ABOUT THIS REPORT:

Traditional final average salary defined benefit (DB) plans are declining, largely because of the volatility of costs. However, the defined contribution (DC) design has significant and important deficiencies, including uncertain and potentially inadequate retirement income. In this paper we adapt results from theoretical, stylized work on pension design, to explore a form of target benefit plan that allows for structured, transparent intergenerational risk sharing (IRS). We compare the IRS plan design with the traditional DB design, based on five broad areas of comparison: affordability (average cost), sustainability (volatility of costs), efficiency, adequacy of benefits, and fairness.
Risk Sharing Pension Plans
Sustainability, Affordability, Adequacy, and Fairness

A Report for the National Pension Hub of the Global Risk Institute

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Abstract

Traditional final average salary defined benefit (DB) plans are declining, largely because of the volatility of costs. However, the defined contribution (DC) style plan has significant and important deficiencies, including uncertain and potentially inadequate retirement income, and, for the individual members, the complexity of managing assets through retirement. In this paper we adapt results from stylized target benefit plans analyzed mathematically in, for example, Cui et al. (2011), Wang et al. (2018), and Zhu et al. (2020a). From that theoretical foundation we explore a form of target benefit plan that allows for structured, transparent intergenerational risk sharing (IRS). We compare the IRS plan design with the traditional DB design. We consider five broad areas of comparison: affordability (average cost), sustainability (volatility of costs), efficiency, adequacy of benefits, and fairness. We assume that, implicitly or explicitly, contributions to the plan come from workers’ salaries, so the problem is to balance the interests of workers and retirees. We find that if we take into consideration the possibility of default, the IRS design can offer a better balance of costs and benefits than the traditional DB, with respect to affordability, sustainability and adequacy, but does not substantially improve fairness. When we move to a career average revalued earnings plan the disparity between membership groups is reduced.

1 Introduction

Recent experience has demonstrated vividly that most private sector employers would prefer not to sponsor traditional Defined Benefit (DB) pensions. The continuing survival of such plans in a few sectors is largely due to the strong preference of labour organisations for DB over the alternatives. But the view that traditional DB is the gold standard of benefit provision largely ignores the real, unacknowledged costs to employees, both in terms of the risk to their job security when a large pension deficit threatens the solvency of their employer, and in terms of their retirement security, if the firm enters bankruptcy or is taken over. Guarantee funds generally offer limited compensation, with caps and haircuts reducing the benefits, especially for pre-retirees. The long period of high equity returns and high interest rates that sustained the relatively generous benefits of traditional DB plans though the 1980s and 1990s can no longer be relied upon to subsidise future costs.

However, there is also an increasing awareness of the problems associated with Defined Contribution (DC) plans, especially since the 2008/9 financial crisis. Many commentators echo the conclusions of Cooper (2013) that DC plans are ‘not fit-for-purpose’. DC benefits are uncertain, annuities are expensive, stock markets are volatile. If you are lucky, you may build up your retirement funds during boom times, and retire when interest rates are high, but the unlucky cohorts may find their retirement funds decimated by market
volatility late in the accumulation phase, and then face the second blow of low interest rates in retirement, creating seriously inadequate retirement income streams. The burden of managing DC assets through retirement is a significant challenge for retirees, who cannot be expected to have the sophisticated financial expertise required for such a complex problem. Governments that encourage employer sponsored pensions, through tax incentives or otherwise, may resent giving up tax income to support plans that are perceived to be unfair or inadequate.

As neither pure DB nor pure DC, in their traditional forms, appears to meet the needs of sponsors, workers and retirees, there is a developing interest in hybrid designs, that combine elements of DC and DB plans. Popular examples include the DB-underpin (also known as floor offset), Cash Balance, and second-election options. However, many DB underpin plans have been wound up as the DB guarantee became too costly; Cash Balance plans carry significant (and largely unacknowledged) risk to sponsors during the accumulation phase, while leaving the members with all the decumulation risk, including interest rate, longevity and dissipation (Hardy et al., 2014). Second election options, which allow a one-time transfer from DC to DB, are more expensive and less sustainable than standard DB plans (Zhu et al., 2018).

More successful developments fall in a very broad category of pension designs designated ‘Target Benefit’ or ‘Defined Ambition’ plans, and which are commonly termed ‘Intergenerational Risk Sharing’, or IRS plans, in the academic literature. These range from plans with fixed contributions, but with some potential risk pooling of benefits (essentially a collective DC plan) to plans which are almost identical to traditional DB, but with the potential to reduce benefits in sufficiently exigent circumstances. In between the ‘almost DC’ and ‘almost DB’ types there is a range of potential forms of IRS plans, where both contributions and benefits may be adjusted in response to investment and demographic experience.

1.1 Pension design criteria

The objective of our work is to explore pension plan designs in relation to the following criteria.

**Affordability** An affordable plan has a total contribution rate that is, on average, within a range deemed acceptable by the sponsor and members.

**Sustainability** A plan that is affordable, based on average costs, may be unsustainable if the volatility of costs is very high. A sustainable plan can be managed such that
costs remain within some predetermined limits even when economic conditions are unfavourable.

**Adequacy** A plan provides adequate pensions if the benefits are predictable and sufficient. Predictability means that employees can plan for retirement, and can reasonably expect that benefit promises and/or projections will be realised. Sufficiency means that, over a full working lifetime, an employee accrues sufficient retirement income to maintain their lifestyle through their retirement, taking into consideration statutory benefits and usual life changes.

**Efficiency** A pension plan is efficient if contributions are used effectively to provide adequate incomes in retirement. It would be inefficient for a plan to carry large surpluses, indicating that too much capital has been collected. It would also be inefficient for a plan to give benefits far above expectations or far above the requirements of the adequacy criterion, as, again, that would indicate that excessive contributions have been collected.

**Fairness** This criterion is probably the hardest to capture quantitatively or qualitatively. There are several different aspects to fairness. Some that we have considered are:

- Variation in costs and benefits for different generations.
- Variations in costs and benefits for different sub-populations, within the same cohort.

## 2 Stylized Intergenerational Risk Sharing (IRS) plans and results

This report is the third and final paper in a series on IRS plans. The first two papers involve fairly abstract, stylized pension design and demographic model, used to develop mathematical insights into hybrid plans. In this final stage, we apply the insights to a more realistic model of a pension plan, and consider the extent to which the criteria can be met under variations of the hybrid design.

The first paper, Zhu et al. (2020a), involves the analysis of a simplified risk sharing plan from a mathematical perspective. The model, which is similar to Cui et al. (2011), and Wang et al. (2018), involves a pension plan with all members entering at the same age, retiring 40 years later, and dying 20 years after retirement. There are no other deaths or
exits. The population is standardized such that at each point there is one life at each age from entry to death.

The pension structure is a risk sharing, target income plan. Targets are set for both contributions and benefits. It is assumed that all contributions are paid from the salaries of the workers, which are constant. The target contribution and benefit are set to be actuarially fair, based on a constant risk free rate of interest assumption.

The actual contributions and benefits differ from targets due to random asset returns. Any surplus or deficit of assets over the value of the target liabilities is partially shared between the workers (through contribution adjustments) and the retirees (through benefit adjustments). To explain the model we will use the following notation.

(i) $c$ and $c_t$ represent the target and actual contributions paid by each worker.

(ii) $b$ and $b_t$ represent the target and actual benefit paid to each retiree, respectively.

(iii) $A_t$ represents the asset value at $t$. Assets are invested in a mix of risky assets (modelled as a geometric Brownian motion) and risk free bonds.

(iv) $L$ represents the liability value, which is the present value of target benefits minus target contributions.

(v) $N_w$ and $N_r$ represent the number of workers and number of retirees, respectively. Because salaries are assumed to be 1 each year for each worker, $N_w$ also represents the total of workers’ salaries.

(vi) The risk sharing is determined by parameters $\alpha$, $\beta$, and $\psi$, as follows$^3$

$$c_t = c - \alpha \frac{(A_t - \psi L)}{N_w}$$  \hspace{1cm} (1)

$$b_t = b + \beta \frac{(A_t - \psi L)}{N_r}$$  \hspace{1cm} (2)

This means that any surplus above $A_t - \psi L$, and any deficit exceeding $\psi L - A_t$, is (partially) shared between workers and retirees, with $\alpha$ and $\beta$ determining the proportion allocated to workers and retirees respectively, and $1 - (\alpha + \beta)$ representing the total proportion of surplus/deficit retained. The parameters, including the optimal investment in risky assets, are determined using an optimization approach.

The key results from Zhu et al. (2020a) that we will make use of in this work are:

$^3$In the paper, there were separate $\psi_w$ and $\psi_r$ parameters for workers and retirees.
The parameters are optimized to maximize the stability of income over all cohorts. This is achieved by minimizing the aggregate squared difference between actual and target income. Income for workers is defined as salary after deduction of pension contributions, and income for retirees is the actual benefit, \( b_t \).

This objective function is different to that used in Cui et al. (2011) and Gollier (2008), who optimize expected aggregate utility of income. The stability measure better reflects the purpose of the pension plan, which is to provide adequate and predictable pensions for all. Expected utility would allow some extra large pensions to offset some small pensions (though with a bias weighting downside results more heavily, because of risk aversion). The stability measure is therefore more consistent with the criteria of efficiency and adequacy, as it penalizes benefits that are far from target on either side, and of sustainability, as it penalizes volatility of contributions.

In order to get reasonable results, there must be some solvency constraint imposed, designed to eliminate a deficit within a specified period. If this constraint is absent, then multiple cohort modelling tends to sacrifice the benefit security of current generations in order to ensure minimal risk for future generations.

The \( \psi \) parameter can be derived under some additional assumptions, but may also be exogenously imposed. For example, it may be interpreted as regulatory solvency or surplus limits.

If \( \psi = 1.0 \), then \( \alpha + \beta \) represents the proportion of deficit distributed, or surplus repaid, each year.

Also, if \( \psi = 1.0 \), then, based on an infinite time horizon, it is optimal for

\[
\frac{\alpha}{\beta} = \frac{N_w}{N_r}.
\]

The optimal proportion of assets invested in equities over the long term is around 10%.

Group optimality will not, in general, correspond to individual cohort optimality.

When we consider a finite horizon, we need an additional parameter that penalizes the end asset-liability ratio if it is not close to 1.0 (otherwise the optimization will just use up all the available assets). The results of the optimization are very sensitive to this penalty parameter, which is a subjective input.
While the first paper considered the optimal plan design from the perspective of all current and future cohorts, the second paper, Zhu et al. (2020b), considers the impact of switching from DB to IRS for cohorts who are already in the plan. The intuition is that the transition to IRS is beneficial for future cohorts, as it reduces the risk of insolvency, which can be catastrophic for members and retirees, and reduces the volatility of contributions. However, if the initial funding level is reasonably healthy, then the value of the protection against insolvency is small for older members and retirees, as it is unlikely that insolvency will occur during their lifetimes, and they gain no benefit from the reduced contribution volatility. For these members, a wholesale switch from DB to IRS will disadvantage them; they will give up benefit stability, but do not gain significant advantage from the protection from insolvency. This is an example of the problem of optimizing for individual cohorts, as compared with optimizing over all cohorts.

In order to create a path for transition that does not (in terms of expected values) disadvantage current retirees, Zhu et al. (2020b) propose a phased transition for older cohorts. The IRS plan in Zhu et al. (2020b) is similar to the IRS plan in Zhu et al. (2020a), but with more realistic assumptions. The parameters for risk sharing (i.e. $\alpha$, $\beta$, $\psi$) are now allowed to differ depending on whether there is a surplus or deficit, based on the target liabilities. The determination of the optimal parameters uses stochastic interest rates and inflation, generated using the Wilkie economic scenario generator (Wilkie (1984), Wilkie et al. (2011)), based on the calibration in Zhang et al. (2018).

Key results from Zhu et al. (2020b) that are relevant to the current work include the following.

(1) Current retirees have no influence on the long term optimization results. It is therefore appropriate to consider their needs separately.

(2) Based on the particular assumptions and parameters of that paper, the results showed that optimal risk sharing of surplus over a range of targets was close to $\alpha + \beta \approx 0.1$ and of deficit was $\alpha + \beta \approx 0.05$.

(3) For both the surplus and deficit, optimal values of $\alpha/\beta$ were close to the ratio of the number of active workers to the number of retirees, as noted in the earlier paper (and in equation (3) above).

(4) For fair treatment of retirees at transition, smaller values of the risk sharing parameter $\beta$ should be applied. An optimization process can be used to determine a scale from the long term value (full participation) to 0 (no participation). Based on the
parameters and assumptions in Zhu et al. (2020b), for example, smaller values of $\beta$
are applied to retirees older than around age 80 at transition.

3 A model defined benefit pension plan

In this paper we replace the stylized pension plans of the earlier work with a more realistic model, based on the traditional final average salary defined benefit pension plan. Our model still involves substantial simplification, but the demographics, assumptions, valuation methods and benefit structure should be familiar to DB plan actuaries and administrators. We will use this plan as a benchmark to compare with the risk sharing design adapted from Zhu et al. (2020a) and Zhu et al. (2020b).

In both Zhu et al. (2020a) and Zhu et al. (2020b), we have followed the academic literature on risk sharing pension plans by assuming that the full contribution risk falls on the workers. That is, explicitly or implicitly, all the contributions are met by taxing the salaries of the active members. We maintain this assumption in this paper. However, we recognise that in private sector plans, shareholders bear a significant amount of risk in practice. In future work we will consider allocating risks to shareholders as well as employees and retirees.

The model described here is a traditional defined benefit plan. It is designed to be relatively straightforward, for ease of interpretation – for example, we assume all cash flows occur at the start or end of each year – but it captures sufficient characteristics of a real world plan to be useful for a broad comparison of different forms of risk sharing, and of the impact of adjusting assumptions.

We have not incorporated longevity or idiosyncratic demographic risk in this model. The comparisons therefore are dependent only on inflation and investment risk.

As we are interested in the perspective of current plan sponsors and members, we have used a time horizon of 30 years. This is very much shorter than most of the academic literature on sustainable design and intergenerational transfers, but is better aligned with the perspectives of current stakeholders.

3.1 Plan benefits

The accrual rate is 1.8%, and the pension is based on the average of the final three years’ salary. On death or withdrawal before retirement age, a lump sum is paid, equal to the actuarial value at exit of the deferred pension. The pension is paid as an annual life annuity-due.
Figure 1: Model pension plan membership information. The left side shows the number of workers/retiree at each age at the start of the projection, and the right side shows salary and service assumptions at the start of the projection, and also shows the age pattern of new entrants.

Cost of living adjustments are funded, up to a maximum of 3%, and are paid in full unless the plan is wound up; there is no cost of living adjustment applied to wind-up benefits.

### 3.2 Demographics

Initially, there are 3456 active members and 1516 retirees.

Members enter at ages between 25 and 55, with the number of new entrants at each age assumed constant throughout the projection.

All lives in force at age 65 retire on that day. Pensions are paid from the plan annually for the remaining lifetime. There are no spousal benefits and no early retirements.

The starting values for the number of members at each age, along with average salaries and average service, are shown in Figure 1. The average salary curve is also used throughout the projection as a promotional salary scale.

The projection of plan membership is deterministic. The pre-retirement service table and post retirement mortality tables are given in the appendix.
3.3 Economic Scenario Generator

We have used the Wilkie economic scenario generator (ESG) (Wilkie (1984)), fitted to US 1951-2014 data, using parameters from Zhang et al. (2018), slightly adapted. The Wilkie ESG generates annual time series for equity prices and long term bond yields, as well as inflation. We do not use a separate series for salary growth within the Wilkie model; instead we assume that salary growth in this plan is 50bp higher than inflation, in addition to a service scale representing promotional increases. We note that the Wilkie model has been shown to generate equity returns that are thinner tailed than the empirical values, so the model should be interpreted as potentially offering an optimistic view of the investment risks involved.

The Wilkie model parameters are given in the appendix.

3.4 Contributions and valuation

We assume that the plan is funded using a Projected Unit Credit (PUC) method for the going-concern, funding valuation, and a Traditional Unit Credit (TUC) valuation for solvency. Employees pay the Normal Contribution indicated by the going-concern valuation (as noted above, we assume that employer contributions are implicit salary deductions), plus 10% of any deficit from the solvency valuation; this is designed loosely to represent a 10-year window to recover from deficit. If the PUC asset liability ratio is greater than 1.2, 20% of the surplus is released as a contribution reduction, with a minimum contribution of 0%.

The Normal Contribution Rate is the contribution payable based on the PUC valuation assumptions, before adjusting for surplus or deficit, expressed as a % of payroll. The Total Contribution Rate is the rate after adjustment.

The going concern (PUC) valuation projects all salaries to retirement, and discounts at a valuation rate of interest which is set using a 3-year moving average of long term risk free rates, with an addition of 200 bp. The valuation salary inflation assumption is 200 bp below the interest rate, with a minimum of 1.0%, and the (price) inflation assumption is 250 bp below the interest rate, subject to the 3% plan maximum, and 0% minimum. We assume a promotional salary scale, as illustrated by the initial salary curve in Figure 1, but the salary projection in the going concern valuation only includes inflationary increases, not promotional increases, making the going-concern, technically, a partly-projected unit credit valuation.

The solvency (TUC) valuation does not project salaries to retirement. The interest rate
is set equal to the current long term bond yield with a deduction of 50bp, representing a margin for wind-up expenses. It is possible for the solvency interest rate to be negative; given that the margin allows for the costs associated with wind-up, this possibility is not excluded. As noted above, it is assumed that cost of living adjustments are funded, and are paid in full (up to the 3% maximum) while the pension plan is a going concern, but are not paid on wind-up benefits. This means that the cost of indexation is included in the going concern valuation, but omitted from the solvency valuation.

3.5 Assets

The plan is assumed to be 100% funded on the going concern basis at the start of the projection.

The assets are assumed to be invested in a mix of equities and long-dated risk free bonds, rebalanced to maintain the proportions at each year end. For the benchmark results, we assume that the equity weighting is 60% of the portfolio value, but in later sections we will allow this parameter to vary.

3.6 DB plan results

In Figure 2 we show the results of 10,000 projections of the assets and liabilities of the model plan. Each plot shows 5%, 25%, 50%, 75%, and 95% quantiles for the metrics at each time period, and, in addition, each plot shows 30 individual paths. The same 30 paths are used in each of the quantile plots in this paper.

The top two figures show the asset liability (A/L) ratios, for the going concern valuation (top left) and for the solvency valuation (top right). The results are similar, as we might expect. The going concern valuation is more conservative with respect to salary growth and cost of living adjustments, but less conservative with respect to the discount rate, compared with the solvency valuation. Asset values are identical.

It appears that the pension plan is reasonably well controlled, with the median asset-liability ratio close to 1.0 throughout, and the lower 5% quantile remaining relatively stable at around 0.7.

However, from the bottom figure we see that the cost of maintaining this stability can be extreme. The total contribution rate, shown in the lower plot, is very volatile, with peaks of over 40% of salary occurring regularly. The median normal contribution rate is shown by the broken line; it lies slightly below the median total contribution rate, indicating that the going concern valuation assumptions are slightly optimistic compared with the
(a) Assets/Liabilities: Going Concern (left side) and Solvency (right side)

(b) Total Contribution Rates, with median normal contribution rate (broken line)

Figure 2: DB Results; 10,000 projections, 5%, 25%, 50%, 75% and 95% quantiles, with 30 sample paths.
simulated experience. However, the difference is not that great. The main problem is not inadequate normal contributions, it is the very high volatility of contributions. In terms of the criteria listed in Section 1.1, the plan may be affordable (assuming that the average cost of around 19% of salary is within acceptable limits), but does not look sustainable, at least, considering this simple investment strategy. In the traditional DB plan, the contribution rate is the only control variable, and it is often a very blunt instrument.

3.7 Incorporating wind-up risk

In literature on workplace pension design, it is common to ignore the risk of DB plan failure. However, default is a real possibility, as we have seen from several examples in recent history. Although winding-up an underfunded plan is legally complicated, it is likely to happen when a sponsor is liquidated, and may be done even when the sponsor continues. For example, United Airlines was permitted to default on its pension obligations as part of a restructuring under bankruptcy protection laws. In Ontario, multi-employer and jointly sponsored plans can wind up without requiring any liquidation, and all accrued benefits, for in-force and retirees, may be reduced. Allowing the plan to wind up underfunded is a ‘control’ variable, albeit even more blunt than the additional contribution rate. When assessing the comparative attractions of different plan designs, the potential for benefit reduction on wind-up should be incorporated.

If the deficit contribution is substantial, then the current workers are (to some extent) subsidising the cost of benefits of earlier cohorts. Some intergenerational transfer is an accepted part of the DB pension framework; but the implicit contract involved in intergenerational risk sharing will break down if the costs to current members become so high that they refuse to participate – by opting out of the plan, or working under non-pensionable temporary contracts, for example.

In order to incorporate wind-up risk in the pension model described in the previous section, we set thresholds for the solvency asset-liability ratio, and for the total contribution rate.

- We impose a cap of 30% of payroll on the total contribution rate.
- We assume that the plan will wind up if the solvency A/L ratio falls below 50%. In this event, active members will be provided with a deferred, un-indexed pension equal to their accrued benefit, reduced in proportion to the solvency A/L ratio. Pensions in payment will be reduced in the same proportion. We assume these benefits are

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4 See Davis (2011) for more details about the Canadian system, and Lowenstein (2005) for information on the United Airlines case.
provided through a bulk buy-out, so that there are no further contributions from active members, and no further risk of benefit reduction.

Our wind-up and maximum contribution criteria are intended to be illustrative rather than descriptive or prescriptive. The point is to explore the probability and impact of wind-up, under relatively simple assumptions. We assume no recourse to a public Pension Guarantee Fund, nor to the assets of the sponsor at default.

The risk of wind-up is low, based on our economic scenario generator (which is on the optimistic side with respect to equity risk). With no cap on the total contribution rate, the estimated probability of wind-up within 30 years is around 1.2%. With a cap of 30% on the total contribution rate, the estimated probability is around 4.9%.

In Figure 3 we show the going concern and solvency A/L ratios where we have imposed the 30% maximum total contribution rate, and the 50% wind-up threshold for the solvency A/L ratio. The cap on contribution rates has reduced the A/L ratios for each of the quantiles shown. Also notice that in three cases, from the 30 sample paths, the plan has been wound-up; this is indicated by an A/L ratio of zero.

The total contribution rate plot shows the impact of the maximum contribution rate of 30%, which corresponds to the 95% quantile from around the 5th projection year. All lower quantiles are very similar to those in Figure 2.

Allowing for a maximum contribution rate and for wind up of the plan will impact the income and benefits of plan members. In Figure 4 we show 100 paths of the inflation-adjusted income, after deduction of pension contributions, for a life who is age 45 at the start of the projection, who remains in service until age 65, and who survives to the end of the projection. The initial income, after deducting contributions, is set to 100. Before retirement, income is the salary net of the total contribution rate; after retirement, the income is the pension benefit.

We make the following observations and comments.

- Although the values are inflation adjusted, the salary trends upwards because of the promotional salary scale used, and because salaries are assumed to increase slightly more than inflation each year (by 50 bp).

- The range in salary values at each age between 46 and 64 arises from variations in the total contribution rate; since that rate is constrained to lie between 0% and 30%, the salaries after pension deductions are also constrained.

- The highest income values before retirement arise either because the pension plan has already been wound up, so there are no further contributions, or because the
Figure 3: DB plan with maximum 30% total contribution rate, and with wind-up triggered by solvency A/L below 50%. 1000 projections, 5%, 25%, 50%, 75% and 95% quantiles, with the same 30 sample paths as in Figure 2.
The asset/liability ratio is so healthy that no contributions are required. The lowest income values before retirement arise when the pension contribution is at its maximum, because the pension deficit is large.

- The pension income (adjusted for inflation) is highly predictable, except in the few cases where the plan is wound up.

- There are six cases of wind-up within these simulations, three occurring before the life reaches age 65 (giving higher income immediately before retirement, but lower benefits), and three arising during the retirement phase.

The plan managers can impact the default risk by changing the proportion of funds invested in equities. However, since, on average, equities generate higher returns than fixed income, the average contribution rate will increase with lower equity investment. We consider this further in the next section.

### 3.8 Varying the equity weighting

To assess the benefits and costs of different values for the equity allocation, we let the proportion of assets invested in equities vary from 0% to 100%, and consider the impact on three metrics:
1. The **average total contribution** rate across all projections. For each individual projection, we average the total contribution rate over the 30 year projection, or up to the time of default if that occurs. This measures affordability.

2. The **estimated probability of default**; this is the number of projections where default occurred, divided by the total number of projections. This measure relates to adequacy and efficiency.

3. An **income stability measure** derived from Zhu *et al.* (2020a). For each cohort, and for each individual projection, we find the average squared difference between the target and actual income, across the 30 projection years. We set the target income for workers to be the projected salary after deduction of the normal contributions. We assume the normal contribution rate is the same for all workers at any given time. The target income for retirees is the pension based on the salary at age 65, with full indexation. Income is normalized to be 100 at the start of the projection for workers, and 50 at the start of the projection for retirees.

The stability measure illustrated in the figures is the square root of the average aggregate squared difference between the target and actual net incomes, where the aggregation applies separately to each individual age group, summed over the 30-year projection. For each cohort, the measure applies to lives who remain in the plan to retirement; values after retirement are discounted for survival. Lower values of the stability measure are preferable to higher values.

As discussed in Section 2, upside divergence from the target income is penalised equally to downside divergence in the stability measure. Upside divergence, in some cases, indicates that the cohort is particularly lucky – for example, if they can take an extended contribution holiday, and still get the same (real valued) benefits as earlier and later cohorts, who are required to contribute over 20% of their pay. In other cases, salaries are high only because the pension plan has already failed. In either case, the principles of efficiency and fairness indicate that upside variation is not desirable; an equitable pension plan should minimize upside and downside discrepancies.

The default risk and average total contribution rates for different equity weightings are shown in Table 1, along with approximate standard errors (in parentheses). The values are derived from 10,000 simulated paths. It is worth noting that the Wilkie model tends to underestimate equity price downside risk, so the true default risks for higher equity weightings are probably larger than indicated here.
Equity Weighting & Probability of Default & Mean contribution rate

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<th>Equity Weighting</th>
<th>Probability of Default</th>
<th>Mean contribution rate</th>
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Table 1: Default risk and mean total contribution rate, by portfolio equity weighting, for the DB plan with 30% contribution cap, and 50% solvency A/L ratio wind-up threshold. 10,000 projections; estimated standard errors in parentheses.

We notice that minimizing the default risk is achieved with an equity weighting of around 15% to 25%; this is much lower than most pension plans would use in practice, but it is quite close to the range derived from the optimization in Zhu et al. (2020a). However, the average total contribution rate when the equity weighting is set to 20% is 265 bp higher than the average contribution rate when the equity weighting is 60%, illustrating the trade-off between default risk and affordability.

We illustrate the impact of the equity weighting parameter on the stability measure in Figure 5. We show separate results for different cohorts, as their interests are different. Active workers age 30 will not retire before the end of the 30-year projection, so their stability measure relates entirely to the impact of contribution volatility. Those age 60 or older at the start of the projection are primarily concerned with the impact on benefits, which predominantly relates to default. Because each cohort spends a different amount of time in the active and retired categories, the numbers are not comparable across cohorts. What is interesting is how the equity proportion impacts the income stability for each cohort. Note that the optimal outcome with respect to stability is a low value for the stability measure. For all cohorts, the optimal equity weighing is around 20% to 35%.
Figure 5: DB plan with 30% contribution cap, and 50% solvency asset-liability ratio threshold. Stability measures by equity weighting, for different cohorts (by starting age). 10,000 projections.
4 The IRS plan

In this section we consider how the risks and benefits of the DB plan would change if it were reconfigured to a risk sharing plan, with target benefits which are identical to the original DB plan, with one major change. When the A/L ratios fall outside the set thresholds for deficit reduction or surplus distribution, both benefits and contributions are adjusted, rather than only contributions as in the traditional DB design.

4.1 Risk sharing mechanics

The adjustments are designed to mimic the stylized risk sharing plans outlined in Section 2 above, as used in Zhu et al. (2020a) and Zhu et al. (2020b), as well as in Cui et al. (2011) and Wang et al. (2018).

Let $\psi_l$ and $\psi_h$ represent the deficit and surplus asset-liability ratio thresholds, respectively. Note that $\psi_l$ is based on the solvency A/L ratio, while $\psi_h$ is based on the going concern A/L ratio. Our benchmark assumptions are that $\psi_l = 1.0$ and $\psi_h = 1.2$, consistent with the thresholds used in previous sections.

When the deficit threshold is breached, contributions and benefits are adjusted such that a portion, $1/\tau_l$, of the deficit is recovered from the year end cash flows. The interpretation of $\tau_l$ is that it loosely represents the time allowed to recover the deficit; in the previous section we assumed deficits were recovered over 10 years, so here we use $\tau_l = 10$. When the surplus threshold is breached, similarly, contributions and benefits are adjusted such that a portion, $1/\tau_h$, of the surplus is distributed from the year end cash flows, where $\tau_h$ is, loosely, the time allowed for spreading surplus payments. Consistent with the assumption of previous sections, we assume here that $\tau_h = 5$.

Let $V_s(t)$ denote the value of the liabilities at $t$ on the solvency basis, and let $V_g(t)$ denote the value of the liabilities at $t$ on the going concern basis. The value of the assets at $t$ (assumed the same for both valuations) is $A(t)$. The normal contribution rate due at $t$ is denoted $\pi(t)$, and the aggregate salary of active employees at $t$ is $S(t)$. Then the aggregate normal contribution due at $t$ is $\pi(t) S(t)$. Finally, let $B(t)$ denote the aggregate target pension benefit due at $t$.

When the pension plan is in deficit, that is, when $A(t)/V_s(t) < \psi_l$, then the employees, in aggregate, pay total contributions of

$$\pi(t) S(t) + \alpha(t)(\psi_l V_s(t) - A(t))$$
and the retirees, in aggregate, receive benefits of

\[ B(t) - \beta_l(t)(\psi_l V_s(t) - A(t)) \]

These are adapted from equations (1) and (2) above. Similarly, when the pension plan is in surplus, the employees, in aggregate, pay contributions of

\[ \pi(t) S(t) - \alpha_h(t)(A(t) - \psi_h V_g(t)) \]

with a lower bound of 0% of pay. The retirees in aggregate, receive benefits of

\[ B(t) + \beta_h(t)(A(t) - \psi_h V_g(t)) \].

When neither threshold is breached, then the retirees’ aggregate benefits are \( B(t) \), and the employees’ aggregate contributions are \( \pi(t) S(t) \).

We assume that the contributions and benefits of each individual member are adjusted in the same proportions, so that the contribution rate paid by each employee when the plan breaches the lower threshold is

\[ \pi^*(t) = \pi(t) + \alpha_l(t)\frac{\psi_l V_s(t) - A(t)}{S(t)} \]

and similarly, when the funding level exceeds the upper limit, we have

\[ \pi^*(t) = \pi(t) - \alpha_h(t)\frac{A(t) - \psi_h V_g(t)}{S(t)} \]

The benefit paid for each retiree will be a multiple, \( h(t) \), say, of their target benefits, where, if the funding is below the lower threshold,

\[ h(t) = 1 - \beta_l(t)\frac{\psi_l V_s(t) - A(t)}{B(t)} \]

and if the funding is above the upper threshold,

\[ h(t) = 1 + \beta_h(t)\frac{A(t) - \psi_h V_g(t)}{B(t)} \]

To set the values for \( \alpha_l \) and \( \beta_l \), we use the fact that when \( A(t)/V_s(t) < \psi_l \), then the total proportion of deficit distributed is \( 1/\tau_l \), so

\[ \alpha_l(t) + \beta_l(t) = \frac{1}{\tau_l} \]
and similarly in the surplus case,
\[
\alpha_h(t) + \beta_h(t) = \frac{1}{\tau_h}
\]

Also, based on the optimization results from Zhu et al. (2020a), specifically the result reproduced above in equation (3), we set
\[
\frac{\alpha_h(t)}{\beta_h(t)} = \frac{\alpha_l(t)}{\beta_l(t)} = \frac{S(t)}{B(t)}
\]

\[
\Rightarrow \alpha_h(t) = \frac{S(t)}{\tau_h(S(t) + B(t))}, \quad \beta_h(t) = \frac{B(t)}{\tau_h(S(t) + B(t))}
\]

and similarly
\[
\alpha_l(t) = \frac{S(t)}{\tau_l(S(t) + B(t))}, \quad \beta_l(t) = \frac{B(t)}{\tau_l(S(t) + B(t))}.
\]

With this risk sharing, we no longer apply the 30% maximum contribution to the plan. We do retain the wind-up feature, so that if the solvency A/L ratio falls below 0.5, the plan is wound up and all accrued benefits are reduced in proportion to the solvency A/L ratio.

### 4.2 IRS plan results

In Figure 6 we show the quantiles and sample paths for the IRS plan A/L ratios and total contribution rates. The A/L ratio quantiles are very close to those in Figure 3, but viewing the individual paths (which are from the same scenarios for all diagrams), we see that none of the paths in this figure involves a wind-up, compared with three wind-ups, from the sample of 30 paths, in Figure 3.

When we consider the total contribution rates, again, the quantiles are very similar to the constrained example from Figure 3. Even though there is no constraint on contribution rates in this case, the 95% quantile is very close to the maximum rate (30%) in Figure 3, but there are occasional paths with higher rates, and in some cases, the contributions remain above 30% for several consecutive years. However, the upside volatility is very much reduced from the DB plan in Figure 2.

In Figure 7 we show 100 sample income paths for the cohort with starting age 45, using the same economic scenarios as in Figure 4. In the working phase, the high side paths are similar; in this figure, these are cases where the contribution rate is zero because surplus is high. The low side scenarios are more variable here, as there is no maximum contribution rate. In Figure 4, which was produced using the same economic scenarios, there were three pre-retirement wind-ups and 3 post-retirement. In this figure there are no pre-retirement
(a) Assets/Liabilities: Going Concern (left side) and Solvency (right side)

(b) Total Contribution Rates, with median NCR (dashed line)

Figure 6: IRS plan, 50% wind-up A/L threshold; 10,000 projections, 5%, 25%, 50%, 75% and 95% quantiles, with 30 sample paths.
wind-ups, and two post-retirement. Apart from the cases of wind-up, the benefits here are more volatile, as we would expect, given that the retirees are participating in deficit and surplus reduction in this case.

In Table 2 we show the default probabilities and average total contribution rates for the IRS DB plan, to compare with the results in Table 1. We see that the default risk for both the DB and IRS plans are very low for equity weightings between 10% and 35%, but that the IRS default risk stays low up to an equity weighting of around 50%. The average total contribution rates are lower for the IRS plan, except for high equity weightings, where the IRS total contribution rate becomes slightly higher.

The relationships in the table are shown graphically in Figure 8. On the left side we see default probabilities by equity weighting. Although the IRS risk starts increasing for weightings over 55%, the risk at 60%, which is a standard industry weighting, is still much lower than for the DB plan, at only 1.2%. In the right hand plot, we show the average contribution rates for the two plans, for different equity weightings. At 60%, the contribution rates are very close, but as the default probability is substantially lower, the IRS plan appears more suited to the higher equity investment proportions which have been so popular.

At very high equity weightings, the traditional DB plan has a lower average contribution rate, because of the cap of 30%, and because the traditional DB plan is more likely to wind up when it is in deficit, whereas the IRS plan is more likely to continue, with high
Equity Weighting | Probability of Default (SE) | Mean contribution rate (SE)
---|---|---
0.00 | 1.04% (0.03%) | 21.68% (0.01%)
0.10 | 0.16% (0.01%) | 20.89% (0.01%)
0.20 | 0.03% (0.00%) | 20.18% (0.01%)
0.30 | 0.00% – | 19.62% (0.02%)
0.40 | 0.00% – | 19.14% (0.02%)
0.50 | 0.11% (0.02%) | 18.69% (0.02%)
0.60 | 1.19% (0.03%) | 18.30% (0.03%)
0.70 | 4.78% (0.08%) | 17.96% (0.04%)
0.80 | 10.82% (0.11%) | 17.67% (0.04%)
0.90 | 19.84% (0.10%) | 17.36% (0.04%)
1.00 | 30.34% (0.10%) | 17.05% (0.04%)

Table 2: Default risk and mean total contribution rate, by portfolio equity weighting, for the IRS plan with 50% solvency A/L ratio wind-up threshold. 10,000 projections; estimated standard errors in parentheses.

High equity investment is a tenacious practice in pension investment management, presumably representing an acceptable balance point between cost and risk. It appears that the IRS plan allows a higher equity investment, closer to the 60% that is common for DB plans, without significantly increasing the wind-up risk, relative to the minimum, and at a slightly lower average contribution rate compared with the DB plan. The IRS plan thus appears to dominate the traditional DB with respect to affordability and sustainability. This is not surprising, given that surplus and deficit are spread over all plan members, rather than only the actives. However, the IRS plan may lack the security of pension income provided by the traditional DB plan. To investigate this, we consider the benefit stability metrics for a range of cohorts, identified by their age at the start of the projection.

In Figure 9 we show the curves of the stability measure for the same cohorts as in Figure 5. We see that for the age 30 cohort, the IRS has improved the stability metric, at all equity weightings. The same is true for the age 40 cohort, but the difference is small. At age 50, the curves are almost indistinguishable. At age 60, the difference is more significant. Only a small number of new entrants to the plan will be over age 50, so the IRS plan appears superior to the DB plan going forward. New entrants at all ages up to 55 will be no worse off, based on this metric, than if they had joined the traditional DB plan – and at younger entry ages they are considerably better off. However, if the DB plan were to
transition to the IRS plan, the older cohorts would be disadvantaged. This was the main focus of Zhu et al. (2020b), which was described briefly in Section 2. The results of that analysis showed that phased transition from the DB plan to the IRS plan could maintain the stability for older plan members. In the following section we explore phased transition for these more real world examples, in more detail.

5 Transition from DB to IRS

The IRS plan operates under a different informal social contract to the DB plan. Under a funded DB plan, in principle, workers pay for their own retirement benefits through their own working lifetimes, so that they should not need to be funded by contributions from the cohorts that follow them. However, in practice, shortfalls and surpluses in the assets supporting accrued benefits do arise, as a result, for example of mismatched assets, or over-optimistic valuation assumptions. This creates an intergenerational transfer of risk and funds as shortfalls in investment returns on retiree funds are offset by additional contributions paid by current workers. On the other side, surplus built up while a person is working may be distributed after they have retired, in which case, under the traditional DB system, it is distributed to the then current workers.

Under the IRS plan, workers and retirees share deficits and surpluses. This means that intergenerational transfers are used to smooth the benefits and contributions, rather than just the benefits, and that the mechanism for transfer is more transparent. The result,
Figure 9: Stability metrics for DB plan and IRS plan by equity weight (lower values indicate more stable income streams); 10,000 projections.
as we have illustrated in the previous section, can be beneficial from the perspective of reduced risk and improved income stability. However, at transition, some cohorts will be disadvantaged if the new plan is not phased in. Those near or in retirement have, through their working lives, effectively underwritten the benefits of the cohorts before them. After transition, they must take part in the risks of the plan. They may be seen as getting the worst of both systems.

The response to this problem, developed in Zhu et al. (2020b), is to allow for a phasing in of the risk sharing, by allocating different values of $\beta$ to the different cohorts in place at transition. In that paper, the phasing-in took effect for lives around age 80 at transition, for a plan that is fully funded at transition. In this study, the age appears to be lower, based on the age 60 plot in Figure 9.

We assume that the traditional DB plan described in Section 3.7 is planning a transition to the IRS plan design described in Section 4. We assume an equity weighting of 60%, and that all those in employment at the time of transfer participate fully in the contribution side of the risk sharing of the IRS plan immediately on transition, but may only partially participate in benefit risk sharing. Those in or near to retirement at the time of transition will have benefit adjustments (positive or negative) reduced by a participation factor, $\gamma(x)$, where $x$ is the cohort age at transition, and $0 \leq \gamma(x) \leq 1$. This means that the adjustment to benefits for retirees aged $x+t$ at $t$ is determined by the parameter $\beta(t) \gamma(x)$.

The purpose of phasing in the new plan is to reduce the downside risk for existing cohorts. We therefore use the same stability measure as described in previous sections, except that here only downside stability is measured. For all ages where the expected downside stability is improved by the transition to the IRS plan, we set $\gamma(x) = 1.0$. For all cohorts for whom the expected downside stability measure is increased (i.e. worsened) after transition to the IRS plan, we set $\gamma(x)$ such that the expected downside stability is constrained to be equal to the value under the DB plan. Then in benefit risk sharing, the relevant cohorts would have benefits paid at age $x+t$ adjusted by a factor of

$$h(x+t, t) = 1 - \beta_l(t) \gamma(x) \frac{\psi_l V_s(t) - A(t)}{B(t)}$$

for deficit distribution, and by a factor of

$$h(x+t, t) = 1 + \beta_h(t) \gamma(x) \frac{A(t) - \psi_h V_g(t)}{B(t)}$$

for surplus distribution.

The values of $\gamma(x)$ are found by iteration. The results for our benchmark example are shown in Figure 10.
Because, under this scheme, older cohorts do not fully participate in the risk sharing in the years immediately following transition, this arrangement means that surplus and deficits are paid down more slowly than for the case of full participation, creating slightly higher stability measures for younger cohorts. Nevertheless, the expected downside stability measure is still improved compared with the DB plan. The overall impact on solvency over the 30 year projection of phasing in the IRS plan is an increase from an estimated default probability of 1.2% for the plan without phasing in, to 1.4% with the phasing in adjustments. The overall impact on contribution rates is an increase from 18.30% to 18.35%.

Improving the stability measures for older cohorts results a slight worsening of the stability measure for younger cohorts; the impact is shown in Figure 11.

We make the following observations.

1. The phasing in adjustments apply at much younger ages in this more realistic plan than in the stylized plan of Zhu et al. (2020b). However, it is notable that the pattern of phasing in adjustment parameters is very similar to that in Zhu et al’s paper.

2. Although the phasing in results are slightly less favourable from the perspective of default and total contribution rate compared with the unphased in plan, the results overall have significant advantages over the traditional DB plan, particularly in terms of default risk.
3. It might be possible to adjust some other parameters through the phasing in period to recover the lower default risk. Possible candidates are the upper threshold for distributing surplus, and the period of surplus or deficit distribution ($\tau$).

6 Impact of the initial funding level

It is interesting to repeat some of the analysis from the previous section for the case where the initial funding level is below 100%. In this section, we consider the DB and IRS plans exactly as above, but with starting asset-liability ratio (based on the going-concern liabilities) of only 80%.

In Figure 12 we show the quantiles and sample paths for the solvency A/L ratio, together with total contribution rates. For the DB results, we assume, as in Figure 3, that the total contribution rate is capped at 30% of pay, and that the plan will be wound up if the solvency asset-liability ratio falls below 50%. We see that it is difficult for the plan to recover to a fully funded position – after 30 years, the median solvency funding level is only around 84% – because the additional contributions are constrained by the 30% cap, which is insufficient to make up ground. By the 22nd year of the projection, there is more than a 5% chance that the plan has been wound up, as indicated by the lower 5% quantile path. Overall, the risk of wind-up is around 9.4%. From the total contribution plot, we see...
that the 75% quantile path for contributions remains close to the maximum contribution rate of 30%.

From the IRS plots, we see that insolvency risk is much reduced, but the median funding position is only slightly better than the DB case. The 75% and median contribution rates are lower than the DB case, but the 95% quantile is higher, at around 31.5%.

Given the risk of higher contributions under the IRS plan, it might be conjectured that workers would prefer to be in the DB plan than the IRS plan, but that is not the case, based on the stability metric. In Figure 13 we show the stability metrics for different equity weights when the starting A/L is 80%. For the age 30 and 40 cohorts, the curves are similar to the fully funded case in Figure 9, although shifted up (note the y-axis scale changes); for the age 50 cohort the IRS plan now appears more distinctly preferable to the DB plan, and similarly for the age 60 cohort, for equity weightings greater than around 25%. We note also a slight shift in the optimal equity weighting (for both DB and IRS) in terms of optimizing stability, from around 25% to between 30% and 45%.

These results indicate that workers, from the youngest ages right up to retirement, are better off in the IRS plan than the DB plan, if the funding level is low. After retirement, the DB plan is preferred, despite the increased probability of wind-up, as the default risk under the DB plan is outweighed by the benefit adjustment risk under the IRS plan. When a DB plan is underfunded, if default is avoided, then the retirees are the beneficiaries; having under-contributed to the plan, they are now subsidised by the current workers. In Figure 14 we show the stability measures by cohort age, assuming a 60% equity investment proportion. The left side plot shows the curves for plans which are fully funded at the start of the projection, and the right hand plot shows the curves for the plans which are 80% funded at the start of the projection. We see that for a fully funded plan, with a 60% equity investment weighting, the IRS plan gives better stability measures at all ages up to around 50, but for the plans which are only 80% funded at the start, the IRS gives better results at all ages up to the retirement age, 65.
(a) DB plan: Solvency A/L (left side); Total Contribution Rates, with median normal contribution rates (dashed line) (right side)

(b) IRS plan: Solvency A/L (left side); Total Contribution Rates, with median normal contribution rates (dashed line) (right side)

Figure 12: DB and IRS results with starting A/L=80%; 10,000 projections, 5%, 25%, 50%, 75% and 95% quantiles, with the same 30 sample paths as in previous figures.
Figure 13: Stability metrics for DB plan and IRS plan by equity weight (lower values indicate more stable income streams); 80% initial A/L; averaged over 10 repetitions of 1000 projections.
7 Changing dependency ratio

Figure 14: Stability measures for DB plan and IRS plan by age, assuming 60% equity investment. Left side fully funded, right side 80% funded at start; 10,000 projections.

If a pension plan is to be sustainable, it should be robust to demographic shifts. In this section, we compare the two plans under a scenario where the new entrants are fewer and older. The unit credit funding method creates a normal contribution rate which is an average of the rates for individual cohorts. The cost of accruals at younger ages is considerably less than the cost at older ages. If the age profile of the plan shifts, then the normal contributions will also shift. The DB plan will be more likely to hit the 30% maximum rate, leaving the plan more exposed to the risk of wind-up. The benchmark DB and IRS models used in this paper have a relatively stable population and age profile of active workers, and a slightly increasing population of retirees.

In this section we examine the impact of a changing demographic profile for the plan membership. We start with the same population, but assume a smaller and older pattern of new entrants to the plan. The result is that the average age of the active plan membership increases from 47.0 to 53.2 over the course of the 30-year projection. In comparison, there is an increase of only 0.1 in the average age under the benchmark model.

In Table 3 we show statistics for the DB and IRS plans, comparing the benchmark model with the ageing demographics model. Figures assume an equity investment weight of 60%. 

33
<table>
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<th></th>
<th>DB Benchmark</th>
<th>Ageing</th>
<th>IRS Benchmark</th>
<th>Ageing</th>
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</thead>
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<td></td>
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<tr>
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<td>18.3%</td>
<td>19.5%</td>
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<tr>
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<tr>
<td>Age 75</td>
<td>6.6</td>
<td>7.0</td>
<td>14.2</td>
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</tr>
</tbody>
</table>

Table 3: Comparison of benchmark plan with plan using aging workforce; 10000 projections, 60% equity investment weighting.

We see that in terms of default risk, the IRS plan is more robust than the traditional DB plan. In both cases, the aging workforce model creates increases in the average contribution rates, and in the stability measures. The impact on plan members is most significant for younger workers, and for those near retirement for the traditional DB plan.

8 Heterogeneous plan membership

For many organisations that have retained DB plans, a single plan covers all pensionable appointments, from the lowest to the highest ranks. There are differences in the average experience of salaried and non-salaried employees that impact the costs and risks of the plan.\(^5\) In this section we explore the implications of the following two sources of heterogeneity, both associated with a non-salaried (eg hourly paid) work force.

(1) Non-salaried workers have a much flatter earnings progression than salaried employees. Typically, there is a short initial period of increases, after which wages are relatively flat.

(2) Non-salaried employees typically have higher mortality than salaried employees.

\(^5\)We use ‘non-salaried’ to refer to lower paid workers, typically employed in manual or junior clerical roles, while ‘salaried’ refers to employees in managerial streams.
To illustrate the effect of salary heterogeneity on plan costs, we run the DB and IRS models, assuming a 100% non-salaried workforce. We assume promotional salary increases apply up to age 30, after which only inflationary salary growth applies. The in-force salary and pension assumptions are adjusted in proportion. The effect is that a non-salaried worker starting employment at age 25, on the same starting income as a salaried worker, earns around 40% of their salaried colleague at retirement.

In Figure 15 we show quantiles and sample paths for the solvency asset-liability ratios, and for the total contribution rates, for a 100% non-salaried workforce. Compared with the equivalent plots for the salaried workforce, in Figure 3 and Figure 6, we see much lower downside risk in the solvency ratio, and much lower upside variability in the contribution rates. We also notice that the DB and IRS plots, especially for contributions, look similar. This is because if the A/L ratios stay within the upper and lower bounds, the contributions and benefits under the two plans are identical. With a non-salaried workforce, the upper and lower thresholds are less frequently breached, and when they are it is usually by a small amount, so the adjustment to contributions is small.

Additional summary results are shown in Table 4. We show the default rates, average total contribution rates, and average replacement rates for the non-salaried employees, together with the results for salaried employees. The table shows that the default risk and contribution rates are substantially lower for the non-salaried workforce. The difference in the average total contribution rate is 410 b.p. for the DB plan, and 360 b.p. for the IRS plan. The implication of this is that where non-salaried and salaried employees are in the same plan, and paying the same contribution rates, there is a significant subsidy of the salaried workers by the non-salaried. Furthermore, sharing the plan with the salaried workers significantly increases the default risk for the non-salaried workers – their plan could wind-up, and their benefits could be reduced, because of the additional cost and volatility created by the salaried workers. Table 4 illustrates that neither the traditional DB plan nor the IRS plan satisfy the fairness criterion, which says that heterogeneous groups should be treated equitably within the plan. This feature of traditional DB plans is quite well known, at least anecdotally, amongst pension actuaries. It is therefore surprising that labour unions representing the non-salaried workers are champions of the traditional, final average salary DB plan.
Figure 15: DB and IRS results with non-salaried workforce assumptions; 10,000 projections, 5%, 25%, 50%, 75% and 95% quantiles, 30 sample paths.
The disparity becomes even more acute if the non-salaried workers experience a higher mortality rate after retirement. In Table 5 we show default rates and average total contribution rates for the non-salaried workforce, assuming that the expected number of years of retirement is 4 years less than that of the salaried workforce. This difference in expected lifetimes by socio-economic group is consistent with the empirical results in Bushnik *et al.* (2020).

There are several ways to mitigate the disparity between salaried and non-salaried workers, with respect to the cost and security of their pension benefits.

1. Run two separate plans. This is a fairly common solution, but can lead to problems of governance. At least if all employees are in the same pension plan, then the interests of managerial level employees are aligned with those of the non-salaried employees. Often with two plans, the non-salaried employee plan is less well-funded than the salaried employee plan.

2. Impose a cap on pensionable salary. If most salaried employees exceed the cap over the last few years of employment, then their results will look more like the non-salaried employees. The maximum benefit payable under the Canadian Income Tax laws creates a natural cap on pensionable salary, although some plans (including, for example, the University of Waterloo plan, which covers employees at all ranks) offer top up benefits for the highest paid workers, thereby undoing the mitigating effects of the cap.

3. Use a career average revalued earnings (CARE) plan design. In the traditional DB plan, and in the IRS plan discussed above (which is very close to a traditional DB plan), the pension benefit is based on the earnings near retirement, while the contributions are based on the average earnings over the worker’s whole career. It is easy to see why a steeper salary scale will give more expensive, and more variable benefits than a flat salary scale, relative to the contributions. Under the CARE design, the benefits are based on the worker’s career average earnings, revalued for inflation to retirement. In the following section, we explore the CARE design in more detail.
Table 4: Default rates, average total contribution rates and average replacement rates for DB and IRS plans, assuming 100% non-salaried workforce; based on 10,000 projections.

<table>
<thead>
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<th>Default Rate</th>
<th>Ave. Contn Rate</th>
<th>Ave. Replacement Rate</th>
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<tr>
<td>DB, Salaried</td>
<td>4.9%</td>
<td>18.5%</td>
<td>42.5%</td>
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<tr>
<td>DB Non-salaried</td>
<td>0.2%</td>
<td>14.4%</td>
<td>43.5%</td>
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<td>IRS Non-salaried</td>
<td>0.04%</td>
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<td>43.5%</td>
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Table 5: Default rates, average total contribution rates and average replacement rates for DB and IRS plans, assuming 100% non-salaried workforce with reduced life expectancy; based on 10,000 projections.

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<th>Ave. Contn Rate</th>
<th>Ave. Replacement Rate</th>
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<td>IRS Non-salaried</td>
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<td>44.3%</td>
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9 Career Average Revalued Earnings Model Plan

To illustrate the characteristics of the CARE design, we have re-run the model plans (DB and IRS) from the previous sections, but assuming benefits are now based on Career Average Revalued Earnings. The main points to note about the CARE model and assumptions applied in this section are as follows.

- The annual pension at retirement is \( B_{65} = \alpha (TPRE)_{65} \), where \( \alpha \) is the accrual rate, and \( (TPE)_{65} \) represents the total past earnings, revalued for price inflation from the end of the relevant year of employment to age 65.

- The replacement rate for new retirees from the Final Average Salary (FAS) plan is around 43%, for both salaried and non-salaried employees. Using the same accrual rate (1.8%) for the CARE plan gives an average replacement rate of 33% for the salaried employees, and around 41% for the non-salaried employees. To achieve the same replacement rate for salaried employees under the CARE design as they receive under the FAS design requires the accrual rate to be increased to 2.33%, but this rate is significantly higher than those used in practice; the maximum permissible accrual rate under Canadian tax laws is 2.0%. In the remainder of this section, we have
used an accrual rate of 2.0% for the CARE plans, but we also give some summary statistics for $\alpha = 1.8\%$ and $\alpha = 2.33\%$ in Table 6.

- We assume that the revaluation rate each year is the inflation rate, bounded below at 0%, and capped at 3%, which means that accruals are adjusted for COLA identically to pensions in payment.

- Commuted values paid on death or withdrawal before age 65 do not include any revaluation to age 65, but do allow for COLA in payment.

- The input values for $(TPRE)_x$ at the start of the projection are consistent with the starting salaries, service, and salary scale assumptions used as inputs for the FAS plans in previous sections.

- The going-concern (Projected Unit Credit) valuation now projects accrued benefits for inflationary increases to retirement, not for salary growth rates. This means that the PUC valuation liability is generally closer to the TUC liability.

- In the figures and tables in this section, we assume that non-salaried workers have a flat salary structure, after age 30, but we do not incorporate any extra mortality, so the salary progression is the only difference between the salaried and non-salaried results.

We note from Table 6 that the disparity between salaried and non-salaried members with respect to default rates and the average total contribution rate has been reduced using the CARE benefits, both for the DB and IRS plans, and for each of the accrual rates illustrated. The difference between the contribution rates for the two groups is between 1.2 and 2.2 percentage points, roughly half of the difference for the final salary plans. However, a new disparity has emerged, based on the average replacement rates. At the 2.0% accrual rate, the non-salaried workers would achieve an average replacement rate of around 46% under the CARE plan, compared with 36% for the salaried employees.

At first sight, the difference in replacement rates may seem unfair to the salaried workers, but the benefit formula is exactly the same for the two groups; the non-salaried replacement rate is higher because their final year’s salary is lower. Furthermore, there is a strong argument to be made that lower paid workers (who are mostly in the non-salaried ranks) require a higher replacement rate than salaried employees, as their outgoings after retirement are closer to their outgoings before retirement, and their additional available resources in retirement (savings, housing equity) are likely to be more limited, so that their capacity to meet unexpected shocks in living expenses, or to absorb the effects of inflation exceeding
### Table 6: Default rates, average total contribution rates and average replacement rates for DB and IRS CARE plans, varying accrual rates, based on 10,000 projections.

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The 3% cap on COLA, is much more limited than for the salaried workers. Furthermore, as we see from the results in Table 6, in a shared plan, the salaried total contribution rate is still higher than the non-salaried. Hence, the non-salaried employees in a shared plan pay more than the expected cost of their benefits. That is, sharing the plan with non-salaried employees reduces costs for the salaried employees, even though their replacement rate is smaller. Finally, we reiterate that the expected lifetime of the non-salaried workers may be several years lower than the salaried workers, as noted in the previous section.

In Figure 16 we show quantile plots for the solvency asset-liability ratio for (a) the DB CARE plan, assuming a salaried workforce (right side) and a non-salaried workforce, with (left side) and (b) the IRS CARE plan assuming a salaried workforce (right side) and a non-salaried workforce, with reduced life expectancy (left side), and in Figure 17 we show the total contribution rate graphs for the same design/population combinations.
From the A/L ratios we see that for either group, switching to the CARE plan, with a 2% accrual rate, has reduced the downside risk, as measured by the lower quantiles. This is partly because the benefits for salaried workers are lower, but also because the impact of salary volatility is very much mitigated in this design. We also see that the A/L distributions for salaried and non-salaried workers are very similar, supporting the notion that the CARE plan reduces disparate treatment of the different groups. Viewing the total contribution rate gives a similar picture; the figures for salaried and non-salaried workers are very similar, although the rates for salaried workers are slightly higher.

10 Conclusion

In this paper we have adapted the theoretical results from Zhu et al. (2020a) and Zhu et al. (2020b) in order to apply them to a realistic pension plan model. We have compared a traditional final average salary (FAS) DB plan to an IRS (Defined Ambition) plan, with benefits based on final average salaries. We also introduced both DB and IRS versions of a career average revalued earnings (CARE) plan.

The comparisons between the different designs were based on the five loose criteria outlined in Section 1.1 – that the design should be affordable, sustainable, efficient, adequate and fair. These criteria address both the needs of the contributors to the plan, and the beneficiaries. We found that the traditional FAS plan may be deemed affordable (based on average costs), but is not highly sustainable, taking volatility of costs into consideration. Furthermore, the main selling point of the traditional DB plan, the adequacy and predictability of the benefits, is severely undermined by the possibility of default. In contrast, the IRS plan offers a more stable income, taking both pre- and post-retirement periods into consideration. More stable costs improve the efficiency and sustainability of the pension plan.

In reviewing the fairness of the pension design, we considered two aspects. The first is the effect of transitioning from DB to IRS, which is beneficial for younger lives, but not for older. Following Zhu et al. (2020b), we showed that this can be mitigated with a phased transition. The second aspect of fairness considered is a more embedded problem, that of equal treatment of different employee groups. We examined this by comparing costs and benefits for a non-salaried workforce, which is defined to be an employee group with a flat salary structure. Both the traditional, final average salary DB plan and the final average salary IRS plan failed to treat the two groups equitably. However, the difference was mitigated with a CARE plan design.
Figure 16: CARE plan solvency asset liability ratios; 1000 projections, 5%, 25%, 50%, 75% and 95% quantiles, with 30 sample paths.
Figure 17: CARE plan total contribution rates; 1000 projections, 5%, 25%, 50%, 75% and 95% quantiles, with 30 sample paths.
The traditional DB pension has proved more resilient in Canada than in the USA or UK, for example, but it has been in decline for some time. The plans developed in the 1970’s are not necessarily suitable for today’s economic or demographic climate. The benefits are not portable, but today’s workers are likely to change jobs far more frequently than those in DB plans 30 years ago. Even the standard benefit incorporating a 60% survivor’s pension seems old fashioned in a modern era of dual earning couples, and, incidentally, increases the longevity risk exposure of the plan. Most of all, the reliance on an equity risk premium to foot a large part of the pension cost, which paid off handsomely in the 1980’s and 1990’s, has not proved to be realistic over the past 13 years. If we must resort to equities and other risky assets to fund pensions, then we need to insert more flexibility into the funding and benefit structure. As we have mentioned above, the 60% equity investment benchmark is very tenacious in pensions management, which leads to the conclusion that the traditional DB guarantee needs to be updated.

We see this increased flexibility being introduced in some places. The Defined Ambition plans implemented in The Netherlands are seen as early examples of risk sharing plans Bovenberg et al. (2016). The University Superannuation Scheme (USS) in the UK closed its final average salary plan in 2014, replacing it with a career average revalued earnings plan, with a cap on pensionable salaries. The New Brunswick Public Sector Pension Plan (NBPSPP) is a well known Canadian example of a target benefit plan, implemented to replace an underfunded traditional final average salary DB plan. The target benefits use a career average revalued earnings formula, with a 1.4%/2% tiered accrual rate. The NBPSPP risk sharing protocol is more complicated than the IRS design in this paper, with a series of controls that starts with increasing contributions, and proceeds through reducing COLAs, and ultimately, if necessary, reducing the non COLA benefits in payment. In contrast, the new Ontario University Pension Plan, a ‘jointly sponsored’ pension plan developed in response to critical underfunding of benefits in several Universities and colleges, has replaced the generous benefits of the distressed plans it encompassed with a new, guaranteed benefit final average salary DB plan, with a high accrual rate, (2.0% on earnings above the YMPE\(^6\) and 1.6% below), but with the small additional control of discretionary cost of living adjustments. As a jointly sponsored plan, if things do not work out, the UPP may institute a wind-up without fully funding the benefits, which is not true of the single employer plans that it has replaced. In contrast, the UK’s USS has an accrual rate of approximately 1.6% on a CARE plan.

The IRS design presented in this paper is simple and transparent, especially compared with some of the target benefit plans that have been implemented, and yet works well to meet the

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\(^6\)The Yearly Maximum Pensionable Earnings (YMPE), currently $58,700, is the maximum pension for Canada Pension Plan salary related benefits.
needs of both contributors and beneficiaries. With CARE type benefits, it also achieves a more equitable treatment of different employee groups than the FAS benefits. Unlike many current DB plans, the cost of living adjustment is not treated as an expendable addendum to the pension, and although the pension is adjustable, the value of this flexibility in terms of avoiding insolvency makes the trade-off worthwhile to members, based on the lifetime income stability measure developed in this paper.

11 Acknowledgements

This project was supported by Global Risk Institute, through the National Pension Hub. Mary Hardy and David Saunders also acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC), funding reference numbers 312618 (Saunders) and 203271 (Hardy).

References


**Appendix**

**Wilkie model parameters**

These parameters are derived from Zhang *et al.* (2018), with some minor adjustments.

\[
\begin{align*}
\mu_q & = 0.0244 & a_q & = 0.5000 & \sigma_q & = 0.0111 \\
\mu_y & = 0.0252 & a_y & = 0.9112 & \sigma_y & = 0.1159 & \sigma_w & = -4.5762 \\
\mu_d & = 0.0 & \sigma d & = 0.1097 & w_d & = 1.0 & d_d & = 0.38 & y_d & = 0.0 \\
\mu_c & = 0.0238 & \sigma c & = 0.28 & d_c & = 0.058 & a_c & = 0.9 & y_c & = 0.0
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### Inforce survival rates

All decrements including death. All active lives who reach age 65 are assumed to retire immediately.

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