

Striking a Balance:

The Optimal Hedging Ratio and Cost Trade-Offs in Global Currency Risk Management



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EXECUTIVE SUMMARY:

This paper explores the effectiveness of global currency hedging, assessing both its risk-reducing benefits and cost efficiency. Our research suggests that while currency hedging can help to reduce volatility and minimize downside risk, it can also come with a cost, particularly in emerging markets. Furthermore, the efficacy of hedging strategies can vary depending on the prevailing market conditions, and different approaches may be more suitable for different asset classes. Our findings can serve as a valuable resource for investors looking to optimize their currency hedging choices in international portfolios. The study conducted an in-depth historical analysis of market rates, currency hedging strategies and portfolio returns over the period of January 1975 to October 2021 covering Canada, United States, United Kingdom, Germany, Japan, Australia, China, Brazil and India.

Key takeaways for Canadian investment managers:

- When hedging currency risk in emerging markets, investors should carefully consider the cost effectiveness of using forward contracts versus a borrow/lend strategy. There can be a significant cost difference between these two approaches.
- While the Covered Interest Parity (CIP) may hold true in developed markets, the discrepancy between the hedging cost obtained through “borrow/lend” and “forward” approaches suggests that CIP is less likely to hold true in emerging markets.
- For Canadian investors, the cost of currency hedging is generally lower than for US investors, regardless of whether they are investing in developed or emerging foreign stock markets. This highlights the importance of considering the investor’s base currency when hedging currency risk.
- Neglecting currency risk can lead to a loss of potential gains, as demonstrated by the optimal long CNY position for Canadian investors in the Chinese stock market. Investors should carefully consider the potential benefits of currency hedging for each market in which they are invested.
- Investing in multi-country portfolios can be a way to lower investment volatility and increase diversification. The cost of minimizing currency fluctuation for a Canadian investor in a multi-country stock portfolio is relatively low, at less than 1%.
- The benefits of currency hedging can vary depending on the type of investment, the currency being hedged, and the investor’s base currency. It’s important for investors to carefully consider their individual circumstances and investment goals before deciding whether to hedge currency risk.

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- Neglecting currency risk can lead to a loss of potential gains, as demonstrated by the optimal long CNY position for Canadian investors in the Chinese stock market. Investors should carefully consider the potential benefits of currency hedging for each market in which they are invested.
- Investing in multi-country portfolios can be a way to lower investment volatility and increase diversification. The cost of minimizing currency fluctuation for a Canadian investor in a multi-country stock portfolio is relatively low, at less than 1%.
- The benefits of currency hedging can vary depending on the type of investment, the currency being hedged, and the investor's base currency. It's important for investors to carefully consider their individual circumstances and investment goals before deciding whether to hedge currency risk.

1. Introduction

The currency risk associated with emerging market investments can greatly influence the overall risk and return profile of the underlying assets once they are converted to the investor's home currency. Large Canadian pension funds have doubled their asset allocation to emerging markets over the last decade.

Given this, Canadian asset managers may need to consider hedging their currency exposure, particularly when it comes to emerging market exposure. Previous studies have suggested different hedging strategies, including no hedging (e.g., Solnik, 1974), full hedging (e.g., Perold and Schulman, 1988), uniform hedging ratios (e.g., Eun and Resnick, 1988), and partial hedging (e.g., Campbell, Serfaty-De Medeiros, and Viceira, 2010) for certain currencies and asset classes. There has been a lack of focus on emerging country currencies in this literature. Recent studies (e.g., Ang et al., 2021) have addressed the issue of optimal currency portfolios and hedging costs for developed market currencies, but there is still a lack of research on the hedging cost for emerging market currencies. Emerging market currencies are known for their higher volatility and risk, which makes the task of currency allocation and hedging more challenging for investors. It is important for investors to have a better understanding of the risk-return trade-offs associated with different currency portfolios and hedging strategies in emerging markets.

In light of the aforementioned gaps in the literature, our study aims to investigate optimal hedging policies and the cost of currency hedging for institutional investors in the US and Canada, with a focus on emerging country currencies. Our study seeks to fill the gaps in the literature by providing insights into the optimal hedging policies for emerging markets and the potential impact on overall portfolio performance.

This study extends the framework of Campbell, Serfaty-De Medeiros, and Viceira (2010) by adding emerging markets to the equity and bond portfolio and examining the cost of currency hedging until the end of 2021. Our goal is to reduce the impact of foreign exchange fluctuations through standard global currency hedging while minimizing portfolio return variance in relation to currency demand. We focus both on emerging markets such as Brazil, China, and India, and developed markets like the US, Canada, Germany, UK, Euro zone, Japan, and Australia. The time series data spans from 1975 to 2021, and we obtain the optimal currency position by regressing portfolio excess returns on a constant and a set of currency excess returns.

The findings of this study suggest that optimal currency hedging strategies can significantly reduce the volatility of foreign long-term bond investments, particularly in developed markets, for both USD and CAD-based investors. However, when it comes to emerging market stocks and bonds, employing a currency hedge can potentially reduce the Sharpe ratio of returns. It is important for investors to carefully consider the benefits and costs of currency hedging before making investment decisions.

Currency hedging costs can vary depending on the specific currency being hedged and other market factors. Our study found that using currency forward contracts can achieve similar hedging results compared to borrowing and lending in foreign currencies, with lower transaction costs. However, there is a significant cost difference between the two methods when it comes to hedging emerging market currencies. In such cases, investors should weigh the potential benefits and costs before making investment decisions. Our empirical results demonstrate that in most cases, the actual forward exchange rate for emerging market currencies is greater than the one implied by the covered interest parity, indicating that hedging can result in arbitrage free gains when using forward contracts.

Furthermore, the study discusses the impact of hedging on multi-country portfolios, highlighting the benefits of diversification. Multi-country portfolios demonstrate lower volatility and hedging costs compared to most single-country portfolios. However, the benefits of currency hedging may vary depending on the type of investment, currency, and the investor's base currency. Therefore, it is important for investors to carefully evaluate the potential benefits and costs of currency hedging based on their individual circumstances before making any investment decisions.

The remainder of this paper is structured as follows. In Section 2, we present the general framework, which provides us with closed-form solutions to help illustrate key concepts, including the definition of the cost of currency hedging. Section 3 discusses the data used in the study, and Section 4 presents the empirical results. Finally, in Section 5, we conclude our study and discuss the implications of our findings for institutional investors seeking to optimize their currency hedging strategies.

2. General Framework

International investors investing in assets or securities outside of their home country typically want to hedge currency risk because fluctuations in foreign currency exchange rates can have a significant impact on their investment returns. For example: a Canadian investor buys shares of a Brazilian company in Brazilian real. If the value of the real depreciates relative to the Canadian dollar, the investor's returns in Canadian dollar terms will be reduced.

Hedging currency risk allows international investors to mitigate this risk by using financial instruments such as forwards, futures, options, or swaps to lock in a specific exchange rate at which to buy or sell the foreign currency. This can help to protect the investor's returns against adverse currency movements and provide greater certainty and predictability in their investment outcomes.

In this section, we lay out the general framework for the global currency hedging model. We denote R_{t+1}^c the gross return of risky assets denominated in local currency c , where $c = 1$ represents the domestic country and $c = 2, \dots, n$ represents the $n - 1$ foreign countries. We define S_{t+1}^c as the spot exchange rate measured in domestic currency per unit of foreign currency c with the domestic exchange rate constant $S_{t+1}^1 = 1$ for all time t . By indexing the domestic and foreign countries in this way, we establish a clear and consistent notation for the rest of the paper.

2.1 Gross Portfolio Returns

To invest in the risky assets of a foreign country, an investor first exchanges one unit of home currency for $1/S_t^c$ units of currency c at the spot exchange rate S_t^c , and then invests in country c 's risky assets. At the end of the investment period, the investor can exchange the foreign currency back into their home currency at the spot exchange rate S_{t+1}^c to earn an unhedged return given by:

$$R_{u,t+1}^c = R_{t+1}^c (S_{t+1}^c \div S_t^c)$$

For international investors with a portfolio of foreign assets, the gross return of the unhedged portfolio can be calculated by:

$$R_{u,t+1} = \omega_t' R_{t+1} (S_{t+1} \div S_t) \quad (1)$$

where ω_t refers to a vector of n portfolio weights, while R_{t+1} is a vector of gross returns for both domestic and foreign assets in local currency terms. S_{t+1} is a vector of spot exchange rates, with the domestic exchange rate constant $S_{t+1}^1 = 1$ for all time t , and \div indicates the element-by-element ratio operator.

Suppose an investor wishes to hedge against currency risk in a portfolio containing foreign assets. To accomplish this, we introduce $F_{t,t+1}^c$ as the one-period forward exchange rate in domestic currency per unit of foreign currency c , and θ_t^c as the value in domestic currency of the amount of forward exchange rate contracts for foreign currency c . At the end of the investment period $t + 1$, the investor can convert θ_t^c / S_t^c units of the foreign-currency based return $R_{t+1}^c \omega_t^c / S_t^c$ back into the domestic currency at the predetermined exchange rate $F_{t,t+1}^c$. The remaining amount of the returns, which is $\frac{R_{t+1}^c \omega_t^c}{S_t^c} - \frac{\theta_t^c}{S_t^c}$, is exchanged back into the domestic currency at the market spot exchange rate S_{t+1}^c . Collectively, the gross return of the hedged portfolio is given by:

$$R_{h,t+1} = \omega_t' R_{t+1} (S_{t+1} \div S_t) - \theta_t' (S_{t+1} \div S_t) + \theta_t' (F_{t+1} \div S_t) \quad (2)$$

where $\Theta_t = (\theta_t^1, \dots, \theta_t^c, \dots, \theta_t^n)$ is a vector of parameters representing the proportion of foreign currency exposure hedged using forward contracts, commonly known as the currency risk hedge ratio. This ratio determines the extent to which an investor is protected against potential losses due to adverse currency movements, and can be adjusted based on the investor's risk appetite and market condition. For equation (2), $\theta_t^c = 0$ corresponds to a no hedging position and $\theta_t^c = \omega_t^c$ represents a fully hedging position to currency c .

All hedge ratios should add up to 1, $\theta_t^1 = 1 - \sum_{c=2}^n \theta_t^c$, indicating the choice of domestic hedging ratio θ_t^1 is arbitrary. The reason why the sum of the hedge ratios in a portfolio adds up to 1 is that these ratios represent the percentage of the foreign currency exposure that is hedged using forwards. Since all of the foreign currency exposure must either be hedged or unhedged, the sum of the hedge ratios and the percentage of the exposure that is not hedged must add up to 1.

The vector $\mathbf{F}_{t+1} = (F_{t,t+1}^1, \dots, F_{t,t+1}^c, \dots, F_{t,t+1}^n)'$ represents the set of one-period forward exchange rates at time $t + 1$ measured in domestic currency per unit of foreign currency c . The domestic forward exchange rate is constant with $F_{t,t+1}^1 = 1$.

Covered Interest Parity

As per Campbell, Serfaty-De Medeiros, and Viceira (2010), we utilize the concept of covered interest rate parity (CIP) to indirectly infer forward exchange rates. CIP stipulates that the difference between the forward and spot exchange rates must equal the interest rate differential between two currencies in the cash money markets. This is because any deviation from CIP would create a riskless profit opportunity for arbitrageurs. As a result, the fair price of a forward contract is given by:

$$F_{t,t+1}^c = S_t^c \frac{(1+I_t^1)}{(1+I_t^c)} \quad (3)$$

where I_t^c denotes the risk-free interest rate for country c at the end of period t . As shown in (3), entering into a forward contract is equivalent to borrowing or lending in the foreign currency and lending or borrowing in the domestic currency, depending on whether the investor is buying or selling the foreign currency. Forward exchange rates are not publicly quoted like the spots rate, and may not be available at all for certain currencies or time periods. This relationship is particularly useful when dealing with less liquid or exotic currencies for which forward rate data may be sparse or unreliable. By using CIP, investors can estimate forward exchange rates and use them in currency hedging strategies.

The hedged portfolio return can be expressed as follows by substituting equation (3) in equation (2):

$$R_{h,t+1} = \omega'_t R_{t+1} (S_{t+1} \div S_t) - \theta'_t (S_{t+1} \div S_t) + \theta'_t [(1 + I_t^1) \div (1 + I_t)] \quad (4)$$

where the vector $I_t = (I_t^1, \dots, I_t^c, \dots, I_t^n)$ represents the vector of risk-free interest rates for each country in the portfolio, and $I_t^1 = I_t^1 \mathbf{1}$ denotes the vector of risk-free interest rates of the domestic country.

Cost of currency hedging

In reality, frictions such as transaction costs, bid-ask spreads, and other market imperfections can cause deviations from covered interest parity. These deviations can lead to a cost of currency hedging, as the hedging investor may have to pay a premium to enter into a forward contract that reflects the expected future spot exchange rate.

To calculate the impact of the cost in currency hedging, one can compare the returns of a portfolio with and without currency hedging. The difference between these two returns can be attributed to the cost of currency hedging:

$$Cost = R_{u,t+1} - R_{h,t+1} = \theta'_t (S_{t+1} \div S_t) - \theta'_t [(1 + I_t^1) \div (1 + I_t)]$$

The CIP assumes that hedging through borrowing/lending and using forwards contracts should yield equivalent results. However, the riskless gains appears when CIP does not hold, and the forward exchange rate deviates from the interest rate differential. In particular,

- if the forward exchange rate is larger than the one implied by CIP, (e.g., $F_{t,t+1}^c > S_t^c \frac{(1+I_t^1)}{(1+I_t^c)}$) then hedging becomes a riskless gain.
- On the other hand, if the forward exchange rate is smaller than the one implied by CIP (e.g., $F_{t,t+1}^c < S_t^c \frac{(1+I_t^1)}{(1+I_t^c)}$), hedging will be costly.

Therefore, the cost of currency hedging depends on whether the forward exchange rate deviates from the interest rate differential implied by CIP.

Currency exposure

The currency exposure variable, denoted by ψ_t^c represents the difference between the foreign currency holdings, ω_t^c , and the amount that is hedged using forward contracts, θ_t^c . In other words, $\psi_t^c = \omega_t^c - \theta_t^c$ captures the degree to which an investor is exposed to the risk of currency fluctuations in currency c .

- If $\psi_t^c = 0$, it means that the investor has fully hedged their exposure to currency c . In this case, the hedge ratio is equal to the foreign currency holding $\theta_t^c = \omega_t^c$.

- If $\psi_t^c = \omega_t^c$, the portfolio is completely unhedged. In this case, the hedge ratio is zero, $\theta_t^c = 0$.
- If $0 < \psi_t^c < \omega_t^c$, it means that the investor wants to hold some exposure of currency c , and only hedge a portion of the foreign currency risk.
- If $\psi_t^c > \omega_t^c$, it means that the investor is short currency c (or net long domestic currency) and has committed to sell those currencies forward. This is a position associated with negative hedge ratio $\theta_t^c < 0$.
- If $\psi_t^c < 0$, it means that the investor has over-hedged their exposure to foreign currency c . In this case, the hedge ratio is greater than ω_t^c .

For an international investor with multiple foreign currencies, the currency exposure sums up to zero ($\psi_t^1 + \dots \psi_t^c + \dots \psi_t^n$) = 0. This is because the investor's portfolio can be expressed as a combination of long and short positions in different currencies. For every long position in a foreign currency, there must be a corresponding short position in another currency, which ultimately cancels out the total currency exposure.

Therefore, the expression for the hedged portfolio return (4) can be reformulated using the concept of currency exposure:

$$R_{h,t+1} = \omega_t' R_{t+1} (S_{t+1} \div S_t) - \omega_t' [(S_{t+1} \div S_t) - (1 + I_t^1) \div (1 + I_t)] + \Psi_t' [(S_{t+1} \div S_t) - (1 + I_t^1) \div (1 + I_t)] \quad (5)$$

where the vector $\Psi_t = (\psi_t^1 \dots \psi_t^c \dots \psi_t^n)'$ represents the currency exposure of the portfolio to each foreign currency c , with $\Psi_t = \omega_t - \Theta_t$.

2.2 Log Portfolio Returns

We will now adopt the approach proposed by Campbell, Serfaty-De Medeiros, and Viceira (2010) to transform the gross hedged portfolio returns (5) into log-normal returns. Converting gross returns to log-returns is a common way to approximate the log-normal distribution and allows for the use of optimization techniques that assume normality. These log portfolio excess returns will be represented with lowercase letters:

$$r_{h,t+1} - i_t^1 = \omega_t' (r_{t+1} - i_t) + \Psi_t' (\Delta S_{t+1} - i_t^1 + i_t) + \frac{1}{2} \Sigma_{h,t+1} \quad (6)$$

where we use the notion $r_{h,t+1}$ to represent the log hedged portfolio return. The variable i_t^1 is the domestic interest rate, while the vector i_t denotes the foreign interest rate, and r_{t+1} is used to represent the log risky asset returns in local currencies.

Additionally, we use Δs_{t+1} as the log spot exchange rate return between t and $t + 1$. The vector of log spot exchange rate return, denoted as $\Delta \mathbf{s}_t$, represents the change in the log exchange rate for domestic investors. It is a vector composed of n components $\Delta \mathbf{s}_t = (\Delta s_t^1, \dots, \Delta s_t^n)$ with $\Delta s_{t+1}^c = \ln S_{t+1}^c - \ln S_t^c$. The last term $\Sigma_{h,t+1}$ is a Jensen's inequality term.

Equation (6) offers a clear and intuitive breakdown of the portfolio's excess return. The first term $\omega'_t(\mathbf{r}_{t+1} - \mathbf{i}_t)$ represents the excess return on a fully hedged portfolio that has zero exposure to currency risk. On the other hand, the second term, $\Psi'_t(\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^1 + \mathbf{i}_t)$, solely depends on the vector of excess returns on currencies, which indicates pure currency exposure.

2.3 Objective Function

In accordance with the framework proposed by Campbell, Serfaty-De Medeiros, and Viceira (2010), our analysis centers around an investor who aims to minimize the risk associated with foreign currency exposure. Specifically, the investor aims to minimize the variance of the log portfolio excess return, $Var(r_{h,t+1} - i_t^1)$, which represents the foreign exchange risk. The objective function is given by:

$$\min_{\Psi} Var(r_{h,t+1} - i_t^1)$$

By minimizing the variance of the portfolio's excess return, the investor can reduce the impact of currency fluctuations on their overall portfolio performance. Since Ψ_t represents the weights in a zero-investment portfolio, which is designed to have a neutral or zero effect on an investor's overall financial position, the domestic currency exposure ψ_t^1 can be automatically determined as $\psi_t^1 = 0 - (\psi_t^2 + \dots + \psi_t^n)$.

To determine the optimal currency hedging policy for a leveraged investor, it is crucial to identify the vector of foreign currency demands, denoted as $\tilde{\Psi}_t = (\psi_t^2, \dots, \psi_t^n)'$ which represents the weights of each foreign currency in the portfolio.

The vector $\tilde{\Psi}_t$, which minimizes the one-period conditional global variance of the log excess return of the hedge portfolio, is equal to the following

$$\tilde{\Psi}_t^* = -Var(\Delta \tilde{\mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^1 + \tilde{\mathbf{i}}_t)^{-1} [cov(\omega'_t(\mathbf{r}_{t+1} - \mathbf{i}_t), (\Delta \tilde{\mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^1 + \tilde{\mathbf{i}}_t))] \quad (7)$$

where the variables with superscript \sim denotes an $(n - 1) \times 1$ vector, with the domestic component excluded.

According to equation (7), the optimal currency demand $\tilde{\Psi}^*$ can be calculated by conducting a multi-factor regressing analysis of the portfolio excess returns $\omega'_t(r_{t+1} - i_t)$ on a vector of currency excess returns $(\Delta \tilde{s}_{t+1} - \tilde{i}_t^1 + \tilde{i}_t)$, where the slope coefficients of the vector of currency return must be reversed:

$$\omega'_t(r_{t+1} - i_t) = \alpha + \beta'(\Delta \tilde{s}_{t+1} - \tilde{i}_t^1 + \tilde{i}_t) + \epsilon_t$$

where $\tilde{\Psi}^* = -\hat{\beta}'$ with $\hat{\beta}$ the estimator of β .

If asset excess returns and the currency excess returns are uncorrelated, the currency demand becomes zero, which implies a fully hedged position. When asset excess returns and currency excess returns have a positive correlation ($\hat{\beta}^c > 0$), a decline in the stock market can be associated with the depreciation in foreign currency. This could be because during a market downturn, investors tend to sell off their investments and repatriate their funds back to their home currency, increasing the demand for their own currency and decreasing the demand for foreign currency. As a result, the foreign currency tends to depreciate in value. To mitigate this foreign exchange risk, investors can consider over-hedging by shorting foreign currency beyond what is needed for full hedging.

If there is a negative correlation between asset returns and exchange rates ($\hat{\beta}^c < 0$), it means that the foreign currency tends to appreciate when the stock market falls. In such a scenario, investors can consider reducing portfolio return volatility by under-hedging, which involves holding foreign currency instead of fully hedging their foreign currency exposure.

3. Data

In our analysis, either the United States or Canada is considered a domestic country. The other developed economies included in our analysis are Germany, the United Kingdom, the Eurozone (from 1999), Japan, and Australia, along with three emerging markets, namely Brazil, China, and India. For most developed economies, the sample period spans from January 1975 to October 2021, with data series reported at a monthly frequency. Our analysis uses the 90-days T-bill, the 10-year constant maturity yield, exchange rates relative to USD, and Consumer Price Index (CPI) of all items, primarily sourced from the FRED website, with initial sources published by the International Monetary Fund (IMF)'s International Financial Statistics (IFS). We obtained country stock index returns from Morgan Stanley Capital International (MSCI) via Bloomberg. It is worth noting that the emerging market sample series are considerably shorter when compared to the developed markets.

Table [A1](#) displays the starting periods of the unbalanced data series for all the countries in our analysis. As the data for some of the indicators or variables were not available for the entire sample period, the data series are unbalanced, with different start dates for different countries and series.

The international portfolio considered in our analysis comprises two risky asset classes, namely, stocks and long-term bonds. The logarithmic returns on the equity market of country c , denominated in its local currency c , are defined as:

$$r_{s,t}^c = \ln P_t^c - \ln P_{t-1}^c$$

with P_t^c the stock market price denominated in the local currency of country c at time t .

We employed the methodology proposed by Campbell & Viceira (2002) to generate the log nominal bond return series. Specifically, we computed the returns using the 10-year constant maturity yield, represented by the formula:

$$r_{b,m,t+1}^c = D_{m,t}^c y_{m,t}^c - (D_{m,t}^c - 1) y_{m-1,t+1}^c$$

Here, m denotes the bond maturity, $r_{b,m,t+1}^c$ represents the log nominal return on m -period constant maturity yields for country c at time t , and $y_{m,t}^c = \ln(1 + Y_{m,t}^c)$ is the yield on the m -period maturity bond at time t . The duration of the bond is represented by $D_{m,t}^c$, which we approximated using the formula:

$$D_{m,t}^c = \frac{1 - (1 + Y_{m,t}^c)^{-m}}{1 - (1 + Y_{m,t}^c)^{-1}}$$

For the sake of simplicity, we have assumed a fixed maturity of 10 years for all bonds in our calculation. As a result, we will omit the subscript m when referring to bond maturities for the remainder of this paper.

3.1 Summary Statistics

[Table 1](#) presents the rolling annual mean and standard deviation of various financial metrics, including short-term interest rates, log stock and bond returns in excess of their respective local short-term interest rates, changes in log exchange rates, the currency excess return, as well as log unhedged stock and bond returns in excess of domestic short-term interest rates, from both US and Canada perspectives. All returns are expressed as percentages. It is important to note that due to the differing starting dates of the analysis, the statistics presented in [Table 1](#) should be interpreted with some

caution. To adjust for Jensen's Inequality, the mean of log excess returns (geometric average) is transformed into mean simple excess returns (arithmetic average) by adding one-half of their variance.

Log short-term interest rate

The log interest rate is calculated by $i_t^c = \ln(1 + I_t^c)$. Among the countries listed, Brazil had the highest average annualized nominal interest rate of 9.48 percent over the period 2007 to 2021. Brazil has had a history of high inflation rates, which has led to the Central Bank of Brazil raising interest rates to combat inflationary pressure. Additionally, Brazil has had a history of currency depreciation, which can also contribute to inflationary pressures. Furthermore, political instability, corruption scandals, and fiscal deficits have also contributed to economic uncertainty and the need for higher interest rates in Brazil.

Japan has experienced a prolonged period of deflation and economic stagnation since the 1990s, resulting in a low demand for credit and low inflation rates. Furthermore, the Bank of Japan has implemented an accommodative monetary policy to stimulate the economy and address deflation. These factors have contributed to Japan's low interest rate environment, with an average of 0.86% per year compared to other developed countries.

The short-term interest rates in Canada, Australia, and the UK are highly volatile, with an average fluctuation of over 4 percent per year. This could be attributed to various factors, such as economic conditions, monetary policy decisions, and geopolitical events. In the case of Canada, Australia, and the UK, their economies are highly dependent on commodity exports, which can lead to greater fluctuations in interest rates due to changes in global commodity prices. Additionally, their central banks have implemented policies aimed at maintaining low inflation rates, which can also contribute to higher volatility in interest rates. Finally, political and geopolitical events, such as Brexit in the UK or changes in trade policy, can also cause fluctuations in interest rates.

Excess log foreign stock (bond) returns

Excess log stock (bond) returns represent the returns on foreign stocks (bonds) to a fully hedged investor with no exposure to foreign currencies, in excess of the local log nominal interest rate. Specifically, the excess 10-year foreign bond returns for country c are calculated as the difference between the return on a 10-year bond investment and the risk-free rate for country c at time t . This can be expressed as $x_{b,t+1}^c = r_{b,t+1}^c - i_t^c$. Similarly, the excess foreign stock returns are given by $x_{s,t+1}^c = r_{s,t+1}^c - i_t^c$. The average excess log stock (bond) returns reported in [Table 1](#) are adjusted for the Jensen's inequality, by adding one-half of their variance.

The annualized average excess bond returns across countries show similar results, hovering around 3 percent. However, China stands out with the lowest reported annual excess return of 0.58 percent, indicating lower long-term excess bond returns compared to other listed countries. This could be because Chinese bond market is still developing and this may lead to a lower perceived risk differentiation between different maturities or credit qualities of bonds in China. It is also worth noting that the Chinese government may exert greater control over interest rates and bond issuance, leading to a lower perceived risk premium.

China and India have high average excess stock returns compared to developed markets, but with higher volatility. In contrast, the negative excess stock return of -6.12 percent in Brazil can be mainly attributed to the country's high inflation rate. The negative excess return of Brazilian stocks highlights the potential risks and challenges associated with investing in emerging markets.

Exchange rate returns

Changes in exchange rates, Δs_t^c , refer to the fluctuations in the value of one currency relative to another currency over time. A negative average historical change in exchange rate suggests that the domestic currency has appreciated over time over the given sample periods. [Table 1](#) suggests that, on average, all the listed emerging market currencies have depreciated relative to the US dollar over the given sample periods.

When Canada is the home country, the same result of a large negative average change in exchange rate for emerging market currencies is observed. This can be explained by the fact that Canada has close economic ties with the United States, and fluctuations in the US dollar can also affect the value of the Canadian dollar and its exchange rate with emerging market currencies.

Currency excess returns

The currency excess returns $(\Delta s_{t+1}^c + i_t^c - i_t^1)$ are returns from borrowing in domestic currency for one period, lending the proceeds in foreign currency for the same period, and exchanging back into domestic currency after one period to repay the domestic currency loan. We assume that investors can borrow and lend at the same rate. In reality, currency excess returns to investors would be lower because of transaction costs and bid-ask spreads. Excess currency returns are composed of the ex-ante known interest rate differential $(i_t^c - i_t^1)$, and an uncertain currency return component, Δs_{t+1}^c , capturing the change in the value of the long currency relative to the funding (short) currency.

The excess returns to most currencies are generally small for both US and Canadian investors. However, the excess return to the Brazilian real stands out as being substantially large, with an annual excess return of 8.36 percent for US investors and 7.2 percent for Canadian investors. This significant excess return for the Brazilian real may be due to various factors, such as the country's high inflation rate or strong economic growth. Investors should take note of this potential opportunity for higher returns, but they should also consider the risks associated, such as currency fluctuations and political instability.

The annual volatility of excess returns for all currencies is generally similar to that of exchange rates, with the exception of CNY from the US perspective. The excess currency return on CNY for US investors exhibits a high degree of predictability, with a relatively low volatility of 4.18 percent, compared to the corresponding volatility of exchange rate returns. This predictability is largely due to the Chinese government's tight control over its currency exchange rate. The People's Bank of China (PBOC) has a longstanding policy of maintaining the value of the Chinese yuan (CNY) relatively stable against the US dollar (USD), achieved through government intervention in currency markets and strict capital controls. This stability results in a relatively narrow fluctuation band of the CNY/USD exchange rate, making it easier to predict and potentially exploit for excess returns.

Unhedged excess stock and bond returns

The log unhedged excess stock and bond returns, x_{us}^c and x_{ub}^c , are approximated as the sum of the foreign log stock (bond) returns, $r_{s,t}^c$ ($r_{b,t}^c$), and changes in log exchange rates Δs_t^c , in excess of the domestic interest rate. In the case of fully unhedged foreign currency, i.e., $\theta^c = 0$, the log unhedged excess stock returns are given by $x_{us,t+1}^c = r_{s,t+1}^c + \Delta s_{t+1}^c - i_t^1$. Similarly, the log unhedged excess bond returns are given by $x_{ub,t+1}^c = r_{b,t+1}^c + \Delta s_{t+1}^c - i_t^1$. The average unhedged excess stock (bond) returns are adjusted for the Jensen's inequality term.

In general, unhedged foreign excess stock (bond) returns tend to be higher than their hedged counterparts, but they are also associated with higher risk. However, Japan is an exception to this trend. Our analysis indicates that, from both the US and Canada perspectives, the unhedged excess stock (bond) returns in Japan are significantly lower than the hedged returns. This may be due to Japan's unique economic situation, including its long-term economic stagnation, low inflation, and ongoing deflationary pressures, which have contributed to a relatively low demand for credit and lower interest rates.

Cross-Country Return Correlations

[Table 2a](#), Panel A and Panel B show the quarterly correlations of foreign-currency excess returns viewed from the perspectives of the US and Canada, respectively. The sample is truncated to the longest common horizon from 1997 to 2021. When the US dollar is the domestic currency, the excess returns on the Japanese yen (JPY) exhibit a low correlation with the excess returns of other currencies due to the factor that the JPY excess currency return is close to zero, as shown in [Table 1](#).

Correlation of excess currency returns

The excess currency returns of CNY and USD are highly correlated from the Canadian perspective, with a historical correlation coefficient of 90 percent. This indicates that movements in the USD can have a significant impact on the value of the CNY. This correlation is due to China's long-standing policy of pegging the value of the CNY to a basket of other currencies, including the USD. This policy is aimed at promoting stability in the value of the CNY and maintaining its competitiveness in international trade.

From the Canadian perspective, the excess currency returns of the Brazilian real (BRL) exhibit a low to negative correlation with the excess currency returns of other currencies. This suggests that movements in the BRL are largely independent of movements in other currencies. One possible reason for this is Brazil's unique economic and political environment, which can be influenced by factors such as commodity prices, political instability, and social unrest. Additionally, Brazil has a relatively closed economy with significant trade barriers, which may limit the impact of global economic factors on its currency.

From the US perspective, the excess currency returns of the Australian dollar (AUD) and Canadian dollar (CAD) exhibit an unusually high correlation of 78 percent. This is attributed to the fact that both economies are closely linked through trade and investment, particularly in the natural resources sector. As both Australia and Canada are major exporters of natural resources, such as oil, gas, and minerals, changes in commodity prices can impact both their economies in a similar way, leading to a correlation between their respective currencies.

However, from the Canadian perspective, the excess currency returns of AUD and USD exhibit a negative correlation of -22 percent. This may be due to the fact that the Canadian and Australian economies are often competing for similar export markets. When the US dollar strengthens, it can lead to a decrease in demand for exports from both countries, which can lead to a decrease in the value of their respective currencies.

Correlation of excess hedged stock (bond) returns

Panel C of [Table 2a](#) shows the correlation between hedged stock returns across all developed markets to be above 60 percent. One of the most important factors is the increasing globalization of financial markets, which has led to greater integration and interdependence between economies. As a result, changes in one country's stock market can have a significant impact on other countries' stock markets, leading to a high degree of correlation.

Chinese stock market returns demonstrate a weaker positive correlation with other global markets, however, the correlation remains close to or above 40 percent. Among all markets, the correlation between Chinese and EU stock returns is the lowest, with a correlation coefficient of 39 percent. There are several potential reasons for this. Firstly, China's stock market is still relatively closed to foreign investment, which limits the degree of integration with other global markets. Secondly, differences in the industries and sectors represented in the stock markets of China and the EU could also contribute to the lower correlation. For instance, China's stock market is dominated by state-owned enterprises in the financial, energy, and industrial sectors, while the other markets have a more diversified mix of sectors. This could lead to different levels of volatility and performance in the respective markets, resulting in lower correlations.

Panel D of [Table 2a](#) illustrates the correlation between global bond market returns, revealing a clear distinction between developed and emerging markets. Among the seven developed markets, bond excess returns are highly correlated, with one correlation standing out as unusually large. The Canadian bond returns exhibit a very high correlation with the US bond market at 91%, which could be attributed to the two countries' close economic ties and geographical proximity.

On the other hand, the three emerging markets display low or negative correlations with other bond markets. Specifically, the Brazilian bond market only has a correlation of 6% with the Canadian bond market and a negative correlation of 11% with the Chinese bond market. This could be because two countries have different economic cycles and policy environments, which can lead to differences in interest rates and bond yields. Additionally, the two countries may have different levels of exposure to global economic and financial shocks, which can also affect their bond market performance.

Correlation of unhedged stock (bond) excess returns

Panel A and B of [Table 2b](#) presents the correlation between unhedged stock returns from the US and Canada perspectives, respectively. Notably, the correlation between unhedged global stock returns is

marginally lower from the Canadian viewpoint in contrast to that from the US viewpoint, which suggests that international diversification provides some supplementary benefits to Canadian investors.

Panel C of [Table 2b](#) presents the correlation matrix of the global unhedged bond market from the US perspective. It is interesting to note that the Chinese unhedged bond market displays low or negative correlations with all other markets in the sample. This suggests that the Chinese bond market may provide additional diversification benefits to US investors, as it exhibits relatively low co-movements with other bond markets.

Panel D shows that the Brazil unhedged bond market exhibits the lowest cross-sectional correlation with all other markets from the Canadian perspective. One possible explanation for this is that the Brazilian bond market is relatively isolated and less integrated with other global bond markets, leading to lower correlation. However, it is worth noting that this low correlation does not apply to US investors, which may be due to the fact that the US dollar is the world's most widely used currency for international trade and finance.

Table 1. Summary Statistics. The table provides a comprehensive overview of summary statistics for various financial variables including log interest rates i^c , excess log stock x_s^c and bond returns x_b^c , changes in log exchange rates Δs^c , and unhedged excess bond return x_{ub}^c and stock returns x_{us}^c . The data covers the period from 1975M1 to 2021M10 with some series starting at later dates, and we have taken the longest common time frame for each market. The average (avg) and standard deviations (s.d.) are annualized, and the mean log returns (geometric averages) are adjusted by one-half their variance to reflect mean gross returns (arithmetic averages). It is important to note that due to the different starting dates, the statistics should be interpreted with some caution.

		US	Canada	Germany	Australia	Japan	UK	EU	India	China	Brazil
	Start	1975M1	1975M1	1995M1	1975M1	1987M11	1975M1	1996M12	1998M12	2002M7	2007M6
Nominal rate i^c	avg	4.96	5.87	1.99	7.51	0.94	6.47	1.90	7.14	3.31	9.83
	s.d.	(3.83)	(4.32)	(1.80)	(4.56)	(1.41)	(4.44)	(1.83)	(1.23)	(1.05)	(3.04)
Excess bond return x_b^c	adj avg	2.74	2.45	3.67	2.06	3.49	4.09	4.24	3.07	0.15	5.64
	s.d.	(8.75)	(8.21)	(5.35)	(8.24)	(7.11)	(10.50)	(6.18)	(7.91)	(6.26)	(13.76)
Excess stock return x_s^c	adj avg	5.10	2.36	5.75	1.16	3.61	1.57	5.33	7.87	8.36	-2.11
	s.d.	(16.35)	(17.98)	(23.31)	(17.57)	(19.48)	(14.55)	(23.77)	(26.69)	(27.34)	(22.64)
US Perspective											
Exchange rate return Δs^c	adj avg		-0.36	3.89	-0.57	1.56	-0.61	0.15	-2.31	1.31	-4.93
	s.d.		(6.65)	(16.78)	(11.10)	(10.31)	(10.87)	(10.92)	(7.21)	(4.06)	(17.61)
Excess currency returns	adj avg		0.55	3.34	2.03	-0.89	0.95	-0.26	2.69	3.07	3.78
	s.d.		(6.76)	(17.34)	(11.53)	(10.85)	(11.35)	(11.37)	(7.05)	(4.31)	(18.09)
Excess unhedged bond return x_{ub}^c	adj avg		2.95	6.84	3.88	2.94	4.94	2.86	5.79	3.24	10.22
	s.d.		(10.18)	(17.18)	(12.63)	(14.71)	(14.85)	(13.75)	(9.53)	(6.79)	(26.93)
Excess unhedged stock return x_{us}^c	adj avg		3.26	7.99	3.48	-0.02	4.66	1.15	11.71	11.93	3.98
	s.d.		(20.95)	(24.04)	(22.36)	(21.61)	(23.01)	(23.62)	(31.54)	(27.39)	(36.08)
Canada Perspective											
Exchange rate return Δs^c	adj avg	0.81		3.55	-0.30	1.51	-0.06	-0.89	-3.06	0.51	-4.97
	s.d.	(6.65)		(16.40)	(7.78)	(11.48)	(10.58)	(10.09)	(5.70)	(8.61)	(12.12)
Excess currency returns	adj avg	-0.09		2.84	1.37	-1.60	0.59	-1.54	1.75	2.01	3.54
	s.d.	(6.76)		(16.68)	(8.02)	(11.88)	(11.01)	(10.39)	(6.10)	(9.41)	(13.53)
Excess unhedged bond return x_{ub}^c	adj avg	2.68		6.43	3.34	2.08	4.75	1.66	4.56	2.60	9.34
	s.d.	(11.35)		(17.07)	(10.75)	(15.40)	(15.71)	(13.54)	(10.48)	(12.78)	(21.34)
Excess unhedged stock return x_{us}^c	adj avg	4.69		6.64	2.54	-1.14	4.22	-0.76	9.36	9.40	2.22
	s.d.	(15.79)		(21.04)	(19.40)	(20.25)	(22.45)	(20.26)	(26.41)	(22.89)	(29.20)

Table 2a Cross-Country Return Correlations. This table presents cross-country correlations of log excess returns on currencies, stocks and bonds. Panel A and Panel B display the correlations of currency log excess returns ($\Delta s_{t+1}^c + i_t^c - i_t^1$) between row and column currencies from US and Canada perspectives, respectively. Panel C and D report correlations of hedged excess stock returns x_s^c and hedge bond returns x_b^c , respectively.

	US	Canada	Germany	Australia	Japan	UK	EU	India	China	Brazil
Panel A: Currency Returns US Perspective										
Canada		1.00								
Germany		0.28	1.00							
Australia		0.77	0.40	1.00						
Japan		0.09	0.01	0.13	1.00					
UK		0.55	0.39	0.65	0.06	1.00				
EU		0.54	0.53	0.68	0.30	0.71	1.00			
India		0.48	0.22	0.60	0.02	0.41	0.39	1.00		
China		0.18	0.13	0.23	0.11	0.30	0.28	0.16	1.00	
Brazil		0.56	0.33	0.58	-0.06	0.31	0.37	0.46	0.14	1.00
Panel B: Currency Returns Canada Perspective										
US	1.00									
Germany	0.19		1.00							
Australia	-0.20		0.25	1.00						
Japan	0.60		0.10	-0.04	1.00					
UK	0.37		0.34	0.31	0.24	1.00				
EU	0.32		0.49	0.40	0.42	0.64	1.00			
India	0.63		0.19	0.19	0.36	0.38	0.34	1.00		
China	0.90		0.21	-0.12	0.58	0.43	0.38	0.59	1.00	
Brazil	-0.17		0.17	0.31	-0.20	-0.05	0.04	0.10	-0.13	1.00
Panel C: Hedged Stock Market Excess Returns										
US	1.00									
Canada	0.82	1.00								
Germany	0.83	0.76	1.00							
Australia	0.79	0.73	0.75	1.00						
Japan	0.68	0.64	0.69	0.66	1.00					
UK	0.85	0.75	0.81	0.82	0.63	1.00				
EU	0.86	0.80	0.96	0.80	0.69	0.86	1.00			
India	0.59	0.65	0.61	0.59	0.54	0.51	0.61	1.00		
China	0.50	0.54	0.40	0.49	0.48	0.46	0.39	0.55	1.00	
Brazil	0.61	0.70	0.58	0.63	0.55	0.61	0.61	0.65	0.58	1.00
Panel D: Hedged Bond Market Excess Returns										
US	1.00									
Canada	0.90	1.00								
Germany	0.82	0.76	1.00							
Australia	0.80	0.77	0.81	1.00						
Japan	0.64	0.58	0.67	0.54	1.00					
UK	0.87	0.83	0.84	0.81	0.64	1.00				
EU	0.51	0.46	0.66	0.53	0.50	0.56	1.00			
India	0.35	0.26	0.30	0.33	0.15	0.29	0.20	1.00		
China	0.41	0.36	0.45	0.48	0.24	0.43	0.24	0.26	1.00	
Brazil	0.11	0.03	0.10	0.06	0.11	0.14	0.15	0.10	-0.10	1.00

Table 2b Correlation of unhedged returns. This table presents cross-country correlations of unhedged stock and bond returns. Panel A and B report the correlations of unhedged stock market excess returns from the US and Canadian perspectives, respectively. Panel C and Panel D report the correlations of unhedged bond market excess returns from the US and Canadian perspectives, respectively.

	US	Canada	Germany	Australia	Japan	UK	EU	India	China	Brazil
Panel A: Unhedged Stock Market Excess Returns US Perspective										
US	1.00									
Canada	0.81	1.00								
Germany	0.73	0.69	1.00							
Australia	0.80	0.85	0.72	1.00						
Japan	0.67	0.64	0.50	0.63	1.00					
UK	0.85	0.80	0.73	0.87	0.65	1.00				
EU	0.84	0.81	0.83	0.84	0.67	0.90	1.00			
India	0.62	0.72	0.64	0.73	0.57	0.66	0.71	1.00		
China	0.57	0.62	0.49	0.64	0.57	0.57	0.59	0.58	1.00	
Brazil	0.63	0.81	0.65	0.78	0.53	0.68	0.74	0.66	0.59	1.00
Panel B: Unhedged Stock Market Excess Returns Canada Perspective										
US	1.00									
Canada	0.66	1.00								
Germany	0.64	0.60	1.00							
Australia	0.65	0.75	0.64	1.00						
Japan	0.58	0.46	0.37	0.45	1.00					
UK	0.78	0.66	0.65	0.79	0.53	1.00				
EU	0.76	0.72	0.79	0.77	0.55	0.85	1.00			
India	0.45	0.61	0.55	0.63	0.42	0.52	0.61	1.00		
China	0.41	0.49	0.38	0.53	0.43	0.43	0.47	0.47	1.00	
Brazil	0.39	0.72	0.54	0.69	0.30	0.50	0.61	0.55	0.47	1.00
Panel C: Unhedged Bond Market Excess Returns US Perspective										
US	1.00									
Canada	0.32	1.00								
Germany	0.38	0.67	1.00							
Australia	0.22	0.86	0.66	1.00						
Japan	0.59	0.34	0.49	0.33	1.00					
UK	0.17	0.61	0.62	0.63	0.18	1.00				
EU	0.19	0.63	0.88	0.64	0.38	0.64	1.00			
India	0.18	0.36	0.27	0.33	0.13	0.04	0.32	1.00		
China	0.27	-0.14	-0.04	-0.16	0.24	-0.07	-0.07	-0.04	1.00	
Brazil	-0.09	0.64	0.42	0.67	0.15	0.47	0.43	0.32	-0.22	1.00
Panel D: Unhedged Bond Market Excess Returns Canada Perspective										
US	1.00									
Canada	0.80	1.00								
Germany	0.72	0.68	1.00							
Australia	0.40	0.71	0.53	1.00						
Japan	0.79	0.65	0.69	0.41	1.00					
UK	0.59	0.58	0.65	0.49	0.45	1.00				
EU	0.50	0.52	0.86	0.47	0.54	0.60	1.00			
India	0.56	0.44	0.41	0.25	0.41	0.19	0.36	1.00		
China	0.78	0.43	0.49	0.11	0.64	0.45	0.33	0.45	1.00	
Brazil	-0.30	0.01	-0.10	0.32	-0.14	-0.04	-0.05	-0.07	-0.41	1.00

4. Empirical Analysis

4.1 Single Country portfolio

We begin our empirical analysis by considering a scenario where an investor is fully invested in a single-country equity portfolio and is contemplating whether exposure to other currencies would aid in mitigating the volatility of the quarterly portfolio return. [Table 3](#) provides the optimal currency hedging strategies for various countries' stocks and bonds from both the US and Canadian perspectives. Each panel of the table represents a different perspective (either the US or Canada) and asset class (either stocks or bonds).

Panel A in [Table 3](#) focuses on a US investor's currency exposure when investing in a foreign stock market. Assume a US investor is investing in the Canadian stock market. In this case, the regression coefficient is 1.0698, indicating a strong positive relationship between the Canadian stock market and the currency excess return from CAD. This suggests that if the Canadian stock market performs well, there is a higher likelihood that the CAD will appreciate in value relative to the USD, resulting in higher returns for the US investor when converting their CAD-denominated investments back into USD. However, if the Canadian stock market performs poorly, the US investor may face losses due to a decline in the CAD's value relative to the USD. Given this positive relationship between the Canadian stock market and the CAD, a US investor who is investing in the Canadian stock market may benefit from a currency hedging strategy that reduces exposure to the CAD. Based on the full sample analysis, the optimal hedging strategy suggests an over-hedge strategy with a net long USD exposure of 107% of the principal investment.

Panel A recommends that US investors consider taking full or overhedging positions in all foreign currencies, except for JPY, where a partial hedge is recommended. The negative correlation between the Japanese stock market and its excess currency return implies that an increase in the Japanese stock market is typically associated with a depreciation of the Japanese yen. Therefore, investors who hold exposure to the Japanese stock market may want to avoid overhedging their exposure to the JPY to capture potential currency gains, while still hedging some of their currency risk through a partial hedge.

Canadian stock investors have unique considerations when it comes to currency hedging. Unlike their US counterparts, Canadian investors should maintain a close-to-unhedged position in the US stock market due to the strong negative correlation of -0.97 between the USD and US stock market returns, as shown in Panel B. This means that fluctuations in the USD tend to have a significant impact on the returns of the US stock market. By maintaining an unhedged position, Canadian investors can take advantage of potential gains or losses in the USD, which may offset any gains or losses from the US stock market.

Secondly, Panel B also suggests that Canadian stock investors should hold long positions in Chinese yuan (CNY) and not hedge their exposure to it. This is because there is a significant negative relation of -1.9303 between CNY excess returns

and Chinese stock market returns. Furthermore, unhedged Chinese stock returns are less volatile than hedged equity returns. These unique features are likely driven by China's currency pegging policy, which prevents CNY from depreciating when the Chinese market falls, as shown in [Table 1](#). However, Canadian investors should short AUD and BRL in excess of their long equity position, as these currencies are positively correlated with Canadian stock returns.

[Table 3](#) presents the results of our analysis on individual optimal currency demands for a portfolio invested entirely in a single foreign bond market, from both US and Canadian perspectives (Panel C and Panel D, respectively). While most of these demands are lack statistical significance when considering developed bond markets, some interesting patterns emerge for emerging markets. Notably, US investors show a small yet significant positive demand for CNY, which tends to depreciate as the Chinese long-term bond return increases. Conversely, Canadian investors exhibit a significant negative demand for CNY, as they expect its appreciation when the Chinese long-term bond return rises.

Table 3 Optimal currency exposure for single-country stock/bond portfolios. Table shows optimal currency exposures for minimizing portfolio variance via a single currency approach. Panels A/B represent US/Canada stock investors while C/D refer to US/Canada bond investors. Coefficients and p-values for each variable are listed with one/two/three stars indicating significance level (*<0.1, **<0.05, ***<0.01). We conduct monthly regression on overlapping quarterly returns, and correct the standard errors for auto-correlation resulting from overlapping intervals using the Newey-West procedure.

	Obs	Coef	Std. Err	Ψ	θ	Strategy	Detailed Comments
Panel A: US Perspective Stock							
Canada	559	1.0698***	0.1661	-1.0698	2.0698	Overhedge	Net long USD 107%
Germany	321	0.0224	0.1059	-0.0224	1.0224	Fully hedge	Zero exposure to EUR
Australia	559	0.4095***	0.0935	-0.4095	1.4095	Overhedge	Net long USD 40%
Japan	559	-0.2420**	0.1146	0.2420	0.7580	Partial hedge	Hedge 76% of JPY
UK	559	0.0418	0.1247	-0.0418	1.0418	Fully hedge	Zero exposure to GBP
EU	273	0.0430	0.2344	-0.0430	1.0430	Fully hedge	Zero exposure to EUR
India	346	1.9426***	0.2573	-1.9426	2.9426	Overhedge	Net long USD 194%
China	293	0.7141	0.5214	-0.7141	1.7141	Overhedge	Net long USD 71%
Brazil	321	0.5411***	0.1727	-0.5411	1.5411	Overhedge	Net long USD 54%
Panel B: Canada Perspective Stock							
US	559	-0.9700***	0.1548	0.9700	0.0300	Unhedge	Full exposure to USD
Germany	321	-0.2026	0.1849	0.2026	0.7974	Partial hedge	Hedge 80% of EUR
Australia	559	0.2336**	0.1236	-0.2336	1.2336	Overhedge	Net long CAD 23%
Japan	559	-0.3996***	0.1139	0.3996	0.6004	Partial hedge	Hedge 60% of JPY
UK	559	-0.1844*	0.1046	0.1844	0.8156	Partial hedge	Hedge 82% of GBP
EU	273	-0.7573***	0.1815	0.7573	0.2427	Partial hedge	Hedge 24% of EUR
India	346	-0.0708	0.2319	0.0708	0.9292	Partial hedge	Hedge 93% of INR
China	293	-1.9063***	0.3136	1.9063	-0.9063	Over exposure	Net long CNY 91%
Brazil	321	0.2731	0.1953	-0.2731	1.2731	Overhedge	Net long CAD 27%
Panel C: US Perspective Bond							
Canada	559	-0.0441	0.0595	0.0441	0.9559	Partial hedge	Hedge 96% of CAD
Germany	559	0.0145	0.0260	-0.0145	1.0145	Fully hedge	Zero exposure to EUR
Australia	559	-0.0950**	0.0448	0.0950	0.9050	Partial hedge	Hedge 91% of AUD
Japan	407	0.0776	0.0501	-0.0776	1.0776	Overhedge	Net long USD by 8%
UK	559	0.0547	0.0511	-0.0547	1.0547	Overhedge	Zero exposure to GBP
EU	273	0.0129	0.0429	-0.0129	1.0129	Fully hedge	Zero exposure to EUR
India	274	-0.0276	0.1000	0.0276	0.9724	Fully hedge	Zero exposure to INR
China	231	-0.2088***	0.0943	0.2088	0.7912	Partial hedge	Hedge 79% of CNY
Brazil	172	0.3559***	0.0867	-0.3559	1.3559	Overhedge	Net long USD by 36%
Panel D: Canada Perspective Bond							
US	559	0.0546	0.0715	-0.0546	1.0546	Overhedge	Net long CAD by 5%
Germany	559	0.0424	0.0331	-0.0424	1.0424	Fully hedge	Zero exposure to EUR
Australia	559	-0.0680	0.0615	0.0680	0.9320	Partial hedge	Hedge 93% of AUD
Japan	407	0.0665*	0.0373	-0.0665	1.0665	Overhedge	Net long CAD by 6%
UK	559	0.1161***	0.0522	-0.1161	1.1161	Overhedge	Net long CAD by 11%
EU	273	0.0486	0.0475	-0.0486	1.0486	Fully hedge	Zero exposure to EUR
India	274	0.1709***	0.0778	-0.1709	1.1709	Overhedge	Net long CAD by 17%
China	231	0.2374***	0.0821	-0.2374	1.2374	Overhedge	Net long CAD by 23%
Brazil	172	0.3220***	0.1126	-0.3220	1.3220	Overhedge	Net long CAD by 32%

Table 4: Performance of a Single-Country Portfolio Based on Different Currency Hedging Policies. The data presented in this table represents the returns (in percentage), standard deviations (in percentage), and Sharpe ratios of single-country stock/bond portfolios from both US and Canada perspectives under different currency hedging policies. The portfolio labeled "No hedge" refers to the simple equity portfolio shown in Equation (1). The portfolio labeled "Optimal hedge" represents a portfolio in which the optimal portion of currency risk is neutralized. The portfolio labeled "Half hedge" refers to a portfolio in which half of the implicit currency risk is neutralized. Finally, the portfolio labeled "Full hedge" represents a portfolio in which all implicit currency risk is hedged.

Strategy	No hedge			Optimal Hedge			Full hedge		Half hedge	
	Adj. Mean	Std. Dev.	SR	Adj. Mean	Std. Dev.	SR	Std. Dev.	SR	Std. Dev.	SR
Panel A: US Perspective Stock										
Canada	11.23	31.66	0.3548	08.29	24.09	0.3441	26.30	0.3528	28.67	0.3533
Germany	16.51	41.69	0.3961	11.92	34.40	0.3466	34.38	0.3478	36.15	0.3726
Australia	12.17	34.92	0.3486	07.33	25.60	0.2865	26.37	0.3079	29.70	0.3290
Japan	12.95	34.08	0.3799	12.51	31.18	0.4011	31.56	0.4052	31.49	0.3947
UK	12.66	33.29	0.3803	10.57	27.91	0.3789	27.93	0.3794	29.54	0.3816
EU	07.04	35.31	0.1993	06.30	31.01	0.2031	31.03	0.2022	32.28	0.1965
India	18.30	47.82	0.3827	07.42	35.48	0.2092	41.53	0.3260	44.48	0.3549
China	13.96	52.09	0.2680	11.18	52.05	0.2148	52.05	0.2470	52.24	0.2705
Brazil	20.82	60.65	0.3433	01.94	43.12	0.0451	45.29	0.1399	51.56	0.2434
Panel B: Canada Perspective Stock										
US	11.49	21.27	0.5399	11.50	21.27	0.5406	23.37	0.5288	21.78	0.5416
Germany	14.34	37.64	0.3811	11.83	33.42	0.3541	34.06	0.3462	33.78	0.3650
Australia	11.15	30.25	0.3686	08.73	26.18	0.3335	26.25	0.3424	27.56	0.3576
Japan	13.01	32.78	0.3969	12.91	30.44	0.4241	31.56	0.4336	30.50	0.4203
UK	12.53	31.34	0.3998	11.49	27.89	0.4120	27.95	0.4116	28.55	0.4092
EU	04.16	29.62	0.1406	04.54	29.31	0.1548	30.91	0.2091	29.43	0.1720
India	15.80	42.47	0.3721	13.80	41.44	0.3331	41.47	0.3303	41.59	0.3508
China	11.49	47.29	0.2430	13.19	45.52	0.2898	52.09	0.2496	49.27	0.2530
Brazil	16.26	53.66	0.3031	04.67	44.75	0.1043	45.07	0.1411	48.06	0.2202
Panel C: US Perspective Bond										
Canada	08.59	15.12	0.5683	07.80	12.17	0.6413	12.20	0.6386	12.79	0.6304
Germany	12.11	24.04	0.5038	08.39	10.12	0.8296	10.09	0.8334	14.61	0.6590
Australia	09.75	18.55	0.5257	07.44	11.91	0.6249	12.22	0.6010	13.09	0.6243
Japan	07.60	22.49	0.3380	07.42	11.49	0.6456	11.62	0.6292	15.60	0.4533
UK	11.44	23.10	0.4952	09.54	15.61	0.6113	15.59	0.6134	17.79	0.5701
EU	05.99	18.40	0.3252	05.39	09.26	0.5819	09.26	0.5810	12.23	0.4389
India	08.50	15.19	0.5598	05.59	12.16	0.4596	12.19	0.4530	12.74	0.5396
China	05.32	10.51	0.5067	02.81	09.56	0.2936	09.72	0.2245	09.63	0.3846
Brazil	15.40	41.38	0.3721	06.24	18.70	0.3338	20.93	0.3487	29.77	0.3494
Panel D: Canada Perspective Bond										
US	09.31	17.41	0.5349	09.15	13.60	0.6724	0.13.63	0.6698	14.81	0.6140
Germany	12.85	25.04	0.5133	09.30	10.39	0.8949	0.10.39	0.8992	15.56	0.6728
Australia	09.84	16.19	0.6077	08.32	12.28	0.6776	0.12.42	0.6658	12.87	0.6869
Japan	07.78	24.31	0.3202	08.03	11.34	0.7084	0.11.48	0.6873	16.27	0.4519
UK	12.10	23.81	0.5084	10.47	15.65	0.6692	0.15.73	0.6668	18.45	0.5942
EU	04.86	17.83	0.2724	05.70	09.31	0.6124	0.09.36	0.5997	12.35	0.4036
India	07.88	18.29	0.4309	05.35	11.87	0.4510	0.12.15	0.4620	14.48	0.4551
China	05.68	19.48	0.2917	02.17	09.05	0.2403	0.09.61	0.2720	13.67	0.2856
Brazil	13.21	33.17	0.3983	07.04	20.09	0.3503	0.21.14	0.3742	25.99	0.3875

Impact of hedging currency exposure

[Table 4](#) provides a comprehensive in-sample analysis of the impact of different currency hedging strategies on single-country stock/bond portfolio annualized returns, volatility, and Sharpe ratios. The numerator of the Sharpe ratio is given by the log of the mean gross return on each of the portfolios using equation (4), where Ψ is replaced by the currency demands that correspond to each currency hedging strategy. For example, under the “Optimal hedge” strategy, the optimal hedged returns are calculated based on the optimal single-country hedging ratios obtained in [Table 3](#), and the realized returns are based on the full sample used in the regression analysis.

[Table 4](#) provides **three** noteworthy insights. **Firstly**, currency hedging can significantly reduce the volatility of foreign long-term bond investments, while its effectiveness for foreign stock market investments is comparatively lower. For instance, the volatility of Brazilian bond returns decreases from an unhedged level of 41.38% to an optimal hedge level of 18.70%, resulting in a 54.8% risk reduction from the perspective of USD-based investors. Conversely, the volatility of Brazilian stock returns only decreases from an unhedged level of 60.65% to 43.13%, resulting in a 28% decrease in risk for USD-based investors. Furthermore, the risk reduction from currency hedging for most other foreign market stock returns is negligible, including the two other emerging stock markets of China and India.

Second, Currency hedging can have a significant impact on the investment returns of most foreign portfolios. For instance, consider a CAD-based investor investing in the Brazilian stock market, as shown in Panel B of [Table 4](#). In this case, currency hedging can reduce the annual investment return from 16.26% to 4.67%. One reason for this is that currency hedging limits exposure to currency movements that could work in the investor's favor when the value of the foreign currency appreciates. Additionally, currency hedging can also affect the carry return of an investment. The carry return is the return earned from holding an asset based on the interest rate differential between the two currencies.

Although currency hedging can have a negative impact on the investment returns of most foreign portfolios, there are some exceptions for CAD-based investors. By following the optimal currency hedge strategy, CAD-based investors can actually achieve higher stock returns from the Chinese stock market. Specifically, the annual portfolio returns can increase from 11.49% to 13.19%.

Thirdly, it's worth noting that hedging currency risk can have a significant impact on the Sharpe ratio of foreign long-bond markets, particularly in developed markets, for both USD and CAD-based investors. Specifically, hedging currency risk can substantially increase the Sharpe ratio of these markets, resulting in more sustainable returns. However, when it comes to emerging market stocks and bonds, employing a currency hedge can have the opposite effect, potentially reducing the Sharpe ratio of the returns.

Cost of hedging currency risk for a single county portfolio

A common approach to currency hedging is to borrow in a foreign currency for the short term and lend in the investor's base currency, as shown in Equation (4). However, this method can incur additional transaction costs such as interest

rate spreads and credit risks. On the other hand, using currency forward contracts, as shown in Equation (2), is a more cost-effective option. By entering into a contract with a counterparty, investors can lock in a predetermined exchange rate on a specified future date, thereby avoiding the need to borrow and lend in foreign currencies while still achieving the same hedged result.

To analyze the cost efficiency associated with different currency hedging approaches, we regenerate realized in-sample stock/bond portfolio returns using both the borrowing/lending approach and forward contracts. Table 5 displays the average annualized portfolio returns using different currency hedging instruments, and the difference between the unhedged and hedged portfolio returns represents the cost of currency hedging. For example, under the “Optimal Hedge” panel, we use the optimal hedge ratio display in Table 3 to calculate the realized hedged portfolio returns using both a “borrow/lend” strategy and “forward” contracts. Figure 1 and Figure 2 present the cost of employing the optimal currency hedging ratio for USD-based and CAD-based investors, respectively.

To conduct our analysis, we rely on one-month forward exchange rates. For developed markets, such data has been available since December 1988. We also have data for Brazil dating back to 1999, and for China starting from August 2010. Unfortunately, we do not have access to forward exchange rate data for India and Germany at present. To ensure that our analysis is relevant to the specific horizon of the forward contract, we adjust the sample period for each series accordingly. This involves shortening the original time frame to match the new horizon of the forward contract.

Table 5 yields two key takeaways. Firstly, implementing currency forward contracts can achieve similar hedging results compared to the borrow/lend approach. For example, for investors with a USD base currency (Panel A), the hedged Canadian stock returns exhibit an annualized volatility of 22% (7%/0.31) using the borrow/lend approach, which is comparable to the annualized volatility of 21% (8%/0.37) achieved by utilizing currency forward contracts. Both hedging methods result in remarkable reductions in annualized volatility when compared to the unhedged stock returns, which exhibit an annualized volatility of 32% (10.95%/0.34).

Furthermore, when it comes to hedging emerging market currencies, there is a significant cost difference between using forward contracts and implementing a borrow/lend strategy. Figure 1 shows, for USD-based investors, the cost of reducing the risk associated with the Brazilian real (BRL) using the borrow/lend approach is 19% (i.e., the difference between the unhedged and hedged investment returns is 23% minus 4.1%). This cost is more than twice as high as the cost of using forward contracts, which is 6.7% (i.e., the difference between the unhedged and hedged investment returns is 23% minus 16.3%). Therefore, investors should carefully weigh the potential benefits and costs of different hedging strategies before making investment decisions involving emerging market currencies.

When comparing Figure 1 and Figure 2, we observe that the cost of currency hedging is considerably higher for US investors than for Canadian investors, regardless of whether they are investing in developed or emerging foreign stock markets. For instance, the hedging cost for US investors investing in Australian stock market stands at 6.5%, while for

Canadian investors, it is only 2.8%, achieved through the borrow/lend hedging strategy. Interestingly, the Chinese stock market exhibits a negative hedging cost from a Canadian investor's viewpoint, owing to the optimal long CNY position, suggesting that neglecting currency risk could lead to a loss of potential gains.

Based on the data presented in [Figure 1](#) and [Figure 2](#), it can be inferred that the CIP is less likely to hold true in emerging markets. The discrepancy between the hedging cost obtained through "borrow/lend" and "forward" approaches clearly demonstrate the existence of a Covered Interest Arbitrage.

Figure 1. The figure displays the cost of currency hedging of a single-country stock market for USD-based investors using both "borrow/lend" and "forward" approaches. The cost of hedging is calculated by taking the difference between the returns of an unhedged portfolio and a hedged portfolio using the optimal hedge ratio presented in [Table 3](#).

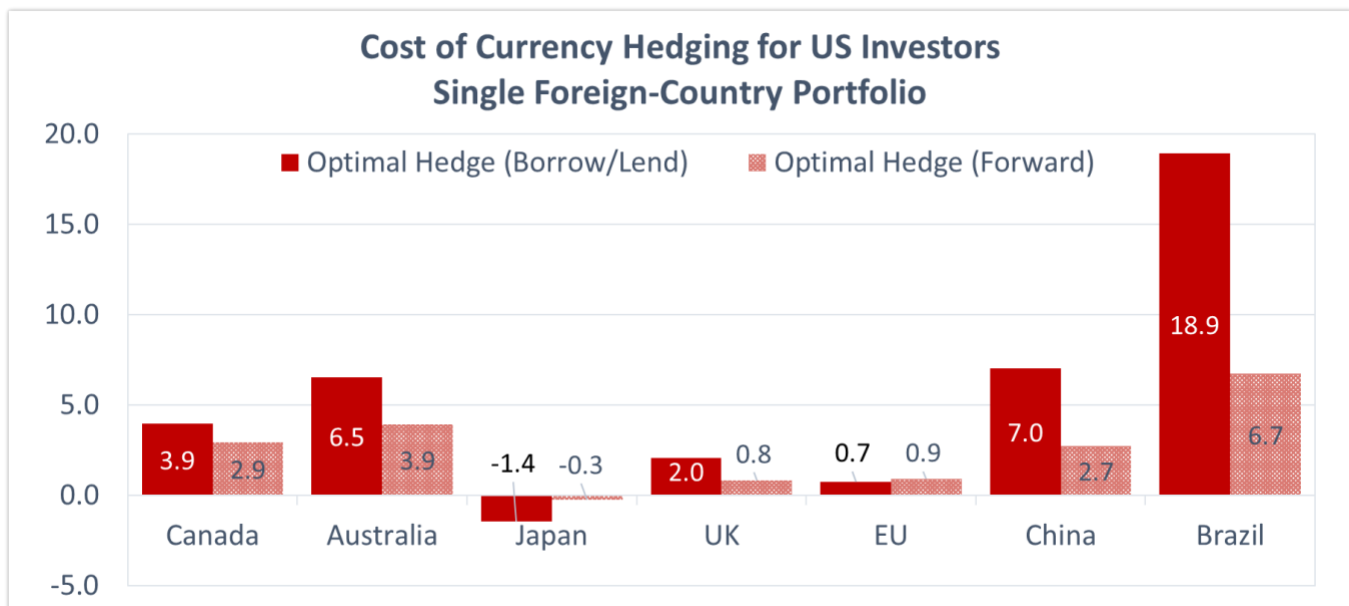


Figure 2. The figure displays the cost of currency hedging of a single-country stock market for CAD-based investors using both "borrow/lend" and "forward" approaches. The cost of hedging is calculated by taking the difference between the returns of an unhedged portfolio and a hedged portfolio using the optimal hedge ratio presented in [Table 3](#).

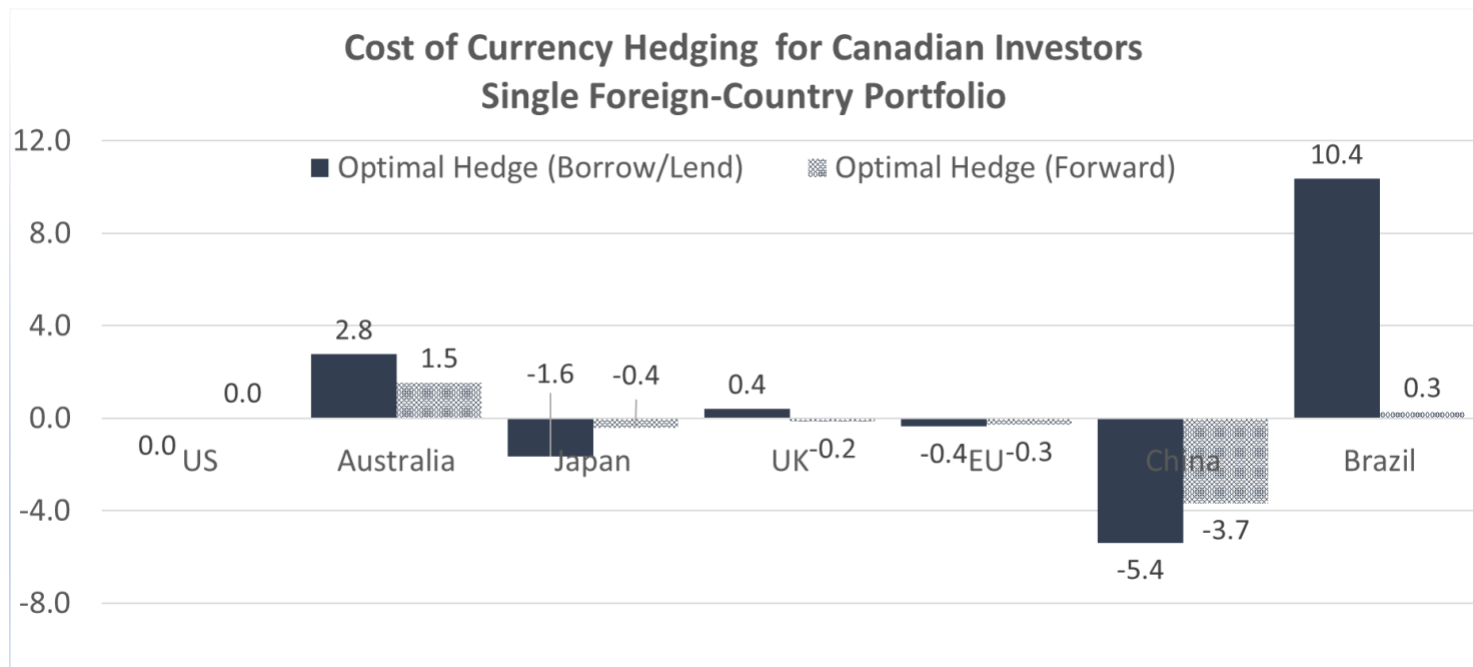


Table 5 Cost of currency hedging. The table displays the impact of currency hedging on the annualized returns (in percentage) of single-country stock/bond portfolios and Sharpe ratios, comparing two approaches: borrowing in foreign currency and lending the proceeds in the base currency, shown in the "Borrow/Lend" columns, and using currency forward contracts to hedge currency fluctuations, shown in the "Forward" columns. The difference between unhedged returns and hedged returns can be attributed to the cost of currency hedging.

	No Hedge		Optimal Hedge				Full Hedge				Half Hedge			
			Borrow/Lend		Forward		Borrow/Lend		Forward		Borrow/Lend		Forward	
	Avg	SR	Avg	SR	Avg	SR	Avg	SR	Av	SR	Avg	SR	Avg	SR
Panel A Stocks US-perspective														
Canada	10.9	0.34	7.0	0.32	8.0	0.37	8.4	0.33	9.0	0.36	9.5	0.34	9.9	0.35
Australi	12.1	0.34	5.6	0.23	8.2	0.34	6.8	0.27	8.7	0.34	9.0	0.31	10.0	0.34
Japan	5.2	0.17	6.6	0.21	5.4	0.17	7.4	0.23	5.8	0.18	5.9	0.19	5.2	0.17
UK	6.8	0.25	4.7	0.22	6.0	0.28	4.8	0.22	6.0	0.28	5.5	0.24	6.1	0.27
EU	7.0	0.20	6.3	0.20	6.1	0.20	6.3	0.20	6.1	0.20	6.3	0.20	6.3	0.20
China	9.7	0.32	2.6	0.10	7.0	0.26	5.4	0.20	8.1	0.29	7.4	0.26	9.0	0.31
Brazil	23.0	0.38	4.1	0.12	16.3	0.49	8.2	0.21	16.9	0.43	14.4	0.30	19.8	0.40
Panel B Stock Canada Perspective														
US	10.6	0.52	10.7	0.52	10.6	0.52	11.8	0.51	11.3	0.49	11.1	0.53	10.8	0.51
Australi	7.8	0.30	5.0	0.23	6.3	0.29	5.4	0.25	6.4	0.29	6.4	0.28	7.0	0.30
Japan	4.5	0.16	6.1	0.21	4.9	0.17	8.0	0.25	5.9	0.18	5.8	0.20	4.8	0.16
UK	5.7	0.26	5.3	0.25	5.8	0.28	5.4	0.25	6.0	0.28	5.3	0.26	5.6	0.27
EU	4.2	0.14	4.5	0.15	4.5	0.15	6.5	0.21	6.1	0.20	5.1	0.17	4.9	0.17
China	10.6	0.39	16.0	0.52	14.3	0.47	5.7	0.21	8.1	0.30	8.0	0.30	9.4	0.35
Brazil	17.4	0.34	7.0	0.19	17.1	0.46	8.4	0.22	16.7	0.42	12.1	0.28	17.3	0.38
Panel C Bond US-perspective														
Canada	7.7	0.57	6.8	0.66	7.3	0.70	6.8	0.65	7.3	0.70	7.1	0.66	7.4	0.68
Australi	10.8	0.57	7.6	0.62	9.2	0.75	7.5	0.59	9.2	0.73	8.7	0.66	9.5	0.72
Japan	6.1	0.30	6.5	0.69	4.8	0.51	6.4	0.67	4.8	0.51	5.8	0.45	5.2	0.39
UK	7.8	0.49	6.3	0.60	7.5	0.71	6.3	0.62	7.5	0.72	6.8	0.62	7.4	0.66
EU	6.0	0.33	5.4	0.58	5.4	0.57	5.4	0.58	5.3	0.57	5.4	0.44	5.4	0.43
China	4.6	0.53	1.6	0.23	3.4	0.52	0.9	0.12	3.2	0.47	2.7	0.38	3.8	0.55
Brazil	15.4	0.37	6.2	0.33	14.4	0.80	7.3	0.35	13.4	0.65	10.4	0.35	13.5	0.45
Panel D Bond Canada-perspective														
US	8.0	0.46	7.7	0.67	7.1	0.63	7.70	0.66	7.1	0.62	7.7	0.56	7.4	0.54
Australi	9.3	0.66	7.7	0.61	8.5	0.69	7.59	0.60	8.5	0.67	8.2	0.69	8.7	0.72
Japan	6.7	0.30	7.1	0.75	4.8	0.53	6.98	0.73	4.8	0.52	6.4	0.44	5.4	0.37
UK	8.0	0.48	6.9	0.66	7.7	0.72	6.93	0.67	7.6	0.73	7.2	0.60	7.5	0.63
EU	4.9	0.27	5.7	0.61	5.5	0.58	5.62	0.60	5.4	0.57	5.0	0.40	4.9	0.39
China	6.8	0.50	0.2	0.02	2.6	0.40	1.31	0.18	3.3	0.48	3.9	0.40	4.9	0.51
Brazil	13.2	0.40	7.0	0.35	14.6	0.75	7.91	0.37	13.7	0.65	10.1	0.39	13.0	0.50

Table 6 Optimal currency exposure for multi-country stock/bond portfolios. The table is divided into four panels: Panels A and C show the optimal currency positions of equally weighted stock/bond portfolios for USD-based investors, while Panels B and D show the optimal currency exposure of equally weighted stock/bond portfolios for CAD-based investors. The "Coef" terms were obtained by regressing the equally weighted excess return onto the vector of all foreign currency excess returns. The home-country currency exposure terms (last row in each panel) were calculated by computing the opposite of the sum of other terms in the same row, along with the corresponding standard deviation. The reported currency positions in the table represent the amount of dollars invested in foreign currency per dollar in the portfolio. To obtain these values, we ran monthly regressions on overlapping quarterly returns. Standard errors were corrected for autocorrelation due to overlapping intervals using the Newey-West procedure.

	Coef	Std. Err	Ψ	θ	Strategy	Detailed Comments
Panel A: US Perspective 9-country equally weighted stock portfolio						
Canada	0.4951***	0.1856	-0.4951	1.4951	Overhedge	Net long 49.5% USD
Germany	-0.1400	0.0939	0.1400	0.8600	Partial hedge	Hedge 86% of EUR
Australia	0.4597***	0.1487	-0.4597	1.4597	Overhedge	Net long 45.9% USD
Japan	-0.3404***	0.0982	0.3404	0.6596	Partial hedge	Hedge 66% of JPY
UK	0.0538	0.1447	-0.0538	1.0538	Overhedge	Net long 5% USD
India	0.4809***	0.1759	-0.4809	1.4809	Overhedge	Net long 48%USD
China	-0.2238	0.2674	0.2238	0.7762	Partial hedge	Hedge 77.6% CNY
Brazil	0.1229**	0.0716	-0.1229	1.1229	Overhedge	Net long 12% of USD
US	-0.9081	0.1595	0.9081	0.0919	Partial hedge	Hedge 9% of USD
Panel B: Canadian Perspective 9-country equally weighted stock portfolio						
US	-0.9180***	0.2678	0.9180	0.0820	Partial hedge	Hedge 8% of USD
Germany	-0.1402	0.0939	0.1402	0.8598	Partial hedge	Hedge 86% of AUD
Australia	0.4600***	0.1490	-0.4600	1.4600	Overhedge	Net long CAD
Japan	-0.3411	0.0984	0.3411	0.6589	Partial hedge	Hedge 65% of GBP
UK	0.0525	0.1444	-0.0525	1.0525	Overhedge	Net long 5% CAD
India	0.4805	0.1758	-0.4805	1.4805	Overhedge	Net long 48% INR
China	-0.2110	0.2685	0.2110	0.7890	Partial hedge	Hedge 79% of CNY
Brazil	0.1229	0.0715	-0.1229	1.1229	Overhedge	Net long CAD
Canada	0.4945	0.1737	-0.4945	1.4945	Overhedge	Lend home currency
Panel C: US Perspective 9-country equally weighted bond portfolio						
Canada	-0.0583	0.0664	0.0583	0.9417	Partial hedge	Hedge 94% of CAD
Germany	-0.1075**	0.0627	0.1075	0.8925	Partial hedge	Hedge 89% of EUR
Australia	-0.0059	0.0576	0.0059	0.9941	Full hedge	No AUD exposure
Japan	0.2503***	0.0448	-0.2503	1.2503	Overhedge	Net long 25% USD
UK	-0.1096**	0.0572	0.1096	0.8904	Partial hedge	Hedge 89% GBP
India	0.1037	0.0758	-0.1037	1.1037	Overhedge	Net long 10% USD
China	-0.0943	0.0820	0.0943	0.9057	Partial hedge	Hedge 91% CNY
Brazil	0.0219	0.0307	-0.0219	1.0219	Full hedge	Hedge 100% BRL
US	-0.0002	0.0616	0.0002	0.9998	Full hedge	Zero exposure to USD
Panel D: Canada Perspective 9-country equally weighted bond portfolio						
US	-0.0002	0.0904	0.0002	0.9998	full hedge	Hedge 100% USD
Germany	-0.1075*	0.0627	0.1075	0.8925	Partial hedge	Hedge 89% EUR
Australia	-0.0059	0.0576	0.0059	0.9941	full hedge	Hedge 100% AUD
Japan	0.2503***	0.0448	-0.2503	1.2503	Overhedge	Net long 25% JPY
UK	-0.1096**	0.0572	0.1096	0.8904	Partial hedge	Hedge 89% GBP
India	0.1037	0.0758	-0.1037	1.1037	Overhedge	Net long 10% INR
China	-0.0943	0.0820	0.0943	0.9057	Partial hedge	Hedge 91% CNY
Brazil	0.0219	0.0307	-0.0219	1.0219	full hedge	Hedge 100% BRL
Canada	-0.0583	0.0653	0.0583	0.9417	Partial hedge	5% exposure to CAD

4.2 Multi-Country portfolio

In the preceding section, we discussed the implications of hedging currency exposure for investors invested in a single foreign market. Now, we delve into the impact of hedging on multi-country portfolios. Unlike single-country portfolios, the risk associated with multi-country portfolios is not just a simple weighted average of individual country risks.

Instead, it depends on various factors, such as the covariances among the stock/bond market returns, the covariances among the exchange rate changes, and the cross-covariances among the stock/bond market returns and the exchange rate changes.

[Table 6](#) presents the optimal currency exposures for investors who are considering multiple currencies simultaneously. In Panel A, the results indicate that USD-based investors should over-hedge their exposure to CAD, AUD, INR, and BRL, as these currencies tend to depreciate against USD when their stock markets fall. Empirically we show the unhedged excess return of the equally weighted 9-country stock is positively correlated with the excess exchange rate returns from these four foreign currencies. To provide an example, if the Brazilian stock market performs poorly, the BRL is likely to depreciate relative to the USD, given the positive coefficient of 0.1229. This means that USD-based investors may face additional losses during the foreign stock market downturn due to a decline in the value of these currencies relative to the USD. Therefore, it is advisable for USD-based investors to over-hedge their exposure to these currencies.

Moreover, the optimal hedging ratio per currency of the multi-country portfolio is smaller than that of the single-country portfolio shown in [Table 3](#). This suggests that investors in multi-currency portfolios can benefit from diversification, as holding a diversified portfolio of currencies helps to reduce the overall currency risk exposure of the portfolio.

In Panel B, we confirm the finding previously observed in [Table 3](#), which is that Canadian investors who invest in US stocks tend to hold almost all (91%) of their US portfolio exposure in USD. This preference for the USD may be attributed to the currency's status as a "safe-haven" asset that investors tend to favor during periods of market uncertainty or economic downturns. During such times, investors may shift their investments towards the USD, leading to an appreciation of the currency.

However, when considering Canadian investors who invest in the Chinese stock market, our findings differ from those in [Table 3](#). Specifically, our analysis suggests that these investors should partially hedge their CNY exposure, rather than

taking a long position in CNY. This indicates that the strong negative correlation previously observed between Chinese exchange rate returns and stock market returns is weakened when considering a multi-currency stock portfolio.

Cost of currency hedging for multi-country portfolios

[Table 7](#) shows the impact of currency hedging on the performance of a multi-country stock/bond portfolio using both the borrow/lend and forward contract approaches under three currency hedging strategies: optimal hedge, full hedge, and half hedge. The optimal hedge ratio was computed using the truncated sample period starting in 2007 for the bond portfolio and 1997 for the stock portfolio. However, unlike [Table 6](#)'s 9-country portfolio, [Table 7](#) includes an 8-country portfolio because the forward exchange rate data for India is not available. To reflect the available forward exchange rate data series, the realized portfolio returns were calculated starting from August 2010 as a subsample period.

[Figure 7](#) provides us with two important insights. Firstly, the multi-country portfolios demonstrate lower volatility compared to most single-country portfolios, highlighting the advantages of diversification. Specifically, without currency hedging, both US and Canadian investors experience an average annual volatility of over 30% for single-country stock returns, and over 25% for single-country unhedged bond portfolios. However, when investing in an equally weighted stock portfolio, the volatility reduces to 22.7% and 17.44% for US and Canadian investors respectively. Additionally, the multi-country bond portfolio experiences a significant reduction in volatility, shrinking by more than half to 11.22% and 10.71% respectively. This underscores the benefits of investing in multi-country portfolios, particularly for those seeking lower volatility and increased diversification.

Secondly, the analysis reveals that the average hedging cost of multi-country portfolios is significantly lower than that of most single-country portfolios. In particular, the cost of minimizing currency fluctuation for a Canadian investor in a multi-country stock portfolio is less than 1%, using the borrow/lend approach (6.50% minus 5.51%). In addition, employing forward contracts to hedge currency risk can lead to additional realized returns. This highlights the potential cost savings and benefits of hedging currency risk in a multi-country portfolio, particularly compared to single-country portfolios.

Another important finding from [Table 7](#) is that USD-based investors tend to benefit more from currency hedging compared to CAD-based investors. For instance, using the borrow/lend hedging strategy, the Sharpe ratio of optimally hedged realized stock return for a CAD-based investor is 0.32, which is even lower than the unhedged realized stock returns. However, for USD-based investors, the Sharpe ratios of hedged stock and bond portfolios are consistently higher than the unhedged ones. This suggests that the benefits of currency hedging may vary depending on the type of investment, currency, and investor's base currency. Therefore, investors should carefully evaluate the costs and benefits of hedging currency risk based on their individual circumstances and investment goals.

Table 7 Cost of currency hedging of multi-country portfolio. The table displays the impact of currency hedging on the annualized returns (in percentage), volatilities (in percentage) and Sharpe ratios of multi-country stock/bond portfolios. comparing two approaches: borrowing in foreign currency and lending the proceeds in the base currency, shown in the "Borrow/Lend" columns, and using currency forward contracts to hedge currency fluctuations, shown in the "Forward" columns.

	US Perspective Multi-country stock		Canada Perspective Multi-country stock		US Perspective Multi-country bond		Canada Perspective Multi-country bond	
No hedge								
Average	5.88		6.50		3.86		5.48	
Std. Dev.	22.70		17.44		11.22		10.71	
SR	0.26		0.37		0.34		0.51	
	Borrow /Lend	Forward	Borrow /Lend	Forward	Borrow /Lend	Forward	Borrow /Lend	Forward
Optimal hedge								
Average	5.13	6.91	5.51	6.97	4.02	5.24	4.40	5.30
Std. Dev.	17.29	17.05	17.25	17.04	7.71	7.72	7.68	7.73
SR	0.30	0.41	0.32	0.41	0.52	0.68	0.57	0.69
Full hedge								
Average	5.37	6.92	5.75	6.98	3.96	5.18	4.34	5.24
Std. Dev.	18.25	18.07	18.22	18.06	8.03	8.03	7.99	8.04
SR	0.29	0.38	0.32	0.39	0.49	0.65	0.54	0.65
Half Hedge								
Average	5.54	6.63	6.08	6.97	3.82	4.60	4.87	5.43
Std. Dev.	20.16	20.10	17.59	17.52	8.79	9.12	8.98	9.22
SR	0.27	0.33	0.35	0.40	0.43	0.50	0.54	0.59

4.3 Subsample Analysis

US Perspective Single-Country Stock Portfolios

To investigate the impact of regime change on optimal currency hedging strategies, we divided the sample into three sub-periods. [Table 8a](#) shows the optimal currency hedging strategies for single-country stock portfolios held by USD-based investors. All three sub-samples exhibit positive coefficient values at "Canada" row, indicating that a decline in the Canadian stock market is likely associated with CAD depreciation. These findings are consistent with the full sample analysis shown in [Table 3](#), which suggests that US investors could benefit from hedging against CAD exchange rate fluctuations when investing in the Canadian stock market.

Unlike other sub-samples or the full sample period, during the 2008 global financial crisis, US investors were advised to increase their exposure to CNY when investing in the Chinese stock market due to the significant negative relationship of -6.17 between CNY excess returns and Chinese stock market returns. During times of economic stress or market volatility, the Chinese central bank may intervene to prevent excessive depreciation of the CNY to maintain stability in

the economy and financial markets. Additionally, the Chinese government may implement policies or regulations to support the stock market during a downturn, indirectly supporting the value of the CNY.

Table 8a Subsample Analysis. US Perspective Single-Country Stock Portfolios

	Coef	Std. Err	Ψ	θ	Strategy	Detailed Comments
Panel A: 2011 – 2021 Post global financial crisis						
Canada	0.7942***	0.2052	-0.7942	1.7942	Overhedge	Net long 79% USD
Germany	0.0346	0.2759	-0.0346	1.0346	Full hedge	No EUR exposure
Australia	0.4388**	0.1820	-0.4388	1.4388	Overhedge	Net long 44% USD
Japan	-1.2811***	0.1487	1.2811	-0.2811	Over exposure	Net long 28% of JPY
UK	0.1416	0.1965	-0.1416	1.1416	Overhedge	Net long 14% USD
EU	0.0834	0.2323	-0.0834	1.0834	Overhedge	Net long 8% USD
India	1.1361***	0.1802	-1.1361	2.1361	Overhedge	Net long 113.6% USD
China	0.9363***	0.3662	-0.9363	1.9363	Overhedge	Net long 93.6% of USD
Brazil	0.7929***	0.1168	-0.7929	1.7929	Overhedge	Net long 79% of USD
Panel B: 2007-2011 Global financial crisis						
Canada	1.7491***	0.1917	-1.7491	2.7491	Overhedge	Net long 175% USD
Germany	0.9833***	0.3633	-0.9833	1.9833	Overhedge	Net long 98% USD
Australia	0.8508***	0.0970	-0.8508	1.8508	Overhedge	Net long 85% USD
Japan	-1.5994***	0.3423	1.5994	-0.5994	Over exposure	Net long 59% of JAP
UK	0.9679***	0.1053	-0.9679	1.9679	Overhedge	Net long 97% USD
EU	1.0646***	0.3122	-1.0646	2.0646	Overhedge	Net long 106% USD
India	2.4979***	0.4826	-2.4979	3.4979	Overhedge	Net long 250% USD
China	-6.1776***	2.2890	6.1776	-5.1776	Over exposure	Net long 518% CNY
Brazil	1.2109***	0.1665	-1.2109	2.2109	Overhedge	Net long 121% USD
Panel C: 1975-1983						
Canada	1.2944*	0.7199	-1.2944	2.2944	Overhedge	Net long 129% of USD
Australia	0.5325	0.3319	-0.5325	1.5325	Overhedge	Net long 53% of USD
Japan	0.0168	0.1128	-0.0168	1.0168	Full hedge	No exposure to JPY
UK	0.2651	0.1994	-0.2651	1.2651	Overhedge	Net long 26 USD

Canada Perspective Single-Country Stock Portfolios

[Table 8b](#) presents the results of optimal currency hedging strategies for single-country stock portfolios held by CAD-based investors over different subsample periods. The findings reveal that CAD-based investors exhibit a strong interest in holding foreign currencies, as the optimal demands “ Ψ ” for most foreign currencies are positive and statistically significant, particularly during the 2008 global financial crisis period.

Our subsample analysis has revealed a robust negative correlation between the stock market performance of the US, China, and the CAD exchange rate. Specifically, we find that both USD and CNY tend to appreciate against CAD when the US or Chinese stock markets experience a decline, regardless of the specific economic regime. These findings have

important implications for CAD-based investors, who may face significant currency risk when investing in foreign stocks denominated in USD or CNY.

It's worth noting that the negative correlation between the stock market performance of the US, China, and the CAD exchange rate appears to have weakened in the past decades. As such, investors should be prepared to reassess their hedging strategies periodically and adjust them accordingly to reflect changing market conditions.

Table 8b Subsample Analysis. Canadian Perspective Single-Country Stock Portfolios

	Coef	Std. Err	Ψ	θ	Strategy	Detailed Comments
Panel A: 2011 – 2021 Post global financial crisis						
US	-0.8331***	0.1873	0.8331	0.1669	Partial hedge	Hedge 17% USD
Germany	-0.5714**	0.2636	0.5714	0.4286	Partial hedge	Hedge 43% EUR
Australia	0.1080	0.2243	-0.1080	1.1080	Overhedge	Net long 10% CAD
Japan	-1.2498***	0.1455	1.2498	-0.2498	Over exposure	Net long 25% JPY
UK	-0.3331**	0.1333	0.3331	0.6669	Partial hedge	Hedge 67% GBP
EU	-0.5520**	0.2480	0.5520	0.4480	Partial hedge	hedge 45% EUR
India	0.1462	0.1636	-0.1462	1.1462	Overhedge	Net long 15.6% CAD
China	-0.6519*	0.3587	0.6519	0.3481	Partial hedge	Hedge 34.8% CNY
Brazil	0.5644***	0.1886	-0.5644	1.5644	Overhedge	Net long 56% CAD
Panel B: 2007-2011 Global financial crisis						
US	-1.7763***	0.1799	1.7763	-0.7763	Over exposure	Net long 77.6% USD
Germany	-1.0821	0.3417	1.0821	-0.0821	Over exposure	Net long 8.2% EUR
Australia	0.9723***	0.2627	-0.9723	1.9723	Overhedge	Net long 97.2% CAD
Japan	-1.2058***	0.1361	1.2058	-0.2058	Over exposure	Net long 20.6% JPY
UK	0.1965	0.4001	-0.1965	1.1965	Overhedge	Net long 19.7% CAD
EU	-0.8399	0.3442	0.8399	0.1601	Partial hedge	hedge 16% EUR
India	-2.6852**	1.0935	2.6852	-1.6852	Over exposure	Net long 168.5% INR
China	-2.9895***	0.2901	2.9895	-1.9895	Over exposure	Net long 198.9% CNY
Brazil	1.4528	0.5061	-1.4528	2.4528	Overhedge	Net long 145% CAD
Panel C: 1975-1983						
US	-1.4732***	0.4075	1.4732	-0.4732	Over exposure	Net long 47% USD
Australia	0.2237	0.2750	-0.2237	1.2237	Overhedge	Net long 22.4% CAD
Japan	0.0266	0.0990	-0.0266	1.0266	Full hedge	No exposure to JPY
UK	0.2653	0.1935	-0.2653	1.2653	Overhedge	Net long 26.5% CAD

US Perspective Single-Country Bond Portfolios

Our analysis of [Table 8c](#) indicates a noteworthy shift in the relationship between Canadian long-term bond returns and the value of CAD against USD over time. Between 1975 and 1983, a decline in Canadian long-term bond returns corresponded with a depreciation of CAD against USD. However, in the more recent subperiods of 2007-2011 and 2011-

2021, a decline in Canadian long-term bond returns has been associated with an appreciation of CAD. This shift in the relationship may be attributed to changes in market conditions and investor sentiment.

For instance, during the earlier period, CAD was closely linked to commodity prices, and a decrease in Canadian bond returns signaled a weakening economy, leading to a depreciation of CAD. On the other hand, during the more recent subperiods, a decline in Canadian bond returns may have been interpreted as a sign of economic weakness, leading to expectations of monetary policy easing and subsequent currency appreciation.

Therefore, US bond investors may find it beneficial to include some CAD in their investment portfolios to diversify their currency exposure and potentially take advantage of CAD's appreciation. A similar pattern can also be observed in the long-term bond market and local currencies of Germany and the UK. These findings emphasize the importance of closely monitoring market conditions when analyzing the relationship between bond returns and currency values.

Table 8c Subsample Analysis. US Perspective Single-Country Bond Portfolios

	Coef	Std. Err	Ψ	θ	Strategy	Detailed Comments
Panel A: 2011 – 2021 Post global financial crisis						
Canada	-0.4204***	0.0786	0.4204	0.5796	Partial hedge	Hedge 57.9% CAD
Germany	-0.2738***	0.0545	0.2738	0.7262	Partial hedge	Hedge 72.6% EUR
Australia	-0.1502**	0.0770	0.1502	0.8498	Partial hedge	Hedge 85% AUD
Japan	0.0531	0.0342	-0.0531	1.0531	Overhedge	Net long 5% of USD
UK	-0.3741***	0.0794	0.3741	0.6259	Partial hedge	Hedge 62.5% GBP
EU	-0.1129	0.0912	0.1129	0.8871	Partial hedge	Hedge 88.7% EUR
India	0.0954	0.0808	-0.0954	1.0954	Overhedge	Net long 9.6% USD
China	-0.2461***	0.0873	0.2461	0.7539	Partial hedge	Hedge 75.4% of CNY
Brazil	0.4006***	0.1046	-0.4006	1.4006	Overhedge	Net long 40% of USD
Panel B: 2007-2011 Global financial crisis						
Canada	-0.3097***	0.0975	0.3097	0.6903	Partial hedge	Hedge 69% CAD
Germany	-0.2782***	0.0609	0.2782	0.7218	Partial hedge	Hedge 72.6% EUR
Australia	-0.3473***	0.0496	0.3473	0.6527	Partial hedge	Hedge 65% AUD
Japan	0.1869***	0.0360	-0.1869	1.1869	Overhedge	Net long 18.7% of USD
UK	-0.2209***	0.0503	0.2209	0.7791	Partial hedge	Hedge 77.9% GBP
EU	-0.1392**	0.0574	0.1392	0.8608	Partial hedge	Hedge 86.1% EUR
India	-0.2960	0.2101	0.2960	0.7040	Partial hedge	Hedge 70% INR
China	-0.0569	0.5160	0.0569	0.9431	Partial hedge	Hedge 94% of CNY
Brazil	0.2198*	0.1205	-0.2198	1.2198	Overhedge	Net long 22% of USD
Panel C: 1975-1983						
Canada	0.8191**	0.3937	-0.8191	1.8191	Overhedge	Net long 81.9% USD
Germany	0.4574***	0.0792	-0.4574	1.4574	Overhedge	Net long 45.7% USD
Australia	0.2201	0.1344	-0.2201	1.2201	Overhedge	Net long 22% USD
UK	0.2750**	0.1193	-0.2750	1.2750	Overhedge	Net long 27.5% USD

Canada Perspective Single-Country Bond Portfolios

Our analysis suggests that Canadian investors may find it beneficial to overhedge their USD exposure in US long-term bond investments during the post-global financial crisis subsample periods. Specifically, as seen in [Table 8d](#), we observe that a decline in US long-term bond returns is associated with a depreciation of the USD against CAD during the low-rate period of 2011-2021, as well as during the global financial crisis of 2007-2011. These findings may suggest that Canadian investors are more sensitive to changes in the USD/CAD exchange rate during these periods and may benefit from overhedging to mitigate currency risk.

In contrast, our analysis of the 1975-1983 subsample (see Panel C) period reveals a strong negative correlation between US bond returns and the USD exchange rate against CAD, indicating that a full exposure to the USD may be beneficial for Canadian investors during this time, as a decline of the US bond market is associated with an appreciated of the USD for CAD-based investors.

Table 8d Subsample Analysis. Canadian Perspective Single-Country Bond Portfolios

	Coef	Std. Err	Ψ	θ	Strategy	Detailed Comments
Panel A: 2011 – 2021 Post global financial crisis						
US	0.2505**	0.1029	-0.2505	1.2505	Overhedge	Net long 25% CAD
Germany	-0.1669**	0.0801	0.1669	0.8331	Partial hedge	Hedge 83% EUR
Australia	-0.0638	0.1202	0.0638	0.9362	Partial hedge	Hedge 94% AUD
Japan	0.0550**	0.0233	-0.0550	1.0550	Overhedge	Net long 5% CAD
UK	-0.1710**	0.0768	0.1710	0.8290	Partial hedge	Hedge 83% GBP
EU	-0.2043*	0.1168	0.2043	0.7957	Partial hedge	Hedge 79.6% EUR
India	0.0452	0.0761	-0.0452	1.0452	Overhedge	Net long 4.5% CAD
China	0.1070	0.0680	-0.1070	1.1070	Overhedge	Net long 10.7% CAD
Brazil	0.3604***	0.1252	-0.3604	1.3604	Overhedge	Net long 36% CAD
Panel B: 2007-2011 Global financial crisis						
US	0.4441***	0.1137	-0.4441	1.4441	Overhedge	Net long 44.6% CAD
Germany	0.0839	0.1132	-0.0839	1.0839	Overhedge	Net long 8.4% CAD
Australia	-0.4193***	0.0991	0.4193	0.5807	Partial hedge	Hedge 58.1% AUD
Japan	0.0936***	0.0231	-0.0936	1.0936	Overhedge	Net long 9.4% CAD
UK	0.0988	0.1704	-0.0988	1.0988	Overhedge	Net long 9.9% CAD
EU	-0.0006	0.0941	0.0006	0.9994	Full hedge	No exposure to EUR
India	0.7584*	0.3942	-0.7584	1.7584	Overhedge	Net long 75% CAD
China	0.4711***	0.1004	-0.4711	1.4711	Overhedge	Net long 47% CAD
Brazil	0.0378	0.2255	-0.0378	1.0378	Overhedge	Net long 37% CAD
Panel C: 1975-1983						
US	-1.0183***	0.3887	1.0183	-0.0183	No hedge	Full exposure to USD
Germany	0.3938***	0.0959	-0.3938	1.3938	Overhedge	Net long 39.4% CAD
Australia	0.1133	0.1409	-0.1133	1.1133	Overhedge	Net long 11.3% CAD
UK	0.2396**	0.1258	-0.2396	1.2396	Overhedge	Net long 24% CAD

Subsample Analysis Multi-Country Portfolios

[Table 9a](#) and [Table 9b](#) display the subsample analysis results for an investor holding an equally-weighted global stock/bond portfolio from both US and Canadian perspectives. The findings are consistent with the single-country subsample analysis conducted earlier. For instance, during the global financial crisis, both US and Canadian stock investors seek net long positions for CNY. Additionally, the other two emerging market currencies are also desired by US and Canadian investors, as they also exhibit significant negative correlation with emerging country currencies returns and their stock market returns.

Table 9a Subsample Analysis. US Perspective Multi-Country Portfolios

Country	Coef	Std. Err	Ψ	θ	Strategy		Coef	Std. Err	Ψ	θ	Strategy
	Stock						Bond				
Panel A: 2011 – 2021Stock Portfolio Post global financial crisis											
Canada	0.6945***	0.2008	-0.6945	1.6945	Overhedge	Canada	0.0148	0.0951	-0.0148	1.0148	Full hedge
Germany	-0.5025**	0.2036	0.5025	0.4975	Partial hedge	Germany	-0.2157***	0.0750	0.2157	0.7843	Partial hedge
Australia	0.3304	0.2308	-0.3304	1.3304	Overhedge	Australia	0.0335	0.0808	-0.0335	1.0335	Overhedge
Japan	-0.4724***	0.1079	0.4724	0.5276	Partial hedge	Japan	0.2339***	0.0489	-0.2339	1.2339	Overhedge
UK	0.2468	0.2077	-0.2468	1.2468	Overhedge	UK	-0.2380***	0.0788	0.2380	0.7620	Partial hedge
India	0.3806**	0.2078	-0.3806	1.3806	Overhedge	India	0.1160	0.1000	-0.1160	1.1160	Overhedge
China	0.0314	0.2494	-0.0314	1.0314	Overhedge	China	0.0990	0.0979	-0.0990	1.0990	Overhedge
Brazil	0.0918	0.1068	-0.0918	1.0918	Overhedge	Brazil	-0.0047	0.0380	0.0047	0.9953	Full hedge
Panel B: 2007-2011 Global financial crisis											
Canada	1.2627***	0.2096	-1.2627	2.2627	Overhedge	Canada	-0.0858	0.1337	0.0858	0.9142	Partial hedge
Germany	-0.2633	0.1637	0.2633	0.7367	Partial hedge	Germany	-0.1065	0.0952	0.1065	0.8935	Partial hedge
Australia	1.1853***	0.1807	-1.1853	2.1853	Overhedge	Australia	0.0141	0.0985	-0.0141	1.0141	Full hedge
Japan	-0.1270	0.1193	0.1270	0.8730	Partial hedge	Japan	0.3485***	0.0703	-0.3485	1.3485	Overhedge
UK	0.2068	0.1248	-0.2068	1.2068	Overhedge	UK	0.0989	0.0881	-0.0989	1.0989	Overhedge
India	-0.4077**	0.1771	0.4077	0.5923	Partial hedge	India	-0.0655	0.1152	0.0655	0.9345	Partial hedge
China	-2.2319***	0.4018	2.2319	-1.2319	Over exposure	China	-0.3416	0.2902	0.3416	0.6584	Partial hedge
Brazil	-0.5496***	0.1374	0.5496	0.4504	Partial hedge	Brazil	-0.0490	0.0877	0.0490	0.9510	Partial hedge
Panel C: 1975-1983											
Canada	0.8285***	0.3862	-0.8285	1.8285	Overhedge	Canada	0.3977	0.3067	-0.3977	1.3977	Overhedge
Australia	-0.3715*	0.2036	0.3715	0.6285	Partial hedge	Germany	0.2453***	0.0895	-0.2453	1.2453	Overhedge
Japan	0.2479	0.1214	-0.2479	1.2479	Overhedge	Australia	-0.1449	0.1362	0.1449	0.8551	Partial hedge
UK	0.2650*	0.1400	-0.2650	1.2650	Overhedge	UK	0.0238	0.1062	-0.0238	1.0238	Full hedge

Table 9b Subsample Analysis. Canada Perspective Multi-Country Portfolios

	Coef	Std. Err	Ψ	θ	Strategy		Coef	Std. Err	Ψ	θ	Strategy
	Stock						Bond				
Panel A: 2011 – 2021Stock Portfolio Post global financial crisis											
US	-0.8006***	0.2300	0.8006	0.1994	Overhedge	US	-0.0388	0.1059	0.0388	0.9612	Full hedge
Germany	-0.5025**	0.2036	0.5025	0.4975	Partial hedge	Germany	-0.2157***	0.0750	0.2157	0.7843	Partial hedge
Australia	0.3304	0.2308	-0.3304	1.3304	Overhedge	Australia	0.0335	0.0808	-0.0335	1.0335	Overhedge
Japan	-0.4724***	0.1079	0.4724	0.5276	Partial hedge	Japan	0.2339***	0.0489	-0.2339	1.2339	Overhedge
UK	0.2468	0.2077	-0.2468	1.2468	Overhedge	UK	-0.2380***	0.0788	0.2380	0.7620	Partial hedge
India	0.3806*	0.2078	-0.3806	1.3806	Overhedge	India	0.1160	0.1000	-0.1160	1.1160	Overhedge
China	0.0314	0.2494	-0.0314	1.0314	Overhedge	China	0.0990	0.0979	-0.0990	1.0990	Overhedge
Brazil	0.0918	0.1068	-0.0918	1.0918	Overhedge	Brazil	-0.0047	0.0380	0.0047	0.9953	Full hedge
Panel B: 2007-2011 Global financial crisis											
US	0.9247***	0.4257	-0.9247	1.9247	Overhedge	US	0.1867	0.3426	-0.1867	1.1867	Partial hedge
Germany	-0.2633	0.1637	0.2633	0.7367	Partial hedge	Germany	-0.1065	0.0952	0.1065	0.8935	Partial hedge
Australia	1.1853***	0.1807	-1.1853	2.1853	Overhedge	Australia	0.0141	0.0985	-0.0141	1.0141	Full hedge
Japan	-0.1270	0.1193	0.1270	0.8730	Partial hedge	Japan	0.3485***	0.0703	-0.3485	1.3485	Overhedge
UK	0.2068	0.1248	-0.2068	1.2068	Overhedge	UK	0.0989	0.0881	-0.0989	1.0989	Overhedge
India	-0.4077**	0.1771	0.4077	0.5923	Partial hedge	India	-0.0655	0.1152	0.0655	0.9345	Partial hedge
China	-2.2319***	0.4018	2.2319	-1.2319	Over exposure	China	-0.3416	0.2902	0.3416	0.6584	Partial hedge
Brazil	-0.5496***	0.1374	0.5496	0.4504	Partial hedge	Brazil	-0.0490	0.0877	0.0490	0.9510	Partial hedge
Panel C: 1975-1983											
US	-0.9698**	0.4643	0.9698	0.0302	Overhedge	Canada	-0.5218*	0.3067	-0.3977	1.3977	Overhedge
Australia	-0.3715*	0.2036	0.3715	0.6285	Partial hedge	Germany	0.2453***	0.0895	-0.2453	1.2453	Overhedge
Japan	0.2479	0.1214	-0.2479	1.2479	Overhedge	Australia	-0.1449	0.1362	0.1449	0.8551	Partial hedge
UK	0.2650*	0.1400	-0.2650	1.2650	Overhedge	UK	0.0238	0.1062	-0.0238	1.0238	Full hedge

5. Conclusion

The risk associated with emerging market investments can be significant, particularly when it comes to currency exposure. In recent years, Canadian pension funds have increased their allocation to emerging markets, and as a result, Canadian asset managers may need to consider hedging their currency risk. However, there is a lack of research on the hedging cost for emerging market currencies. This study aims to fill this gap by investigating the optimal hedging policies and currency portfolio allocation for institutional investors based in the US and Canada, with a specific focus on emerging country currencies.

The study found that optimal currency hedging strategies can lead to a significant reduction in the volatility of foreign long-term bond investments, particularly in developed markets, for both USD and CAD-based investors. However, when it comes to emerging market stocks and bonds, employing a currency hedge can potentially reduce the Sharpe ratio of the returns.

Currency hedging cost is an important consideration, and the study suggests that using currency forward contracts can achieve similar hedging results compared to borrowing and lending in foreign currencies with less transaction costs. However, there is a significant cost difference between the two methods when it comes to hedging emerging market currencies, and investors should weigh potential benefits and costs before making investment decisions.

The study also discusses the impact of hedging on multi-country portfolios, highlighting the benefits of diversification. Optimal currency exposures are presented for investors considering multiple currencies, with USD-based investors recommended to overhedge their exposure to CAD, AUD, INR, and BRL. Multi-country portfolios demonstrate lower volatility and hedging costs compared to most single-country portfolios, but the benefits of currency hedging may vary depending on the type of investment, currency, and investor's base currency.

Overall, this study provides valuable insights into the performance of various hedging strategies and the potential hedging cost associated with foreign exchange markets, particularly in emerging market currencies. The findings can be useful for institutional investors looking to optimize their currency portfolio allocation and hedging policies.

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Table A1 Time Horizon of Sample Data Series

The table displays the sample period of each data series for all countries. The full sample period for all series is from 1975M1 to 2021M10, with each series ending at 2021M10. Among the four countries, namely the US, Canada, Australia, and the UK, all data series have the complete sample period. However, the remaining countries have some series with shorter sample periods.

Country	FX	CPI	90-days T-bill	10-year yield	MSCI
US	NA	Full Sample	Full Sample	Full Sample	Full Sample
Canada	Full Sample	Full Sample	Full Sample	Full Sample	Full Sample
Germany	Full Sample ^a	Full Sample	Full Sample	Full Sample	Start: 1995M1
Australia	Full Sample	Full Sample ^b	Full Sample	Full Sample	Full Sample
Japan	Full Sample	Full Sample	Full Sample ^c	Start:1987M11	Full Sample
UK	Full Sample	Full Sample	Full Sample ^d	Full Sample	Full Sample
Eurozone	Start: 1994M1	Start: 1996M1	Start: 1994M1	Full Sample	Start: 1996M12 ^f
India	Full Sample	Full Sample	Full Sample	Start:1998M12	Start:1992M12
China	Start: 1976M1	Start: 1993M1	Start:1997M6 ^e	Start: 2002M7	Start:1992M12
Brazil	Start: 1994M7	Start: 1979M12	Start: 1995M1	Start: 2007M1	Start: 1992M1

- Following the introduction of the Euro after the Maastricht Treaty in 1999, the exchange rate differentials between Germany and other countries in the Eurozone disappeared.
- The Australian CPI data obtained from the FRED website is based on a quarterly frequency. In order to align with the frequency of other series, we restructured the data into a monthly format.
- The Japanese 90-day T-bills data is only available up to 2002M3 from the FRED website. To fill in the gap, we replaced the missing data with the corresponding OECD interest rate data from 2002M4 onward.
- The UK's 90-day T-bills data is available only up to 1985M12 from the FRED website. To compensate for the missing data, we utilized the corresponding OECD interest rate data from 1986M1 onwards.
- We obtained the China 90-day T-bills data from Russell, covering the period from 2015M5 onwards.
- We utilized the MSCI Europe Index to represent the performance of large and mid-cap equities across 15 developed European countries.