Overview:
A Target GDP Approach to Risk and Return in Climate Change Policy

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1 Introduction

Climate change is widely recognized as a serious risk that requires worldwide governmental action. In a recent speech to Lloyd's of London, Bank of England Governor, Mark Carney, drew attention to the risks of failing to act on climate change including the risk to the world's financial systems. To quote from Carney's speech,

"The combination of the weight of scientific evidence and the dynamics of the financial system suggest that, in the fullness of time, climate change will threaten financial resilience and longer-term prosperity. While there is still time to act, the window of opportunity is finite and shrinking."[1]

Unfortunately, there is no consensus amongst policymakers and researchers on the most appropriate policy responses, including how quickly carbon emissions should be cut. Reaching consensus is made more difficult by the enormous uncertainty inherent in projections regarding the timing and extent of the future warming of the earth's climate, the level of damages, and humanity's ability to adapt to a warmer climate.

Adding to this is the fact that the most serious damages are expected to occur many decades into the future. Using typical discount rates, traditional economic analysis would recommend little action be taken at present. Further, world climate is affected by the cumulative stock of atmospheric carbon, so that any actions taken today to reduce carbon emissions will only have an impact after a considerable time lag. Policies to reduce carbon emissions, whether via a carbon tax, cap and trade, or other regulatory schemes, are very costly as they entail a realignment of the economy away from carbon based fuels. Amongst governments and other decision makers there is a natural tension between the desire to delay the implementation of these costly policies and the concern that a more precautionary approach is warranted in the face of these huge uncertainties and time lags.

2 Current Approaches

A significant portion of current research and policy discussions about climate change is based on analysis from Integrated Assessment Models (IAMs), which integrate climate science and economic models to assess the impact of a warming climate.[2] Of necessity, these models include a large number of parameters and functional relationships about which we have little information. As is pointed out by

[1] From "Breaking the Tragedy of the Horizon - Climate Change and Financial Stability", speech given by Mark Carney, Governor of the Bank of England, Lloyd's of London, 29 September 2015."
[2] See Nordhaus (2008), Hope (2006) and Tol (2001a) and Tol (2001b) for a description of three influential IAMS.

Pindyck (2013), different economic analyses of climate change can come to starkly different conclusions. These differences depend on key factor assumptions such as the levels of future greenhouse gas emissions under business-as-usual versus alternate scenarios, projections of average global temperatures and economic impacts, estimates of the costs of abating greenhouse gas emissions, and the rate of discount. This is exemplified by two very influential studies based on IAM's as described in Stern (2008) and Nordhaus (2008). The former argued that immediate action to reduce global carbon emissions would produce significant positive economic returns, while the latter recommended only limited reduction in carbon emissions. The Stern study was criticized for using a very low discount rate, which Stern justified on the ethical grounds that society should give equal weight to the well-being of current and future generations.

A key component in any economic analysis of climate change is valuing the loss in societal welfare that results from reduced economic output with a warming climate. Societal welfare is assumed to depend on consumption levels via a Constant Relative Risk Aversion (CRRA) social utility function which can be specified as follows:

$$U(C_t) = \frac{C_t^{1-\eta}}{(1-\eta)},$$

where U denotes utility which depends on consumption in period t, C_t, and η is the index of relative risk aversion. The impact of climate change on societal welfare is measured by the discounted sum of annual utilities over a long time period (such as 500 years) in different climate scenarios. The discount rate and the coefficient of relative risk aversion are critical to the results, and there is no general agreement on the correct values.

More recently, research has focused on the treatment of uncertainty in scientific estimates of the response of world climate to the increasing atmospheric concentration of greenhouse gases. Most IAMs are deterministic, with uncertainty analyzed by running Monte Carlo simulations on key model parameters (Pindyck 2013). A criticism of much current work is the failure to deal adequately with possible catastrophic outcomes whereby average global temperatures increase by something of the order of 7 to 8 degrees Celsius by the turn of the next century. The probability of such an event is non-zero, and its impact is unknown. The damage functions used in IAM's are calibrated for small temperature changes and are uninformative about the impact of such an extreme event. Weitzman (2014) has pointed out that if the probability distribution of the negative consequences of climate change has a sufficiently fat tail, and if we adopt a sufficiently high degree of relative risk aversion, the optimal policy for society would be highly precautionary.

3 An Alternative: The Target GDP Approach

An alternative approach to this problem, which we refer to as the 'target GDP approach', involves setting a target level for world income or GDP per capita and choosing policies to minimize the risk of falling short of the target. Using this approach, the trade-off between increased risk versus increased wealth is very clear and intuitive. In addition, it avoids the need to make an arbitrary choice for the discount rate and complements the current IAM model based approaches.

3.1 Intergenerational Fairness

Because of the extreme potential consequences of climate change and the long time frames involved, climate change policy decisions are, at their core, questions about intergenerational fairness. A relatively high discount rate implies that we put less weight on costs and benefits that occur far into the future - i.e. those faced by future generations. The Stern report, referenced above, chose an extremely low discount rate in the interests of intergenerational fairness.

Rather than choosing policies to maximize a sum of discounted social utility, we suggest that from the point of view of intergenerational fairness, we should choose climate change policies which enable future generations to enjoy a reasonable expected level of average real GDP per capita in the distant future. Choosing a climate change policy is equivalent to choosing a future path of emissions for greenhouse gases. While this idea is not in conflict with the social utility function approach, we deal directly with the desired end state, rather than indirectly through the use of a utility function and a discount rate.

We denote the desired level of expected future GDP per capita as G_{des}. We aim to achieve G_{des} by specifying greenhouse gas emissions over time, denoted by $e(t)$ where t refers to time. It turns out that in order to achieve an expected level of future GDP equal to G_{des}, we must aim for a target, G^*, which is slightly higher than G_{des}. Mathematically our objective is to determine a rate of carbon emissions $e(t)$ to minimize the risk that we fall below a particular target, denoted G^*, at time T in the future. More specifically, we desire to minimize

$$min_{e(t)} E\left[\left(min(G_T - G^*, 0)\right)^2\right] \tag{1}$$

$E[\cdot]$ = Expectation
$e(t)$ = emission rate
G^* = Target real GDP per capita at time T
G_T = Observed real GDP per capita at time T

Note that according to Equation (1), a large shortfall is penalized more than a small shortfall. In Equation (1) the expected value of GDP at time T will be less than G^*, i.e., $E[G_T] < G^*$. We therefore choose G^* so that $E[G_T] = G_{des}$. This idea is intuitive as noted above, in order to achieve G_{des} on average, our target GDP must exceed our desired GDP.

While use of the objective function (1) is certainly reasonable by itself, it turns out that any emission strategy $e(t)$ which is optimal for problem (1) is also dynamically mean-variance optimal (Li & Ng 2000, Zhou & Li 2000, Dang & Forsyth 2016). In other words, given a specified mean GDP, $E[G_T] = G_{des}$ and the optimal control $e(t)$ which solves (1), no other strategy has a smaller variance.

This also leads us back to a familiar concept in finance: the mean variance efficient frontier. Higher future expected GDP per capita comes at the cost of greater risk. More certainty about future outcomes is only possible at the expense of lower GDP. The choice of where to be on this frontier is a policy decision. However, given any specified future mean GDP per capita, we can determine the emission strategy which results in this mean value, with the smallest risk.

Objective function (1) is thus simultaneously optimal under two criteria. The optimal emissions policy $e(t)$

- minimizes quadratic shortfall with respect to the target G^*;
- is dynamically mean-variance (M-V) efficient.

In the following, we will refer to the optimal policy which minimizes (1) as M-V optimal.

Note that it is possible (borrowing a concept from the optimal trade execution literature) to add an additional term in the objective function (1) which penalizes the quadratic variation of the GDP per capita, forcing the minimization of the time averaged volatility of GDP per capita in addition to minimizing shortfall. (Forsyth et al. 2012) However, we will keep things simple here, to avoid more parameters.

3.2 Basic Modelling Assumptions

We summarize here some basic modelling assumptions, based on a growing consensus of work.

- Global climate is affected by the mean temperature, which is determined, in the long-run, by the cumulative stock of global atmospheric carbon.
- It is important to take into account the uncertainties in the response of the climate to increasing stock of carbon (i.e. concentration of greenhouse gases). Modelling uncertainty is a familiar concept in finance. In the climate change case, we model the global temperature as a mean reverting stochastic process, which mean-reverts to a level determined by the global stock of atmospheric carbon.
- The rate of increase of global (and country) GDP per capita is
    i. an increasing function of the rate of carbon emissions (due to increasing conventional production, which generates carbon emissions);
    ii. a decreasing function of the global temperature (due to negative environmental effects).
- If the global temperature exceeds a critical temperature, T_{crit}, then there is a non-negligible probability of a catastrophic negative effect on GDP per capita (e.g. extreme weather events may cause large insurance losses, crop failures, water shortages). We assume that once this level of temperature is reached, governments will be forced to take drastic action.
- The global stock of carbon, if no anthropocentric carbon is emitted, decays slowly to a natural background mean level (but still is affected by random variations).

3.3 Climate Change Policy: Optimal Stochastic Control

Our climate policy problem can now be posed as an optimal stochastic control problem. The stochasticity comes about from the uncertain effect on global temperature due to increasing global atmospheric carbon stock. The control in this case is the level of carbon emissions allowed each year. We can imagine that this level can be controlled by a cap and trade or carbon tax policy.

We can see that there is a trade-off in choosing an emissions control policy, $e(t)$. Reducing emissions now will cause an immediate hit to GDP, but lower the carbon stock in the future. The benefit of lowering the carbon stock will take some time to positively affect the GDP growth rate, and with uncertainty about when this positive effect will occur. On the other hand, choosing a larger emission rate now, will allow for short term boosts to the GDP, but will have negative long term consequences.

In addition, we should be cognizant of political realities. One could envision a scenario where emissions are lowered, but the global temperature increases due to random effects. This will cause an immediate short term reduction in GDP. We will add a constraint that in the event that current GDP is below a critical level, G_{crit}, then current emissions are increased to ensure that the minimum GDP is obtained.

However, there is no free lunch with this policy. Once the global temperature exceeds the critical temperature T_{crit}, we impose the constraint that emissions are set to zero. In other words, we assume that once the global temperature has reached an obvious dangerous level, political pressure will require that emissions be drastically reduced.

We will formulate this optimal control in terms of the solution of a Hamilton-Jacobi-Bellman partial differential equation. In general, closed form solutions are not possible. We will determine the optimal control using numerical algorithms.

4 Policy and Financial Implications of the Research

This research will contribute to our understanding of one of the most difficult problems society is grappling with - how much should current generations sacrifice to ensure that climate change does not seriously impair the wellbeing of future generations, given the huge uncertainties involved including the possibility of “tail events”. For several decades climate negotiations have been stalled as world leaders have been unable or unwilling to commit to the large scale reductions in carbon emissions recommended in IPCC reports. The target GDP approach offers a way of determining the best decisions in terms of the risk- return trade-off, making it of particular value to governments, policy makers, businesses and NGOs.

The interest of the financial industry in helping to mitigate climate change is clear. As noted by Mark Carney, reductions in GDP growth due to climate change, as well as increases in the frequency of extreme weather events, will reduce the resiliency of the world's financial systems. This could occur through increased insurance claims, underfunded pension plans, increased mortgage defaults, and lower demand for commodities which would have a large impact on the Canadian financial system. Impairment of the world's financial systems will hinder the ability of policy makers to alleviate the harm from natural disasters as well as to promote adaptation mechanisms for communities experiencing the worst effects of climate change. This research will also have implications for the securitization of climate change risk, whether through reinsurance or weather derivatives.[3]

[3] See recent articles in the Economist. “Compacts of god" (May 30,2015) and “Catastrophe Bonds, Perilous Paper" (October 5, 2013).

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