

# Building Explainable Machine Learning Systems in Finance: Fuzzy Logic-Based Neural Networks

**Author:** Alexey Rubtsov  
*PhD, Senior Research Associate, Global Risk Institute*



## Abstract

A Neuro-Fuzzy System (NFS) is an Artificial Neural Network (ANN) that operates on the principles of Fuzzy Logic. One of the key advantages of NFSs is that they can successfully address the problems of explainability and data scarcity besetting traditional ANNs. Thus, it follows that NFSs can enhance the range of Machine Learning applications in finance and the problems deemed inappropriate for the use of traditional Machine Learning tools can be revisited and perhaps solved using NFSs. This paper discusses NFSs and their merits and challenges for financial services.

## INTRODUCTION

A recent surge of interest in Machine Learning (ML) tools within the financial services industry has exposed many challenges of ML applications in financial modelling. Two of the most prominent challenges are data scarcity and the lack of interpretability of some ML techniques (e.g., Artificial Neural Networks).<sup>1,2</sup> Although a few approaches have been proposed to address these issues, none can yet be regarded as a complete solution.<sup>3</sup> This paper explores an alternative approach with an outstanding record of success in the engineering field: Neuro-Fuzzy Systems, a combination of Artificial Neural Networks and Fuzzy Logic.

Originally postulated by University of California, Berkley Professor Lotfi A. Zadeh in 1965, fuzzy logic has a wide range of applications, from automatic transmissions and washing machines to microwaves and camcorders (see Kosko (1999)). Although it is a relatively new area of science, fuzzy logic is also used in finance to assess credit risk (Baensens et al. (2003)) and asset pricing (Linn and Tay (2007)), among other functions (see Sergueeva et al. (2017) for a review). This paper describes Fuzzy Logic in the context of financial modelling and demonstrates how it can resolve the data scarcity and explainability issues that plague many ML applications in the financial sector. Two case examples are provided to illustrate NFSs in practice: credit spread modelling and the well-known “leverage effect” in equity markets.

<sup>1</sup> See for example: BlackRock shelves unexplainable AI liquidity models (Risk.net, November 12, 2018); Big funds muzzle their AI machines (Risk.net, October 15, 2018); Machine Learning Dangerous When Data Thin (Risk.net, January 24, 2019).

<sup>2</sup> To learn more about ANNs, please see our paper: Artificial Neural Networks in Financial Modelling (GRI, March 21, 2019).

<sup>3</sup> Quants call for better grasp of how AI models ‘think’ (Risk.net, July 11, 2018).

## FUZZY SETS

In contrast to the traditional binary logic that classifies a given piece of information as either True or False (i.e., 0 or 1), fuzzy logic measures the degree of truthfulness and assigns the information a weight in the interval between 0 and 1. For example, it may be difficult to categorize in terms of magnitude an annual stock volatility measure of 20 basis points (bp), but one can at least identify where this value falls in relation to what are clearly high (e.g., 30bp) and low (e.g., 10bp) volatilities. Fuzzy logic would assign 20bp a weight between 0 and 1 with which to determine its proximity to the low versus the high term. For example, in the 10bp (0) to 30bp (1) range, 20bp can be classified as equidistant from the high and low bounds; that is, we can say that we are 50% certain that 20bp volatility is high and 50% certain that 20bp is low. This seemingly innocuous modification of traditional logic allows one to create logical rules that are powerful and easily understood by humans. The following example applies fuzzy logic to the financial context.

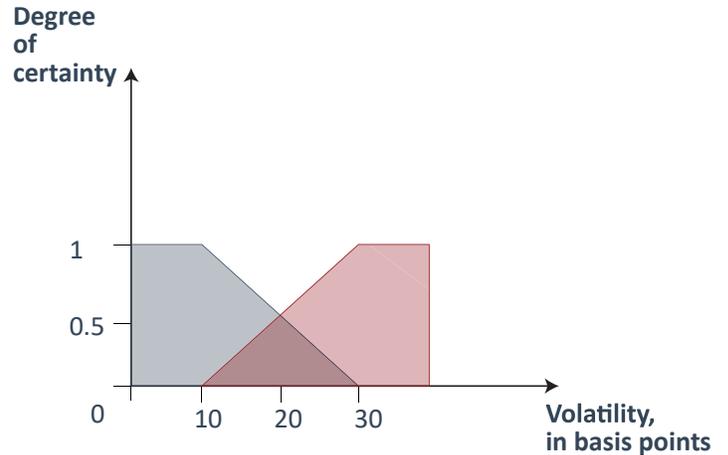
### EXAMPLE 1.

Consider two linguistic rules that describe the well-known “leverage effect” in equity markets, a generally negative covariation between asset returns and their changes in volatility.

**Rule 1:** If volatility is high, asset returns are low

**Rule 2:** If volatility is low, asset returns are high

The precise definitions of “high” and “low” volatility fuzzy sets are given in Figure 1.



**Figure 1.** Fuzzy sets: low volatility (0-30bp) and high volatility (10bp and above). For a given volatility value one can determine the degree of certainty in assigning it to either low or high volatility. Volatility of 20bp is said to be low (with 50% certainty) and high (with 50% certainty).

One interpretation of the “leverage effect” is as follows: when stock prices decrease, firms become mechanically more leveraged because the relative value of their debt increases relative to their equity. As a result of increased leverage, their stock becomes riskier, hence more volatile.

To simplify the exposition, we assume that high/low stock returns are not uncertain in the sense that low stock returns are assumed to be 2% and high stock returns are assumed to be 10%. That is, we do not consider any other values for stock returns.<sup>4</sup> Thus, the rules that we will work with are:

**Rule 1:** If volatility is high, stock returns are 2%

**Rule 2:** If volatility is low, stock returns are 10%

Next, we answer the question:

*According to the above rules, what is the return when volatility is 20bp?*

To answer this question, we will use the degrees of certainty of 20bp to be classified as high and low volatility. This will provide the **weights** (significance) of each rule in determining the return. In particular, the weights of Rule 1 and of Rule 2 are both 50% because we are 50% certain

<sup>4</sup> This approach of constant outputs is known as Sugeno method of fuzzy inference (see Sugeno (1985)).

that 20bp volatility is high (Rule 1) and 50% certain that 20bp volatility is low (Rule 2). In general, the following formula is applied to answer the question:

$$\text{Return} = \frac{\text{Weight of Rule 1} \times \text{Return in Rule 1} + \text{Weight of Rule 2} \times \text{Return in Rule 2}}{\text{Weight of Rule 1} + \text{Weight of Rule 2}}$$

In other words, the return is a weighted average of returns in each rule where the weights are the degrees of certainty. In our example the return is:

$$\text{Return} = \frac{0.5 \times 2\% + 0.5 \times 10\%}{0.5 + 0.5} = 6\%$$

This rather simple procedure of mapping volatility into returns turns out to be quite powerful. Indeed, the fuzzy rules can be regarded as a system (called fuzzy system) that maps inputs (volatility) into outputs (returns) and from this perspective the fuzzy system performs a function similar to the one performed by Artificial Neural Networks (ANNs) with supervised learning. In this respect, the following very important question arises:

*For a given data set, can we determine fuzzy rules that adequately fit the data?*

The positive answer to this question was given by Kosko (1994) who proved that fuzzy systems can approximate any continuous function on a compact set. This fundamental result implies that these systems can be regarded to be as powerful as ANNs when it comes to finding relationships in data. Fuzzy systems that allow one to identify the relevant fuzzy rules based on the available data are known as Adaptive-Network-Based Fuzzy Inference Systems (ANFIS) (Jang (1993)). The advantage of such an approach over ANNs is that the training procedure focuses on fuzzy rules that can be described verbally and in many cases are rather easy to understand. The next example illustrates the use of ANFIS in function approximation.

#### EXAMPLE 2.

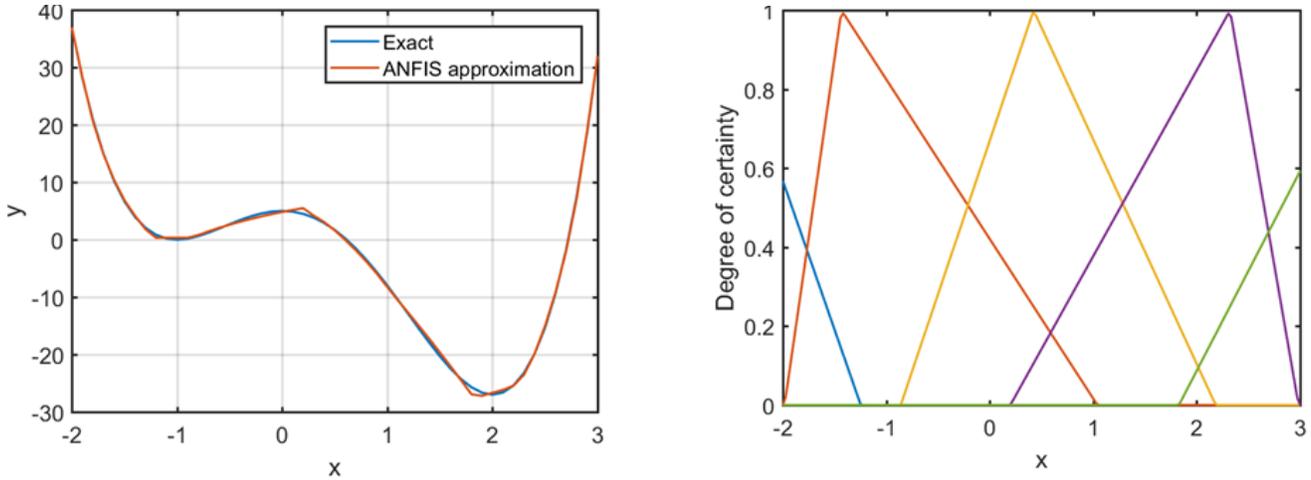
Assume that we want to find linguistic rules to describe the following relationship between  $y$  (output) and  $x$  (input):

$$y = 3x^4 - 4x^3 - 12x^2 + 5 \quad (1)$$

For  $x$  values in the range between -2 and 3, it turns out that an appropriate set of rules that describe the relationship (1) is:

- Rule 1:** If  $x$  is very low, then  $y$  is 36.99
- Rule 2:** If  $x$  is low, then  $y$  is 0.33
- Rule 3:** If  $x$  is about 0.2, then  $y$  is 7.64 (2)
- Rule 4:** If  $x$  is high, then  $y$  is -36.92
- Rule 5:** If  $x$  is very high, then  $y$  is 31.98

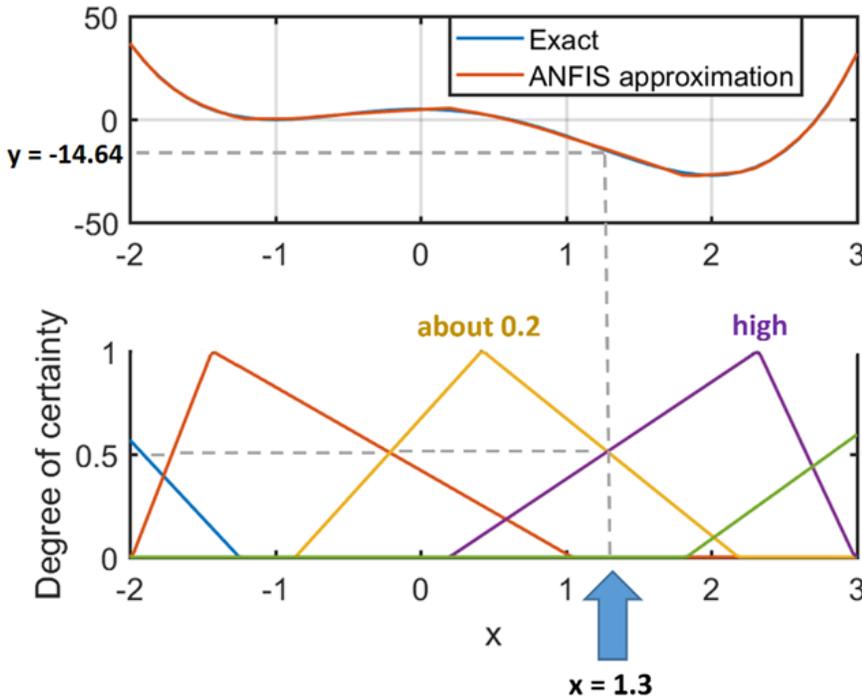
To obtain these rules we used ANFIS as implemented in MATLAB. The accuracy of the rules (2) and the fuzzy sets that characterize input values ( $x$ ) are shown in Figure 2.



**Figure 2.** Function (1) with its approximation (left plot) and the triangular fuzzy sets (right plot) which can be verbally described as (from left to right): very low, low, about 0.2, high, and very high

It follows from the left plot in Figure 2 that fuzzy rules (2) approximate the relationship (1) very well. The linguistic description of fuzzy sets on the right plot in Figure 2 (from left to right: very low, low, about 0.2, high, and very high) is rather subjective and usually follows from the context of the problem. In order to find the value of  $y$  implied by the rules for a given value of  $x$ , we should use the same procedure (formulae) as described in Example 1. That is,

we first determine the certainty of each rule based on the value of  $x$  and then use it to calculate  $y$  as a weighted average of each rule output. The following Figure 3 explains how the fuzzy sets in Figure 2 (right plot) are used to determine the output values ( $y$ ) on the left plot in Figure 2.



**Weighted average output ( $y$ ):**

$$\frac{-36.92 \times 0.5 + 7.64 \times 0.5}{0.5 + 0.5} = -14.64$$

**Active rules when  $x = 1.3$ :**

**If  $x$  is about 2, then  $y$  is 7.64**

**If  $x$  is high, then  $y$  is -36.92**

**Figure 3.** Schematic description of the procedure to evaluate the output  $y$  for a given value of  $x$  based on the system of fuzzy rules (2)

There is no procedure that a-priori determines the appropriate shape of fuzzy sets [e.g. triangular (Figure 2 (right plot)) or bell-shaped (Figure 6)]. However, the criteria that may be used to decide the appropriate form of fuzzy sets could include, for instance, goodness-of-fit, ease of interpretation (in the context of the modelled phenomenon), etc. For example, bell-shaped sets could significantly improve the fit and, thus, allow for fewer rules as compared with triangular-shaped sets.

## LEVERAGE EFFECT AND CREDIT SPREADS

This section provides two examples of ANFIS applications to financial data: modelling of the leverage effect and credit spreads. The examples are chosen to demonstrate the advantages of ANFIS over traditional ML alternatives, and so the model framework is somewhat simplified. The author uses daily data on the iTraxx Europe credit spread index, the 1-year euro swap rate, an equity index constructed from the share prices of the 125 firms that are included in the iTraxx index and the implied volatility index for the Eurostoxx index (Vstoxx). The data span the period from June 2004 to August 2007.<sup>5</sup>

### Leverage effect

Using the available market data, the researcher can derive fuzzy rules that describe the leverage effect: an increase in volatility (Vstoxx) results in a decrease in returns on the market equity index. The rules that are consistent with the data are as follow:

**Rule 1:** If volatility change is about -3%, the index change is 3

**Rule 2:** If volatility change is about 1%, the index change is -1.1

**Rule 3:** If volatility change is about 6%, the index change is -4.9

The goodness-of-fit to the data and the corresponding fuzzy sets for volatility changes are shown in Figure 4.

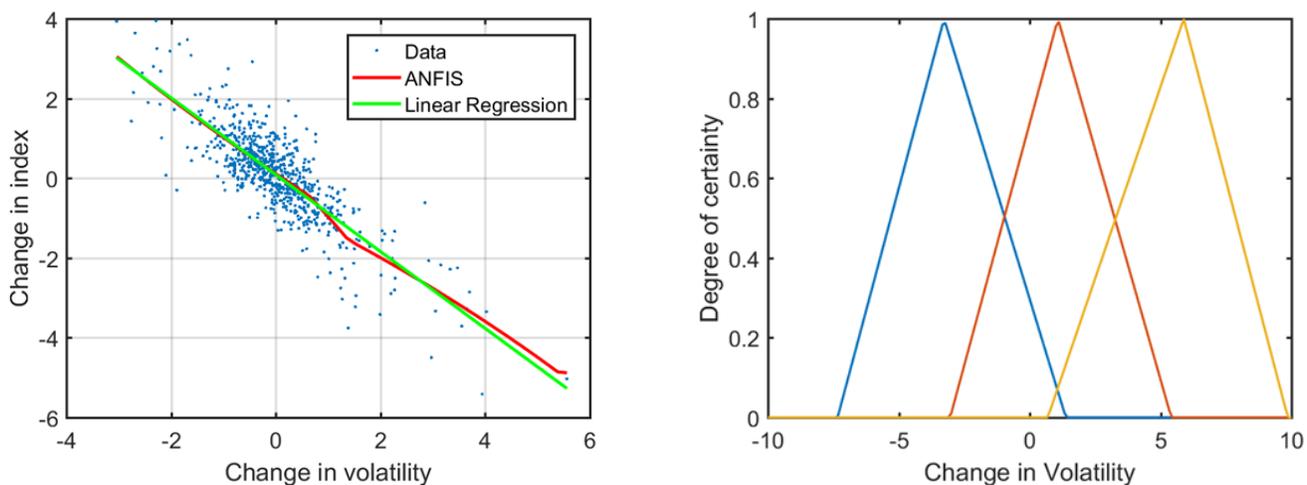


Figure 4. Modelling leverage effect by ANFIS: data with fitted values (left plot) and volatility fuzzy sets (right plot)

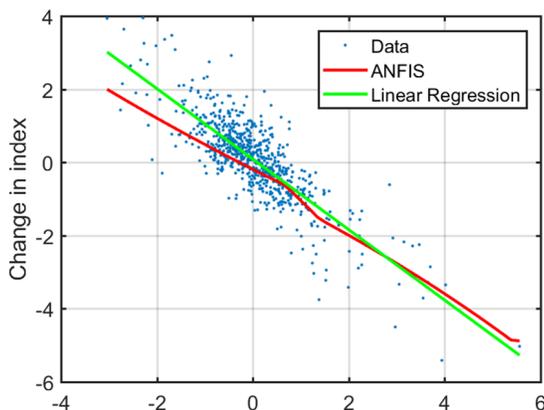
<sup>5</sup> The data is from Carol Alexander’s “Quantitative Methods in Finance: Market Risk Analysis”, John Wiley & Sons Ltd. (2008). This data was also used in Alexander and Kaeck (2008).

It follows from the right panel of Figure 4 that the volatility fuzzy sets can be interpreted (from left to right) as changes in volatility that are: about -3%, about 1%, and about 6%. The left panel of Figure 4 shows that rules provide an approximation that is very close to a linear fit.

As iterated previously, one of the key advantages of fuzzy rules over traditional ANNs is interpretability, which in turn allows the researcher to combine his/her views with the fuzzy rules. For example, it is well known that the leverage effect is generally asymmetric: declines in stock prices are accompanied by larger increases in volatility than the decline in volatility that accompanies rising stock markets (see Nelson (1991), Engle and Ng (1993), Ait-Sahalia et al. (2013), among others). Assume that due to scarce data the researcher believes that this asymmetry is not adequately reflected in the rules. That is, it is not well pronounced in the available data. Since it is Rule 1 that takes care of decreases in volatility, it follows intuitively that in order to reinforce asymmetric index increases for decreases in volatility we can decrease “the index change” in Rule 1 from 3 to, say, 2. With this change, Rule 1 becomes:

**Rule 1:** If volatility change is about -3%, the index change is 2

The result of this change is shown in Figure 5.



**Figure 5.** Modelling asymmetry in leverage effect by decreasing the index change in Rule 1 from 3 to 2.

It is clear from Figure 5 that the change reinforced the required asymmetry in the leverage effect (compare with the left plot in Figure 4).

Once again, it is the interpretability advantage of fuzzy rules that allows researchers to overcome the challenges of data scarcity and adjust the rules according to their own views. It is far from clear how one could achieve the same result with a traditional ANN.

### Credit spreads

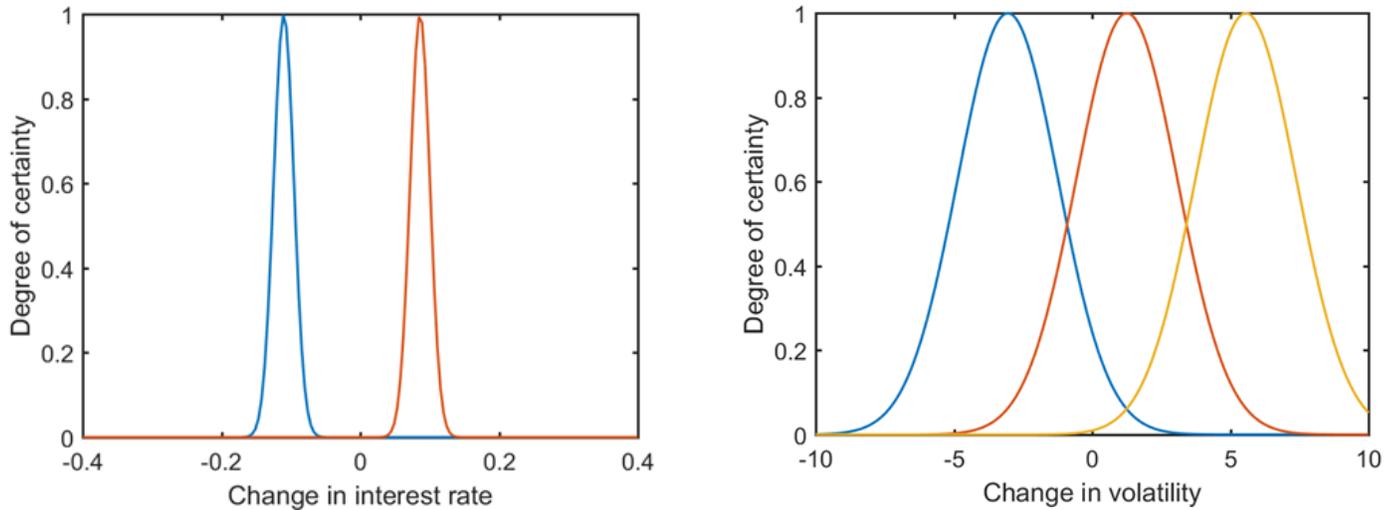
ANFIS can also help to predict changes in credit spreads by the variables determined according to Merton’s (1974) structural model of credit risk. In structural models of credit risk, default occurs when a firm’s value drops below a certain threshold. The theory suggests that interest rates, equity volatility, and equity value are the main determinants of credit spreads. An increase in the interest rate should decrease the probability of default (low interest rates are usually associated with recession). In addition, when the market value of equity of a given firm decreases, the probability of this firm’s default increases. When equity volatility increases, the likelihood of the firm hitting the default threshold increases.

For the sake of illustration, this example uses bell-shaped fuzzy sets (see Figure 6) as opposed to the triangular shapes applied to the leverage effect. Since equity value and equity volatility are (strongly) inversely related (due to the leverage effect), it is sufficient to use only one variable, in this case volatility.

To denote for simplicity:  $\Delta V$  (volatility change),  $\Delta R$  (interest rate change), and  $\Delta S$  (credit spread change). The set of rules that is consistent with the data is as follows:

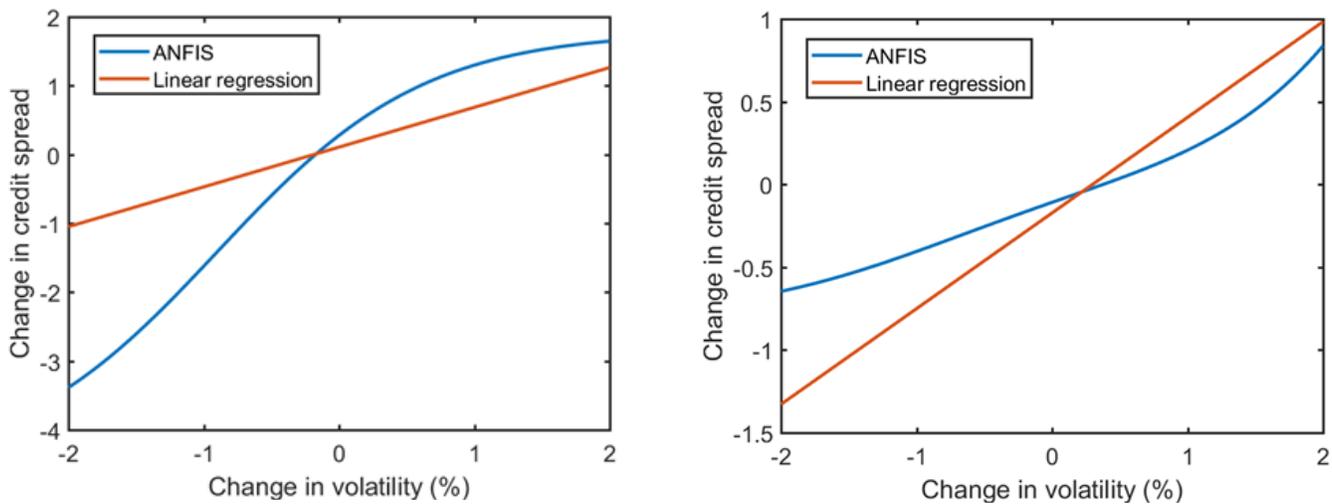
- Rule 1:** If  $\Delta V$  is about -3% and  $\Delta R$  is about -0.11,  $\Delta S$  is - 4.6
- Rule 2:** If  $\Delta V$  is about 1% and  $\Delta R$  is about -0.11,  $\Delta S$  is 1.8
- Rule 3:** If  $\Delta V$  is about 6% and  $\Delta R$  is about -0.11,  $\Delta S$  is 1.7 (3)
- Rule 4:** If  $\Delta V$  is about -3% and  $\Delta R$  is about 0.09,  $\Delta S$  is -0.8
- Rule 5:** If  $\Delta V$  is about 1% and  $\Delta R$  is about 0.09,  $\Delta S$  is 0.3
- Rule 6:** If  $\Delta V$  is about 6% and  $\Delta R$  is about 0.09,  $\Delta S$  is 6

The fuzzy sets for the changes in interest rate and volatility used in the rules (3) are given in Figure 6.



**Figure 6.** Fuzzy sets for changes in interest rate (left plot from left to right): about -0.11, about 0.09; fuzzy sets for changes in volatility (right plot from left to right): about -3%, about 1%, about 6%.

Figure 7 shows the comparison of our model with the linear model.



**Figure 7.** Credit spread as a function of volatility change when interest rate change is -0.1 (left plot) and interest rate change is 0.1 (right plot)

It follows from Figure 7 that the relationship between credit spreads and volatility is quite far from linear. Furthermore, the Mean Squared Error (MSE) for fuzzy rules approximation is 1.37, whereas the MSE for the linear regression is 1.51 implying that fuzzy rules approximation provides a better fit to the data.

In the ANFIS for credit spreads, rules (3) has two fuzzy sets in the “If” part as opposed to only one fuzzy set in the “If” part of rules (2) for modelling the leverage effect (see the previous example). For instance, Rule 1 contains the fuzzy sets “about

-3%" and "about-0.11". In this case, the weight of Rule 1 is determined by multiplying the certainty degrees of  $\Delta V$  and  $\Delta R$ . If a given value of  $\Delta V$  has 30% certainty of being "about -3%" and  $\Delta R$  has 50% certainty of being "about -0.11", then the weight of Rule 1 is 15%:  $0.15 = 0.3 \times 0.5$ .

However, the question remains:

*Are the fuzzy rules obtained consistent with predictions of the Merton's structural model? As mentioned previously, according to the Merton's (1974) model:*

- *An increase in interest rates should decrease default probability (and thus the credit spread);*
- *An increase in equity volatility should decrease equity value (due to leverage effect) and as a result increase default probability.*

These predictions imply that when  $\Delta V$  increases (decreases) and  $\Delta R$  decreases (increases), then credit spread increases (decreases). Evidently, this behavior is well captured by Rules 2, 3, and 4. The credit spread  $\Delta S$  resulting from the other combinations of  $\Delta V$  and  $\Delta R$  (Rules 1, 5, and 6) depends on the magnitude of  $\Delta V$  and  $\Delta R$ .

## CHALLENGES IN MODELLING

It is usually the case that fuzzy rules contain more than one independent variable in the "If" statement. For instance, in credit spread modelling, there are two factors to consider: volatility and interest rates. The weight of each rule is the product of the measures of certainty for each independent variable of interest. However, as the number of variables increases, this operation yields counterintuitive results.

To illustrate, assume there are 10 variables, each with 90% certainty. It may be natural to expect that the weight of the corresponding rule will be similarly large. However, the weight is in fact only 35% ( $0.9^{10} = 0.9 \times 0.9 \times \dots \times 0.9$ ). In the academic literature, this phenomenon is referred to as the "curse of dimensionality", implying that problems appear when dimensionality of the problem gets large (see Mitaim and Kosko (2001)).

Multiplication is not the only way to determine the weight of a rule; the so-called "minimum" operation can be used as well (weight is equal to the smallest degree of certainty). However, this approach is also subject to criticism because a single low degree of certainty can significantly affect the weight of the entire rule regardless of the other, possibly very high, measures (e.g.,  $0.01 = \min(0.01, 0.9, 0.9, \dots, 0.9)$ ). Importantly, multiplication and minimum operations used for the rules are not arbitrary and follow from Mathematical Logic, a subfield of mathematics that explores the application of formal logic to mathematics.

Another challenge with fuzzy systems modelling is the sheer multitude of rules required to obtain an adequate fit for some data sets. Although fuzzy rules are easy to understand, researchers may need a large number of rules to explain applications with fairly complex data relationships, and as a result, analysis becomes very difficult. In such situations the interpretability advantage of fuzzy systems becomes less obvious.

As discussed, there is some uncertainty as to the choice of shape for fuzzy sets, which can potentially take on a variety of forms (triangular, bell-shaped, etc.). Some shapes that provide a better fit to the data could be difficult to describe linguistically, in terms like low, medium, high, etc. (a limitation that is especially true for asymmetric fuzzy sets). Thus, a researcher should find a balance between goodness-of-fit and ease of understanding.

Finally, fuzzy systems can take on a variety of configurations. For example, it is possible to have sets as outputs of rules, not only exact numbers as with the examples considered in this paper. In such a case, a fuzzy rule could resemble the following:

*If volatility is high, returns are low*

where both "high" and "low" are fuzzy sets themselves. This modification provides more flexibility in modelling but could make it more challenging to analyse the rules. In this respect, the linguistic description of fuzzy sets should be more precise to avoid any misunderstanding.

## CONCLUSIONS

NFSs could provide a way to overcome the interpretability problem of ANNs and at the same time maintain the approximation power of ANNs. Furthermore, the author surmises that when the data is scarce, a researcher can appropriately adjust a system to avoid any undesirable behavior. Although the use of NFSs seems promising, a researcher should be mindful of some of the challenges with system modelling, such as multitudinous input variables, the shapes of fuzzy sets, and specific configurations.

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