pricing volatility caused by model risk and it stabilize the liability valuation under different phases of business cycles. Second, it also reduces the required funding under the low rate environment.

For additional detail, please see the **full report below.**

1 *The Canadian Institute of Actuaries (CIA), Accounting Discount Rate Assumption for Pension and Postemployment Benefit Plans, (SEPT 2011)*

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Abstract

Due to the scarcity of the long-term market instruments, the valuation of private defined-benefit (DB) pension liabilities requires an extrapolation of the yield curve. In Canada, corporate yields are adopted to discount the private DB pension liabilities, but the issue on how to extrapolate the yield curve beyond the market liquid point has not been clearly addressed in the regulatory guidance. This paper introduces a macroeconomic extrapolation method called "the Canadian ultimate forward rate" to complete the yield curve. The new method effectively reduces the valuation volatility for it is robust against interpolation models and instantaneous market distortions.

Keywords: corporate yield, extrapolation, ultimate forward rate

1 Introduction

By the end of 2016, 54% of Canadian pension funds were private defined benefit (DB) plans. These private DB funds invest plan assets on behalf of approximately 20% of working and retired Canadian registered pension members, which is equivalent to 1.26 million pension participants. Accordingly, accurate valuation of private DB fund liabilities is of first-order importance for the Canadian pension landscape. In Canada, private DB plans follow Canadian Institute of Chartered Accountants (CICA) guidelines to calculate their accounting liabilities. According

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to regulatory guidance CICA 3461[1], private DB plans should use the yield on high quality [2] fixed income securities that generate cash flows matching the expected amount and timing of payments from the pension plan.

This regulation is challenged by the incomplete long-term bond market. Since DB pension plans are often confronted with ultra-long-term commitments with maturities of more than 70 years, the valuation of DB pension liabilities relies on ultra-long term (30 years and longer) interest rates. The market for ultra-long-term instruments is less liquid. Blommestein (2007) shows that only 1.8 per cent of Canadian government bonds outstanding have residual maturities greater than 30 years by 2005. The "last liquid point" (LLP) on the yield curve is the longest maturity for which the market rates are applied. In Canada, the LLP is 20 years for government bonds and only 10 years for the corporate bonds. In September 2017, 176 high-quality Canadian corporate bonds were traded in the market, of which 41 had maturities longer than 10 years. However, only 2 of these high-quality long-term bonds have more than $ 500 million in value outstanding.

Due to the scarcity of high quality corporate bonds that have maturities of more than 10 years, the Canadian Institute of Actuaries (CIA) released an educational note which provides several methods to extrapolate the long end of the corporate yield curve. These methods seek to obtain more data points beyond the LLP and to supplement the Canadian high quality corporate bonds with U.S. long-term corporate bonds or Canadian long-term provincial bonds followed by an exchange rate adjustment or a spread adjustment. The methods then fit a yield curve to the available bond data using smoothing or regression methods. However, it is still uncertain what should be done to address the mismatch of maturities between pension liabilities and the respective assets used to discount them, as market liquidity beyond 30 years barely exist.

Due to the incompleteness of the long-term bond market, it is necessary to extrapolate the term structure of the corporate bond interest rates beyond the maturity of longest dated market-available instrument. The current extrapolation method (according to CIA Educational Note), extending the smoothed or regressed function beyond the LLP to complete the yield curve, can be very risky and chaotic. Under both interpolation and extrapolation, smoothing the discontinuous bond prices is challenged by both parameter and model risk. Parameter risk is associated with parameter uncertainty (such as regression coefficients) given that the underlying model is specified. Even in the liquid bond market, it is frequently the case that more than

[1] https://www.soa.org/Files/Research/research-2012-pen-valuation-methods.pdf

[2] "High quality" refers to Aa or higher rating based on Moodys rating service.

one deeply liquid instruments that promise the same amount of cash flows in the future, are traded at different prices. Variations in issue size, coupon rate and other bond-specific factors may easily influence the estimation results. Further, many smoothing techniques, such as the Merrill Lynch Exponential Spline (MLES) method employed by Bank of Canada to determine government bond yields, are based on minimizing sums of least square deviations and can give rise to more than one local solution. A small parameter misspecification under the interpolation phase can lead to an explosive impact in the extrapolation phase. Model risk involves fragile beliefs on the probability distribution of the underlying process. Conditional on a perfectly accurate interest rate model, one can derive a term structure of the bond for all maturities. However, there are a large number of models that fit the bond prices up to their LLP while indicating very different prices for longer-term bonds.

This paper proposes to use the ultimate forward rate (UFR) method to extrapolate corporate yields. Under the UFR method, yield curve construction requires completing two fundamental tasks. First, collect bond price data and use this data to interpolate a spot yield curve up to the last available data point. Second, extrapolate this spot yield curve forward such that the corresponding one year forward rate converges to the UFR. Different from the UFR method adopted in Europe, the Canadian UFR includes additional default risk premium so as to adjust the spread between government and corporate long-term yield. The UFR method makes the ultra-long-term pension liabilities much less volatile to the short-term market shocks. Compared with the current regulation, the UFR method is also robust against model risks.

2 Corporate Bond Yield Interpolation and Extrapolation

2.1 Data

The data used for estimating the corporate bond yield is sourced from Bloomberg. The Canadian "high quality" corporate bond market is not as liquid as the government or the provincial bond market. The original life of the Canadian corporate bond is from 1 to 50 years, while deeply traded corporate bonds with outstanding values larger than $500 million only have maturity terms of 10 years or less. Individual bonds selected for calibration must satisfy the following characteristics:

1. Physical bonds, with no embedded derivatives (e.g., callable, puttable, convertible, index-

linked)

2. Payments denominated in Canadian dollars (CAD)
3. Pay fixed (or zero) coupons
4. Maturity terms of greater than one month and less than or equal to 10 years
5. Minimum amount outstanding on an individual security of $100 million
6. Bonds that trade at a premium or a discount of no more than 500 basis points from their coupon rate

The purpose of inclduing the last two filters is to omit bonds that create distortions in the estimations of the yield curve. Eventually, 98 bonds were selected and their last prices on the 24$^{\text{th}}$ August 2017 and the 12$^{\text{th}}$ September 2017 were quoted for the corporate yield calibration.

2.2 Interpolation

The issues in interpolating yield curves have been well studied in scientific literature. Broadly speaking, interpolation algorithms can be classified as either spline-based or function-based. As an example of spline-based method, Bolder and Gusba (2002) argue that the Merrill Lynch exponential spline (MLES) model by Li et al. (2001) is the most desirable term-structure estimation model in terms of the goodness of fit, based on their empirical result using Canadian government bond data. The U.S. Federal Reserve prefers using another exponential spline method by Svensson (1994) to estimate the U.S. government yield curve.[3] The Bank of England employs Waggoner (1997) spline methodology to build the yield curve. Smith and Wilson (2001) and Nelson and Siegel (1987) are the two famous examples of function-based methods on term structure-based modeling. The European Insurance and Occupational Pensions Authority (EIOPA) adopts the Smith-Wilson technique for both interpolation and extrapolation. For the purpose of risk management and specifically to hedge the risk of a long-term liability using a shorter-term bond, Quaedvlieg and Schotman (2016) argue that the Nelson-Siegel model is particularly well-suited because it offers an additional parameter beyond duration hedging, and its parsimony allows tracking of time-variation in the covariance structure.

Vellekoop (2016) claims there exists an unavoidable trade-off between the two types of methods. The function-based approaches are usually based on a limited number of parameters

[3] The U.S. Treasury yield curve data series are released in conjunction with Federal Reserve discussion paper Gürkaynak et al. (2007).

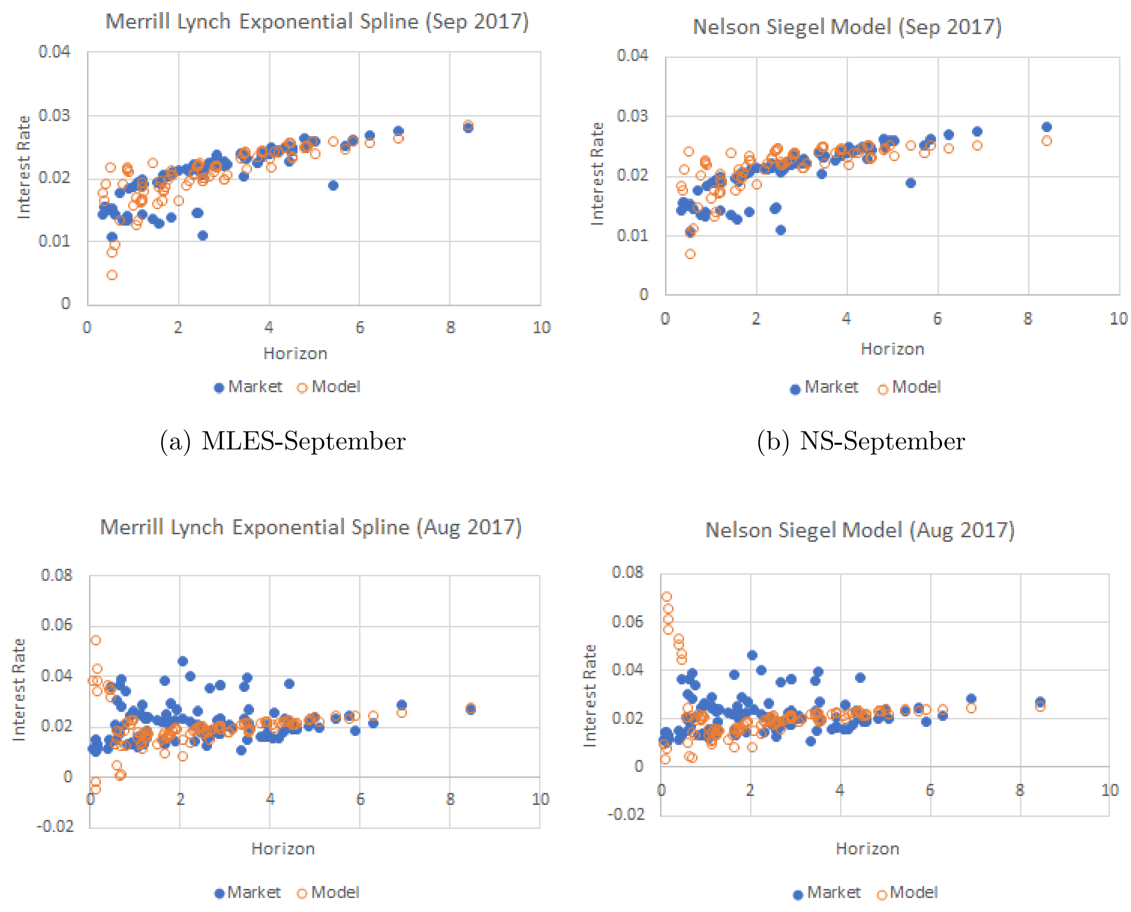
hence the term structure can be infinitely smooth, but market points cannot be fitted precisely. In contrast, spline functions often involve more parameters, allowing a better fit for prices of traded bonds while sacrificing the smoothness. Hibbert (2008) proposes a hybrid of the two methods by introducing a spline-interpolation and function-extrapolation framework, which benefits from the advantages of both methods.

Both spline-based and function-based interpolation approaches are adopted and compared in this paper. Figure 1 plots the observed yield to maturity of the corporate bonds against the model-implied yield-to-maturity of each corresponding bond as a function of the remaining maturity. Both interpolation methods perform reasonably well with goodness of fit higher than 70% for both sample dates. We observe some extreme outliers from Figure 1d for the very parsimonious setup of the Nelson-Siegel model which fails to capture the short-term curvature of the yield curve. In general, for both selected sample dates, the MLSE spline method presents a better interpolation performance with a lightly higher goodness of fit compared with the Nelson-Siegel approach. The trade-off between fit and smoothness when choosing interpolation methodologies is not obvious from Figure 1.

Figure 2 presents the consequence of model risk if no restriction is embedded in the extended yield curve. Figure 2a extends the corporate bond yields of the two sample days beyond the LLP up to 120 years using the two estimated interpolation functions. The MLES method captures more curvature of the yield and the difference between the two sample data is small. The Nelson-Siegel method is smoother and is very sensitive to the sample data indicating the existence of parameter risk. Figure 2b plots the extended forward rate based on the interpolation functions. The volatile patten of the extended curves in both panels indicates a large impact of model and parameter risk on ultra long term liability hedging and pricing when choosing between models and calibration periods.

2.3 Extrapolation

A linear extrapolation of spot rate curves leads to a jump in the instantaneous forward rate, creating potential arbitrage opportunities. A natural way to extrapolate the yield curve is to assume that the last forward rate observed can continue to hold after that maturity. However, Hibbert (2008) and Vellekoop (2016) argue that this simple rule is fragile against instantaneous economic conditions. As demonstrated in Figure 9, which plots forward rates for Canadian government bonds issued between 1996 and 2017, assuming a constant rate beyond the longest maturity often leads to unreasonable-looking yield curves. The large variation of forward rates



(a) MLES-September

(b) NS-September

(c) MLES-September

(d) NS-September

Figure 1: The figure plots the market-based yield to maturity of Canadian corporate bonds against the model-based yield to maturity of the corporate bonds using MLES and Nelson Siegel methods.

at the LLP (30 years in this case) is transferred to the rest of the extrapolated curve which according to Hibbert (2008) makes the long-term bond price five times more volatile than the long-term equity market.

Danish and Dutch regulators proposed to extrapolate liquid market interest rates such that they converge in the long run to an unconditional ultimate forward rate (UFR). The advantages of UFR are twofold. On the one hand, it guarantees market-consistent valuation until the LLP. On the other hand, it maintains pricing stability of ultra long-term cash flows. There is currently a big debate on how to determine the value of UFR, which is a macroeconomic extrapolation issue since the elements used to determine the level of UFR depend on the long-term expectation of some macro factors. In Europe, under the Solvency II regulation (see CEIOPS (2010)), the UFR was set at 4.2% (valid until the end of 2017)[4]. This value was calculated by summing the

[4]One 4 April 2017, EIOPA published its final decision on the methodology for the annual calculation of the UFR for each currency. For most currencies, the UFR will reduce to 4.05% on 1 January 2018.

estimated short term real interest rate of 2.2% and the long-term inflation rate of 2%. Hibbert (2008) argues that a term premium of 1.5% and a convexity adjustment of -0.4% should also be included. The additional elements drive the UFR to 5.3%.

The existing debate in Europe regarding which parameters should be included in the UFR is based on a default-free discount rate. None of these studies consider cases where there exists exposure to credit risk. As previously stated, the discount rate in Canada is calculated using corporate bonds, therefore an element of credit risk should be considered. In this paper, a liquidity risk premium factor is introduced to the Canadian UFR. The purpose of adding the additional term is to distinguish the corporate bond yield from the default-free government bond yield. Hence the Canadian UFR consists of five components:

$$
\begin{aligned}
\text{Canadian UFR} &= \text{Real expected short} - \text{term interest rate} \\
&+ \text{Long term expected inflation} \\
&+ \text{Long term nominal term premium} \\
&+ \text{Long term nominal convexity effect} \\
&+ \text{Long term default risk premium}
\end{aligned}
$$

Details of the motivation and how to estimate each component is discussed in the next section.

2.4 Motivation and methodology for setting a tailor-made Canadian UFR

2.4.1 Real expected short-term interest rate

Expected real interest rates and long-term expected inflation are the two most important factors explaining long-term forward rates. Under the assumption that real interest rates are rather stable in the long run, it is common to use the historical average of real cash returns for expected real short-term interest rates. Dimson et al. (2009) provide a global comparison of annualized bond returns over the last 110 years (1900 to 2009) for 19 countries. For Canada, the average real bond returns over the first half of the 20th century were 1.2 per cent and 2.4 per cent for the second half of the 20th century. In light of these returns, 2.2 per cent would be a reasonable estimate for the expected real interest rate.

2.4.2 Long term expected inflation

Inflation rates are less volatile than equity returns, but forecasting future inflation rates based on past data is still difficult (Stock and Watson (2007) and Ang et al. (2007)). Due to the inflation-targeting framework, inflation in Canada has been remarkably stable since 1991. Therefore, it seems reasonable to adopt the Bank of Canadas current CPI target of 2 per cent.

2.4.3 Long term nominal term premium

The term premium represents the additional return given as compensation for the long-term investment. It measures the difference between the forward rate and the expectation of the future short-term interest rate. Since the term premium can have both a positive and a negative value, this factor is not undertaken under CEIOPS (2010). Hibbert (2008) and Mulquiney and Miller (2014) choose a value of 1.5 per cent under Australian markets. Kim and Wright (2005) and Pandl (2013) show a large decline of the U.S. bond term premium over the past 30 years and Pandl (2013) claims that the risk premia across economics tend to move together. Dimson et al. (2009) report an ex-post bond term premium of around 0.1 per cent for 1900-2000. Based on these studies, a long-term expected term premium of 0.1 per cent seems reasonable.

2.4.4 Long term nominal convexity effect

Convexity means that price appreciation when interest rates fall is greater than the price depreciation of a similar rise in interest rates. To demonstrate the convexity effect, Table 1 provides a simple example. Consider four different zero-coupon bonds maturing in 1, 20, 40 and 80 years. A 100 basis point change in the interest rate from a constant 5 per cent would lead to much larger impacts on price for longer-term bonds. This is a pure technical term. Mulquiney and Miller (2014) assume a convexity effect of -0.2 per cent for Australian market. CEIOPS (2010) assumes away such nonlinear relationship between the interest rates and the bond prices. Hibbert (2008) make the convexity effect equal to -0.4 per cent. In this paper, a convexity effect of -0.2 per cent is quoted.

Maturity (in years)	+100bp	-100bp
1	-0.94 %	0.96%
20	-9.04 %	10.04%
40	-31.56%	46.64%
80	-53.15%	115.02%

Table 1: Price impact of a 100 basis points change interest rate on zero-coupon bonds.

2.4.5 Long term default risk premium

Kwan (1996) argues that corporate bonds are a hybrid of default-free government bonds and stocks, hence are exposed to liquidity shock in both stock and bond markets. Driessen (2004) shows that default risk premium should be included when modelling the expected corporate bond excess returns. De Jong and Driessen (2012) conduct an empirical study on the U.S. corporate bond market and estimate that the total liquidity risk premium is around 0.6 per cent per annum for the U.S. long-maturity corporate bonds. They also argue that liquidity risk premium can help explain the credit spread puzzle which states that corporate bond yield spreads are far wider than historical default losses. Fontaine and Garcia (2011) conducted an empirical study based on the Merrill Lynch high quality bond index data from 1989 to 2007 to show that the historical average liquidity premium is less than 0.5 per cent. Leboeuf and Pinnington (2017) demonstrate that the recent increase in Canadian corporate bond spreads from July 2014 to September 2016 is due to higher default risk premiums. They show that for Canadian high quality corporate bonds, half of the total 0.2 per cent increase in bond spread could be due to higher compensation for liquidity risk. Given that the recent average bond spread of Canadian high rated corporate is around 0.98 per cent with 60 percent from default risk compensation[5], a value of 0.6 per cent is chosen as the default risk premium of Aa rated Canadian corporate bonds.

Adding up all the elements, a UFR of 4.7 per cent seems reasonable for current Canadian markets (see Table 2). The UFR is fixed in the short run but can be adjusted due to the dynamics of macroeconomic factors and market conditions. The beauty of this macro economically assessed UFR is that it makes the pension fund solvency ratio less fragile to potential market turbulence hence is less sensitive to shocks in supply and demand of financial vehicles. As a result, the application of UFR can reduce solvency ratio volatility. Broeders et al. (2014) provide a more comprehensive discussion on the economic impact of adopting UFR.

Component	Rate
Real expected short-term interest rate	2.2%
Long term expected inflation	2.0%
Long term nominal term premium	0.1%
Long term nominal convexity effect	-0.2%
Long term default risk premium	0.6%
Canadian Ultimate Forward Rate	4.7%

Table 2: Components of the Canadian Ultimate Forward Rate.

[5]Assuming 60 percentage is consistent with the empirical result of De Jong and Driessen (2012) and Leboeuf and Pinnington (2017).

2.5 The path towards the UFR

Besides determining the value of UFR, it is almost equally important to investigate the path of forward interest rates beyond the LLP. The EIOPA (2010) assume a convergence speed of 0.1 under the Smith-Wilson framework. Hibbert (2008) use a value of 0.06 as the convergence speed under the Nelson-Siegel framework and they claim that such convergence is consistent with the decaying target forward rate volatility. Based on these studies, this paper adopts the Nelson-Siegel function to extrapolate the forward curve

$$F(t) = \beta_1 + \beta_2 e^{-\lambda t} + t\beta_3 e^{-\lambda t} \tag{1}$$

where β_1 corresponds to the Canadian UFR which is equal to 4.7 per cent. λ controls the convergence speed towards the UFR and is set to 0.06. The remaining two parameters β_2 and β_3 are estimated based on the last market point.

2.6 Implication of UFR

Figure 4 displays the extrapolated forward curve under both the MLES and Nelson-Siegel frameworks and the corresponding extrapolated spot yield curve is shown in Figure 5. As can be seen from Figure 2a and 4, a naive extension of forward rate beyond the LLP may create a lot of noise as different estimation models based on different data sets can imply very different long-term forward rates. The UFR method forces the extrapolated paths to converge to the same ultimate level, 4.7%, hence variation from interpolated curves vanishes in the long run. Figure 5 compares the yield curves with and without implementing the UFR policy. MLES yields are more consistent in terms of the comparison results. For both data sets, MLES yield curves are lower than the UFR-based extrapolated yield curves. The Nelson-Siegel yields present a mixed comparison result, for the Nelson Siegel estimation model is too parsimonious to capture the curvature of the yield curve compared with the MLES method. The extended yield curve using the Nelson-Siegel model fully rely on the three point estimates which controls the level, slope, and curvature of the yield curve and is sensitive the underlying sample set.

The adoption of UFR can benefit the private pension plans in two ways. First it effectively shrinks the pricing volatility caused by model risks and stabilizes the liability valuation under different phases of business cycles, for the long-term expectations of those macroeconomic elements are less sensitive to the instantaneous economy. To see this we work out the implication of the UFR method based on a typical pension fund liability cash flow profile demonstrated in

Figure 6.

Table 3 summaries the liability value and the average discount rate based on naive and UFR extrapolation policies. Without implementing UFR, different estimation models can create dramatic variation in liability valuation even based on the same sample observations. This variation can be as large as $ 17 billions if taking the August 2017 sample as an example. However, this variation caused by model risk almost vanishes if imposing UFR method. The table shows that the average liability valuation mismatch from choosing different interpolation models is $0.22 billions, which is dramatically reduced. Therefore, from a valuation reliability viewpoint imposing the UFR can effectively eliminate the risk of choosing the wrong interpolation model.

Second, the UFR method also reduces the required funding according to our empirical analysis which is based on two random sample sets. For both samples, the average discount rate using the UFR method is around 4.2% in August 2017 and 4.3% in September 2017, which are both higher than the corresponding native average discount rate. Hence, the expected liability value calculated using the UFR yield is much lower for each sample scenario. Although, this advantage can be sensitive to the sample period, imposing the UFR method can effectively protect pension funds from overestimating their pension liabilities due to the underestimated discounted rate.

September 2017	MLES	MLES UFR	NS	NS UFR
Liability Valuation ($ Billions)	45.94	39.45	47.34	39.23
Average Discount Rate (%)	3.37	4.27	4.03	4.28
August 2017	MLES	MLES UFR	NS	NS UFR
Liability Valuation ($ Billions)	43.94	40.21	60.51	40.44
Average Discount Rate (%)	3.61	4.21	2.31	4.19

Table 3: Sensitivity to changed in the yield curve on private pension liability valuation.

3 Conclusion and next steps

In Canada, high quality corporate bond yields are used for discounting the private DB pension liabilities. However, the liquidity of the Canadian corporate bond market fades away after 10 years. To price an ultra long-term pension liability relies heavily on an extrapolated yield curve. Current regulation on yield curve extrapolation does not make the yield extension far enough into the future to meet expected benefit payments and is challenged by both parameter risk and model risk.

This paper introduces a macroeconomic extrapolation method to derive the yield curve beyond the last available data point. The new method assumes a long-term equilibrium ultimate forward rate at a value of 4.7 per cent. Different from the European version of UFR, the Canadian UFR includes a default premium factor because corporate bonds are not default free.

A comprehensive study on estimating macro factors must be conducted. The 5-factor Canadian UFR function is mainly based on dated literature, such as those previously cited. Discussion of the term premium and default risk premium of Canadian high-quality corporate bonds can be an extension of this paper. Another important extension is to investigate the implications for pension plans if Canadian UFR is introduced. An analysis of the impact on the hedging portfolio, solvency ratio and the market effect would be interesting.

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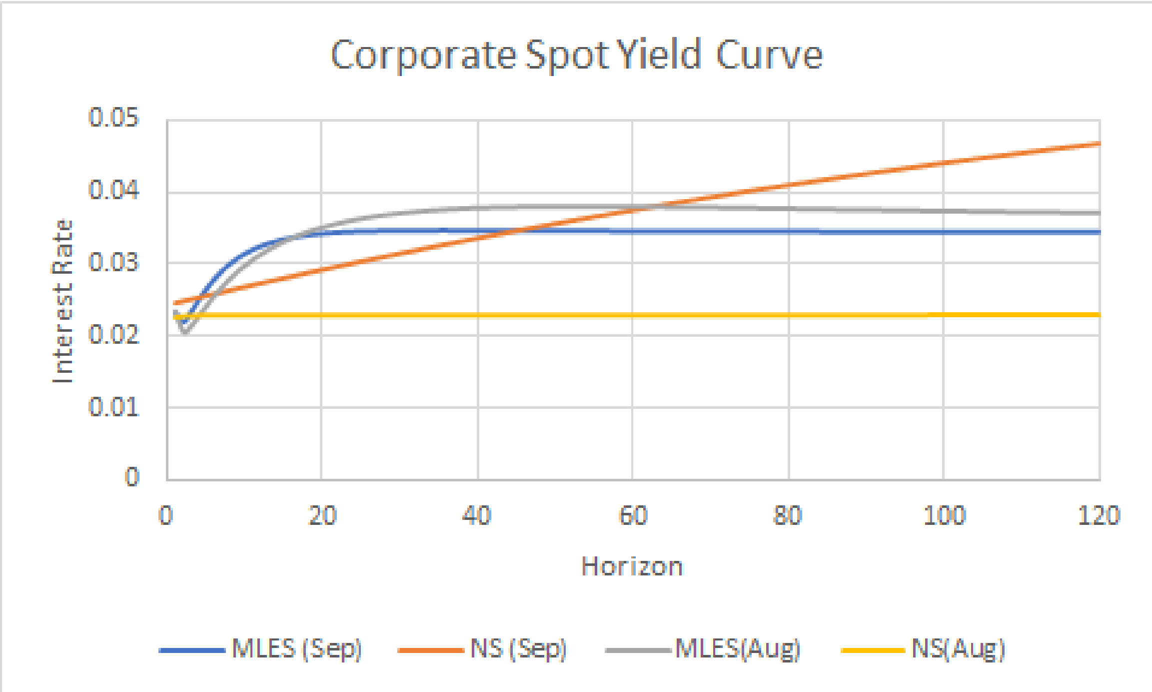
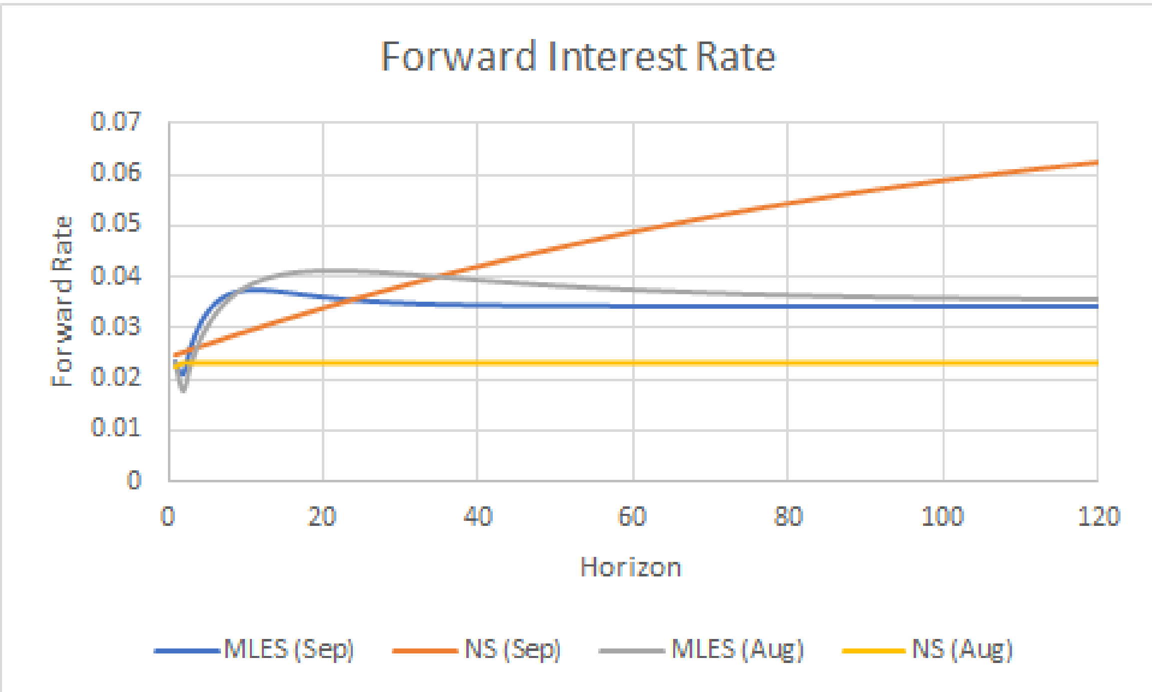


(a) Naive Extension of Spot Yield Curve



(b) Naive Extension of Forward Rate Curve

Figure 2: Extension of spot yield and forward rate curve of Canadian high quality bonds using MLES and Nelson Siegel interpolation methods.

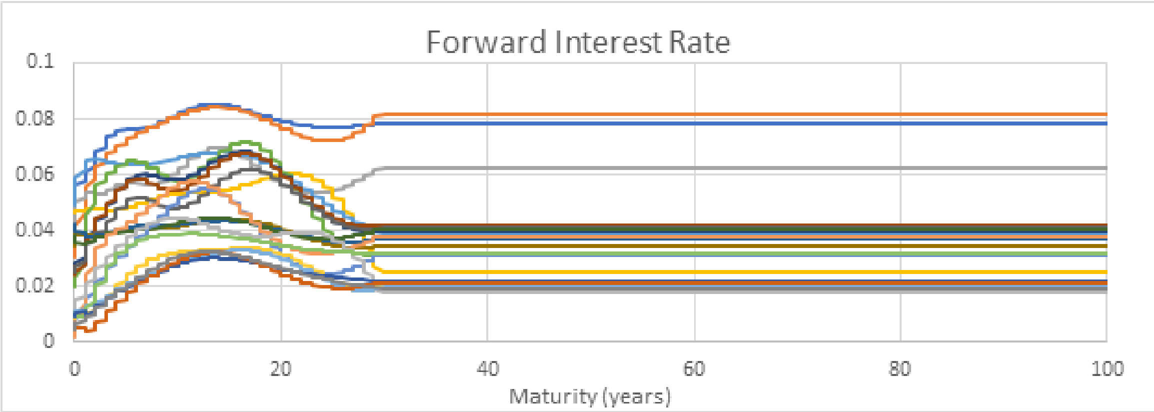


Figure 3: Canadian government forward interest rates assuming constant rate beyond the longest maturity (30 years), beginning of January 1996-2017.

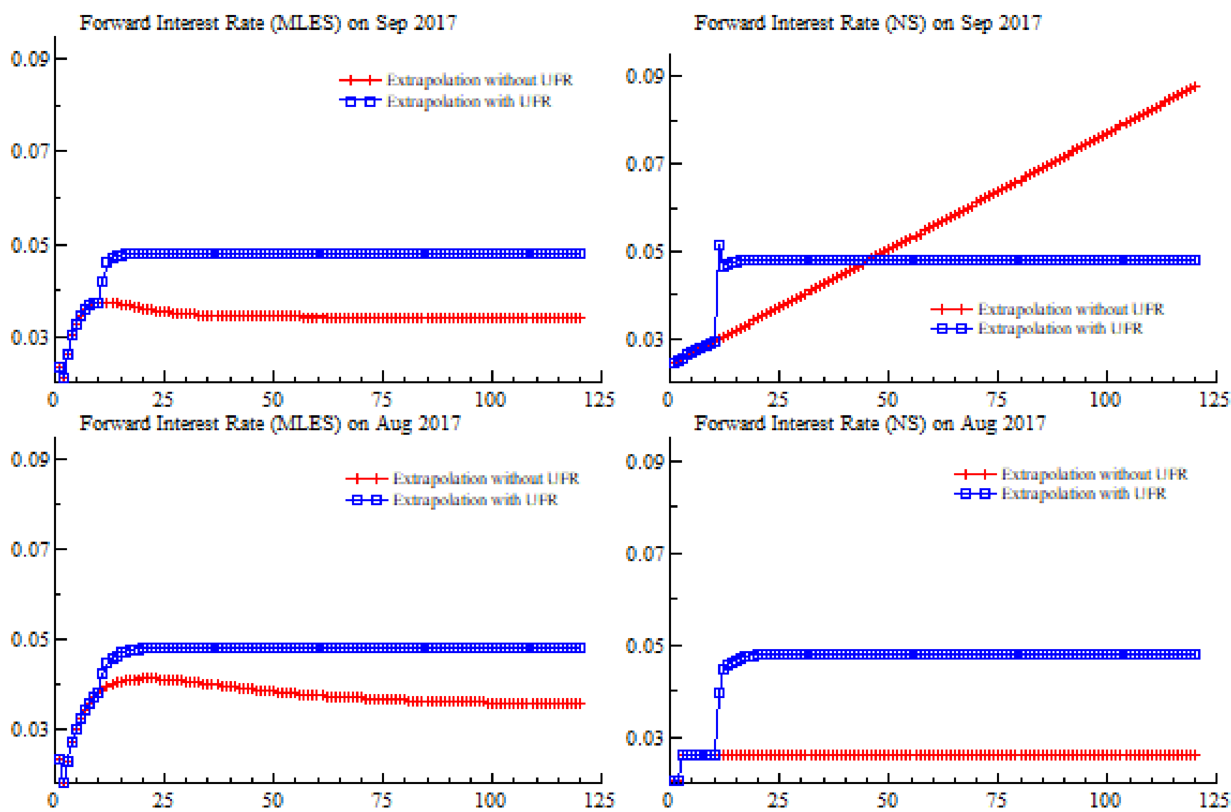


Figure 4: Forward curve extrapolation. Compare the extrapolated forward rate curves with and without implementing UFR policy beyond the MLES and Nelson-Siegel interpolated froward rates curves.

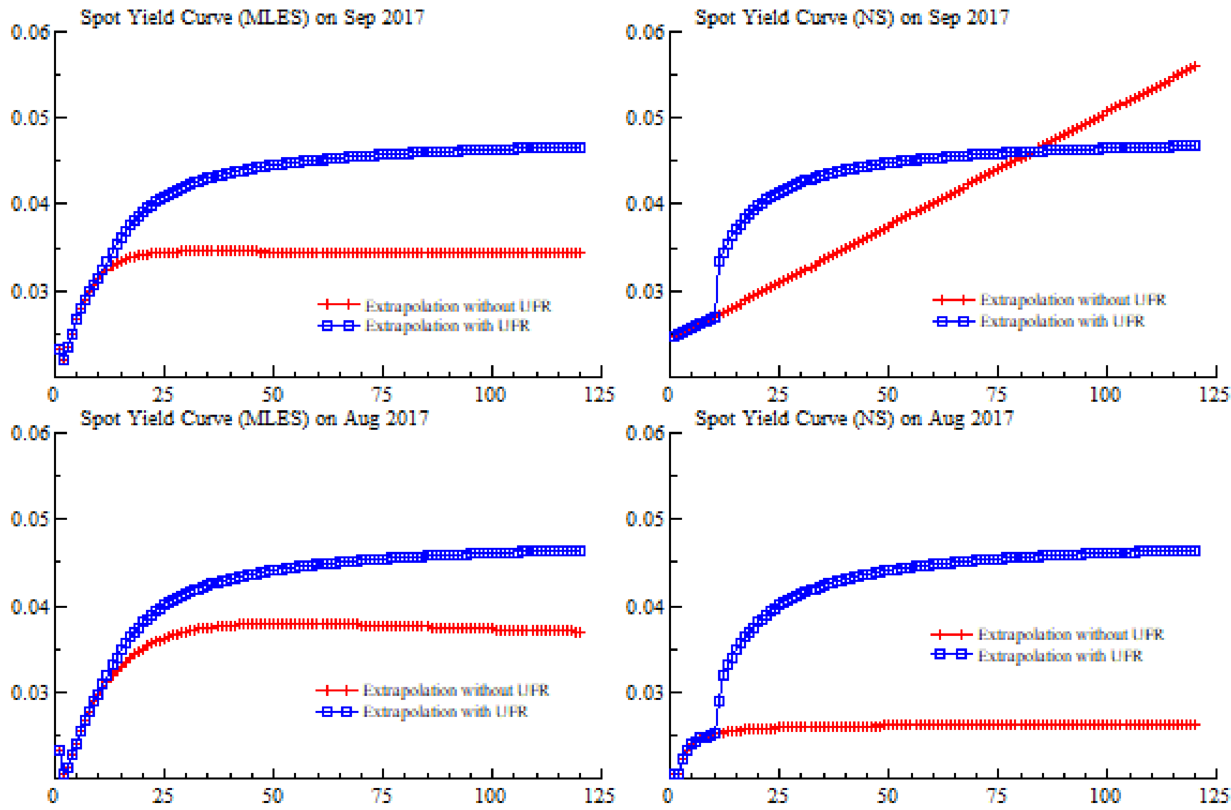


Figure 5: Corporate yield extrapolation. Compare the extrapolated corporate yield curves using native and UFR extrapolation methods.

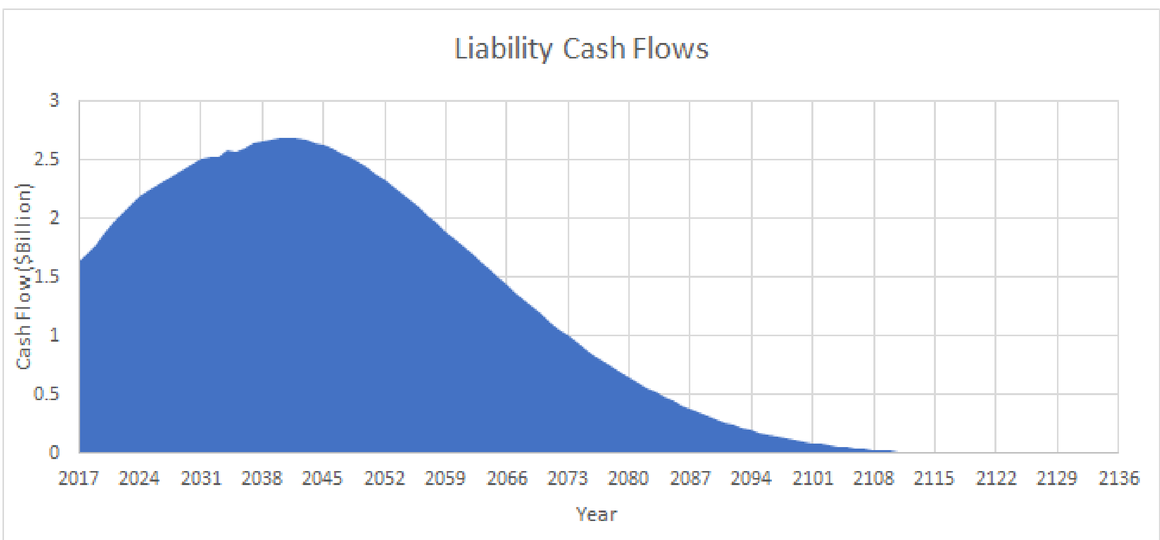


Figure 6: Sample pension liability cash flow profile. Hypothetical example for illustrative purpose only.