

The Other Longevity Risk: Impact of Population Aging on Pension Plan Finances in Canada

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1 Introduction

This paper will focus on what we consider to be “the other longevity risk”, namely the influence of population aging on portfolio returns and the related effect on pension plan finances.

Many businesses incorporate demographic analysis into their long-term strategic plans. Similarly, many sophisticated and long-term investors use demographic analysis in their assessment of stock prices. Generally, these analyses are based on individual companies.

The broad market view from a macro perspective is built up from these company level observations. However, future population structures are not known with certainty, and can potentially be quite different from expectations. This represents a financial risk that a pension plan should consider. In this research, we present a method to incorporate the impact of demographic change on asset values, in order to illustrate the risk this presents for the financing of pension plans.

The effect of population aging on investment returns is the topic of a host of academic literature. Notable papers in this literature include seminal work by Bakshi and Chen (1994) regarding equities and bonds, and Mankiw and Weil (1989) regarding housing¹.

Generally these papers examine a single market only. In contrast, this paper examines the effect of demographic change on a portfolio of bonds and equities, and illustrates the impact on pension plan financial status.

¹A complete review of this literature can be found at this link: <https://www.soa.org/Files/resources/research-report/2018/lit-review-popl-aging-asset-values-impact-pension.pdf>. An overview of this literature was the topic of a prior paper for the National Pension Hub.

As part of this research, our team has built an Economic Scenario Generator (ESG) that reflects the impact of future demographic changes. The demographic variable that we have chosen is the inverse of an old age dependency ratio. In Canada, this variable is 3.7 as of 2012. Over the next 50 years, this variable will decrease. We project that it has a 25% chance of exceeding 2.6 at the end of 50 years, and a 25% chance of being less than 0.4. As this demographic variable moves from 2.6 to 0.4 (i.e., an older population), expected annual equity returns are lower by over 0.75% and expected annual long bond returns are lower by over 0.25%. This has a moderate effect on the financial status of pension plans in the centre of the distribution of results, but quite a stark impact in the tails of the distribution of results.

We start with the plan provisions and membership data of the Ontario Teachers' Pension Plan (OTPP)². To make the results more generalizable to plans in Canada, we present results both with and without inflation protection after termination or retirement.

The base case projections (using the median path of the demographic variable) show that the spread of financial results for the OTPP is very large. In 10% of the economic scenarios, plan assets would need to be increased by 29% or more in order to provide all of the promised benefits. On the other hand, the median result shows that we could remove over a third of plan assets and still be able to pay all of the benefits.

The risk measure that we analyze is based on the run-off of liabilities for current plan members. This is a very long term measure. The results show that shifting more of the asset allocation of the plan into long bonds has an adverse effect on the financial status of the plan, both for the OTPP which

²We are grateful to the OTPP for sharing information with our research team.

is fully indexed and for a plan that does not grant inflationary increases after termination of employment or retirement. This is a much different finding from most asset allocation analyses that typically focus on a much shorter time horizon. The reason for this is that the reduction in expected return by moving part of the asset allocation from equities to bonds has a much longer time period over which to operate.

The rest of this paper is structured as follows. Section 2 describes how we have modified a standard ESG to incorporate the impact of future demographic change. Section 3 describes the approach that we have taken to measuring the financial risk of a pension plan and shows the impact of different future demographic paths on that risk. Section 4 concludes. Additional information about the ESG, the pension plan’s details, and the mortality model are provided in the Appendices.

2 Economic Scenario Generator

Our data ranges from 1950 to 2012. The data used for price inflation, salary inflation, dividend yield, dividend growth and long term bond yield are described in Appendix A. For the demographic factor³⁴, we use the ratio of the number of individuals age 20-64 to the number of individuals age 65-85, defined as follows:

$$\text{Demographic ratio} = \frac{\text{Individuals age 20-64}}{\text{Individuals age 65-85}}.$$

Figure 1 shows the evolution of this demographic ratio over the period 1950 to 2012.

³These data come from the census data available on the Statistics Canada web site.

⁴We have also analyzed data starting in 1935. Using this longer data to fit the ESG, we find that there is no significant relationship between the demographic factor and investment returns. We suspect that the distortion due to additional deaths during WWII is the cause of this.

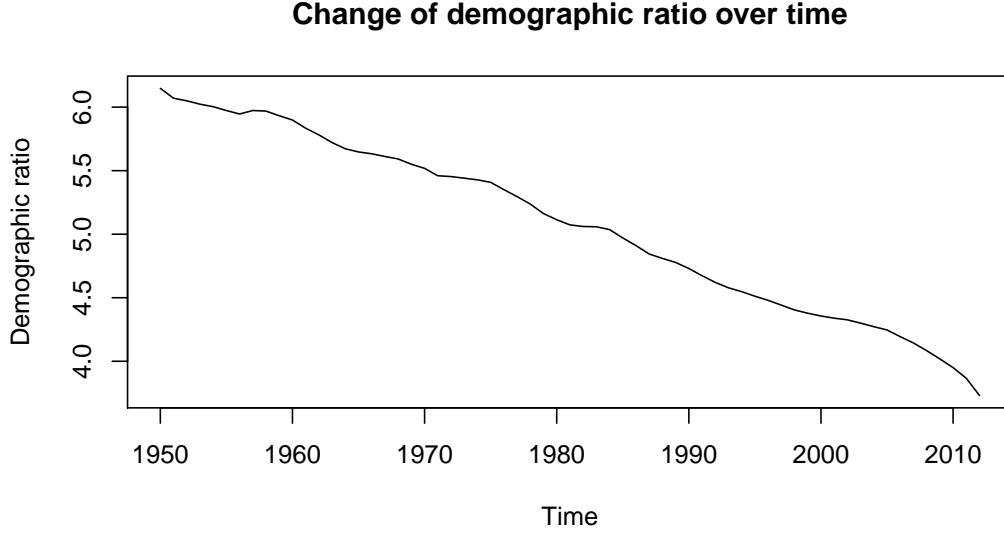


Figure 1: Change of Demographic Ratio

The year-over-year changes in the variables are modelled using a graphical approach. The methodology is described in detail in Appendix A. A graphical model is a dimension reduction tool so that not all pairs of changes need to be modelled. We can ignore those pairs that are conditionally independent, given the other variables. Figure 2 shows the structure of the model when fit to Canadian data⁵.

Introducing the demographic factor into the ESG results in a variation of both returns and yields for bonds and equities, but does not change the projection of the other variables in any material way. As of 2012, the value of the demographic factor is around 3.7. Over the next 50 years, our projection

⁵Using a different breakpoint of 54/55 does not affect the structure of the investment portion of the ESG.

Graphical Model for Canada

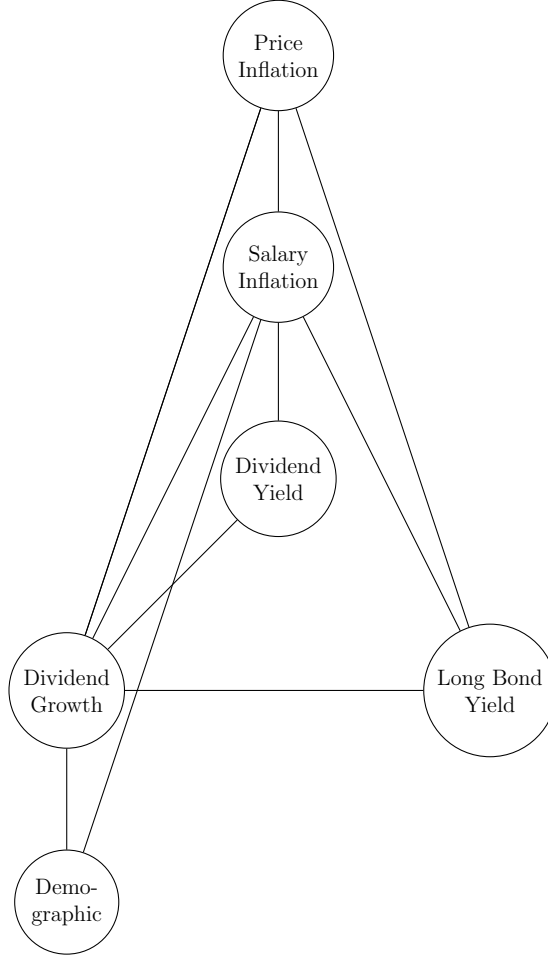


Figure 2: Optimal graphical model for Canada.

of that factor, based on the time series fitted to historic data, is generally expected to decrease. At the 75th, 50th, and 25th percentiles of its distribution, the demographic factor reduces to 2.6, 1.3, and 0.4, respectively⁶. Table 1 shows the impact on returns when the future path of the demographic factor

⁶Of note, the “medium fertility” projection prepared by Statistics Canada from the most recent census generates a value of roughly 2.0 in 50 years time.

moves from the 75th percentile to the 25th percentile.

Table 1: Mean and standard deviation of simulations for different demographic paths using a 50-year projection.

Demographic Factor		Equity Return	Bond Return
75 th percentile	μ	11.32	5.07
(2.64)	σ	17.3	6.8
50 th percentile	μ	10.78	4.90
(1.31)	σ	17.3	6.8
25 th percentile	μ	10.55	4.77
(0.39)	σ	17.4	6.8

3 Results

3.1 Economic Capital Approach

Given the long-term nature of pension risk, we examine the “run-off” liabilities of a plan. These reflect the current plan membership and continued accrual of benefits, but no new entrants to the plan. As such, the time horizon for this analysis is the time until the last of the current plan members dies, perhaps 90 years or so. We examine the present value of the future surplus or deficit of the plan, where the present value is determined based on the future returns on the pension fund. Mathematically, this is expressed as follows:

$$V_0 = A_0 - \sum_{t=0}^T X_t D_{0,t}$$

In this equation, V_0 is the present value of the future surplus or deficit; A_0 is the current value of the assets of the pension fund; X_t represents each year’s net cash flow (i.e., benefit payments less contributions); and $D_{0,t}$ is the return on the pension fund assets over the period from today to time t .

Intuitively, V_0 represents the amount of money that could be taken out of (or deposited into) the pension fund so that the last dollar in the fund is used up when the last beneficiary of the plan receives the last payment.

We run 10,000 simulations⁷ of the pension plan’s finances based on the ESG described in Section 2 in order to look at the distribution of V_0 . So that this value does not depend on the size of the plan (or the currency in an international comparison), we express V_0 as a percentage of the starting assets, A_0 .

⁷In each of these simulations, the contribution rate and the asset allocation are kept constant over time.

In line with the measures under Solvency II for financial institutions, we look at the 0.5th percentile of the distribution of the run-off deficit/surplus. Conceptually, this is the proportion by which the current assets would need to be increased (decreased) so that we have a 199 in 200 chance of having sufficient assets to pay all of the promised benefits. This is a stringent requirement, so we also look at the 10th and 50th percentiles of the distribution of V_0 .

3.2 Base Case Results

As mentioned in the Introduction, we start with the plan provisions and membership data of the OTPP. Further detail is provided in Appendix B. In addition to the results from the ESG, we also need to consider demographic assumptions – termination, retirement, and death. For termination and retirement, we use the assumptions from the OTPP valuation as at January 1, 2018. We use a stochastic mortality assumption where median mortality is scaled to the assumption used by the OTPP. Further detail on the mortality model is provided in Appendix C.

The asset allocation of the OTPP is quite complex. From our ESG, we only have available Canadian equities and long Canada bonds as asset classes. For the base case, we model the plan’s asset allocation⁸ as being 55% in Canadian equities and 45% in long Canada bonds, as an approximation. Also, for the base case we use the 50th percentile path of our demographic factor. A summary of the base case results is provided in Table 2. The full distribution of V_0 is shown in Figure 3.

In Table 2 the two columns show VaR and ES . VaR is value at risk, which is the required amount of economic capital at various percentiles. So, at the median, there is an excess amount of assets of 39%. At the 0.5th

⁸For purposes of modelling, we rebalance the asset allocation to its target each year.

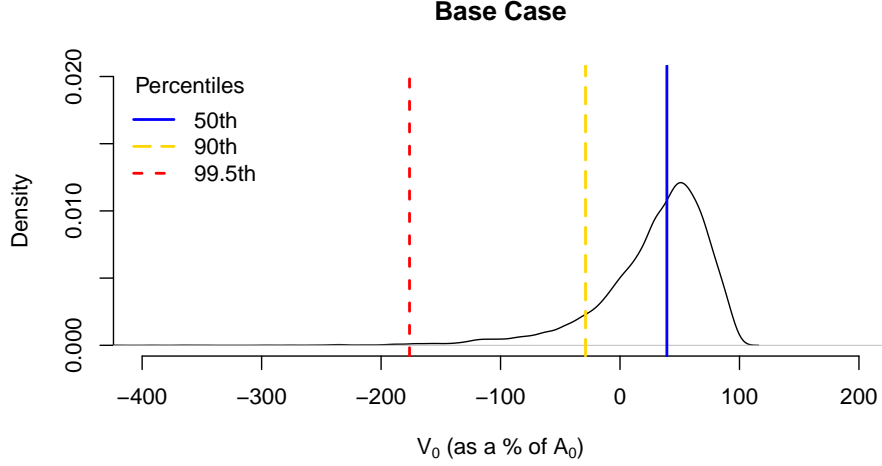
percentile, the assets would need to be augmented by an additional 176% in order to provide for all of the promised benefits. ES is the expected shortfall, which is the average amount of deficit at the various percentiles. For example, the 10th percentile represents all of the scenarios where the shortfall is 29% of starting assets or more. On average, these scenarios have a shortfall of 79% of starting assets.

Table 2: Economic capital for Base Case (as a percentage of A_0)

Percentile	VaR	ES
50	39	-5
10	-29	-79
0.5	-176	-276

Figure 3 shows the full range of outcomes with the 50th, 10th, and 0.5th percentiles shown by a vertical blue, yellow, and red line, respectively. As can be seen, the distribution of results is quite skewed to the left. This is a characteristic of our construction. In the very best news scenario (on the extreme right side of the distribution) we can “withdraw” 100% of the current assets and still provide for the promised benefits, but we cannot go beyond 100%. In other words, future net cash flows and future investment returns on those cash flows would be sufficient to provide for all of the future benefits. On the other hand, in the bad news scenarios, we may need to supplement the assets by well over 100% of their current level.

Figure 3: Base Case distribution (as a percentage of A_0)



3.3 Sensitivity to Future Demographic Path

In the previous section, we examined the financial status of the plan based on the median path of the demographic factor. However, there is a risk that this path does not come to pass. In this section, we examine the impact of this risk on the economic capital of the plan. Specifically, we compare the results at the 75th, 50th, and 25th percentiles of the distribution of the demographic path. Table 3 shows the difference in VaR and ES among the three demographic paths.

Table 3: Economic capital (as a percentage of A_0)

Demographic Percentile	75th		50th		25th	
Percentile	VaR	ES	VaR	ES	VaR	ES
50	42	3	39	-5	36	-12
10	-18	-64	-29	-79	-39	-94
0.5	-151	-237	-176	-276	-203	-313

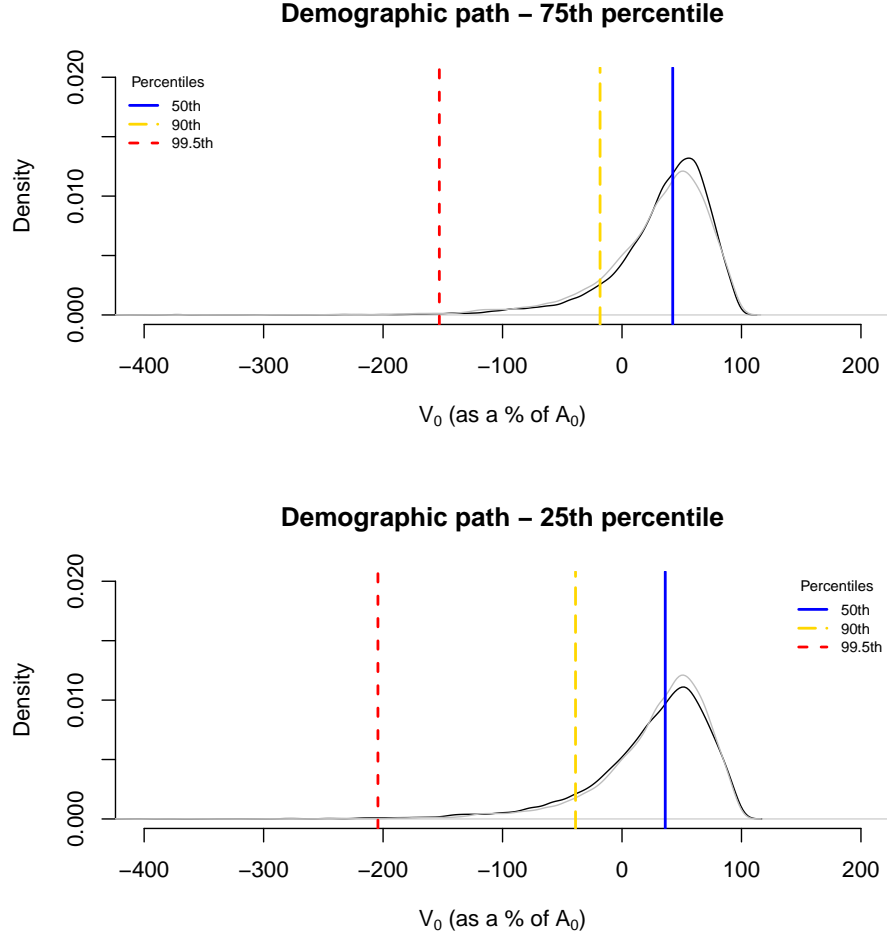
As can be seen from Table 3, the impact of different demographic paths on economic capital requirements is moderate in the centre of the distribu-

tion, but is not trivial at the 10th percentile and is quite stark at the 0.5th percentile. The impact on the full distribution of results may be seen in Figure 4. The grey lines in the two graphs show the distribution based on the 50th percentile demographic path.

Of note, we have not changed the mortality assumption for plan members. Implicitly, we are assuming that the mortality experience of the rest of the population improves toward that of teachers⁹. On the other hand, if the mortality experience of teachers improves in line with the rest of the population, the spread of results across the three demographic paths would widen, as the benefit payments would be made for a longer period of time under the 25th percentile demographic path (and vice versa, for a shorter period of time under the 75th percentile demographic path).

⁹The mortality experience of teachers is significantly better than that of the general population.

Figure 4: Comparison of Demographic Paths



3.4 Sensitivity to Asset Allocation

In this section, we consider two cases: we consider the case where the asset allocation is changed from 55% equities to 75% equities and the case where asset allocation is changed from 55% equities to 25% equities¹⁰. For both cases, we set the demographic path to the 25th percentile and describe that

¹⁰The typical asset allocation for a Canadian plan would lie in the range of 60% to 40% in equities. The range we show is meant to bracket this range.

as “faster aging” in the Figures. Table 4 shows the change in VaR and ES between the various asset allocations.

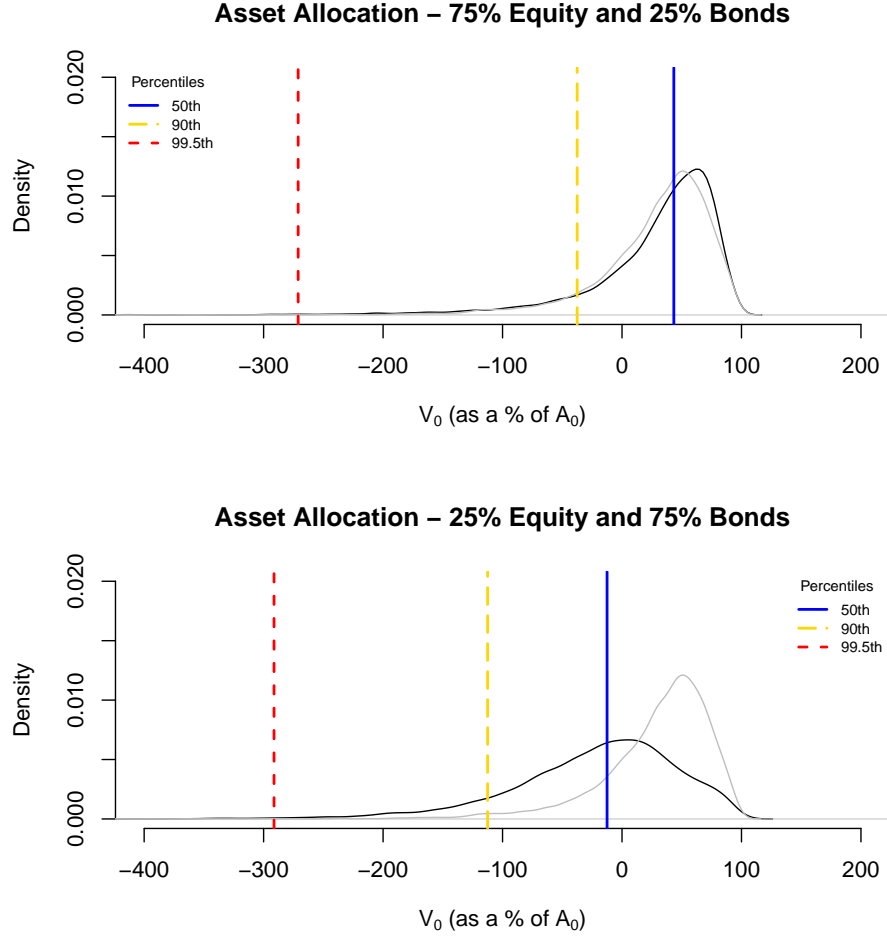
Table 4: Economic capital (as a percentage of A_0)

Equity Allocation	75%		55%		25%	
Percentile	VaR	ES	VaR	ES	VaR	ES
50	43	-11	36	-12	-13	-76
10	-38	-113	-39	-94	-112	-172
0.5	-266	-448	-203	-313	-291	-378

This table shows that there is only a moderate impact to increase the equity allocation to 75%, but a significant impact to reduce it to 25%. There are two factors here that reduce the attractiveness of increasing the allocation to long Canada bonds. One is that the liabilities of the plan are “real” (i.e., inflation protected), so there is no matching benefit to increasing the allocation to long nominal bonds. The other factor is that we are looking at a very long time horizon. With a larger bond allocation, the reduction in expected return over such a long period significantly worsens all of the downside risk measures.

The impact on the full distribution of results is seen in Figure 5. In the lower panel of Figure 5 we can clearly see the shift to the left in the distribution of V_0 relative to the 55% equity allocation shown by the grey line.

Figure 5: Comparison of Equity Allocation under Faster Aging



3.5 Sensitivity to Inflation Indexing

In this section, we examine the impact of removing inflation indexation on the benefits (both immediate and deferred). We also reduce the contribution rate to 14%, which reflects the smaller value of the benefits, though we do not change the starting asset value. As in the previous section, we set the demographic path to the 25th percentile. Table 5 shows the change in VaR

and ES between the various asset allocations.

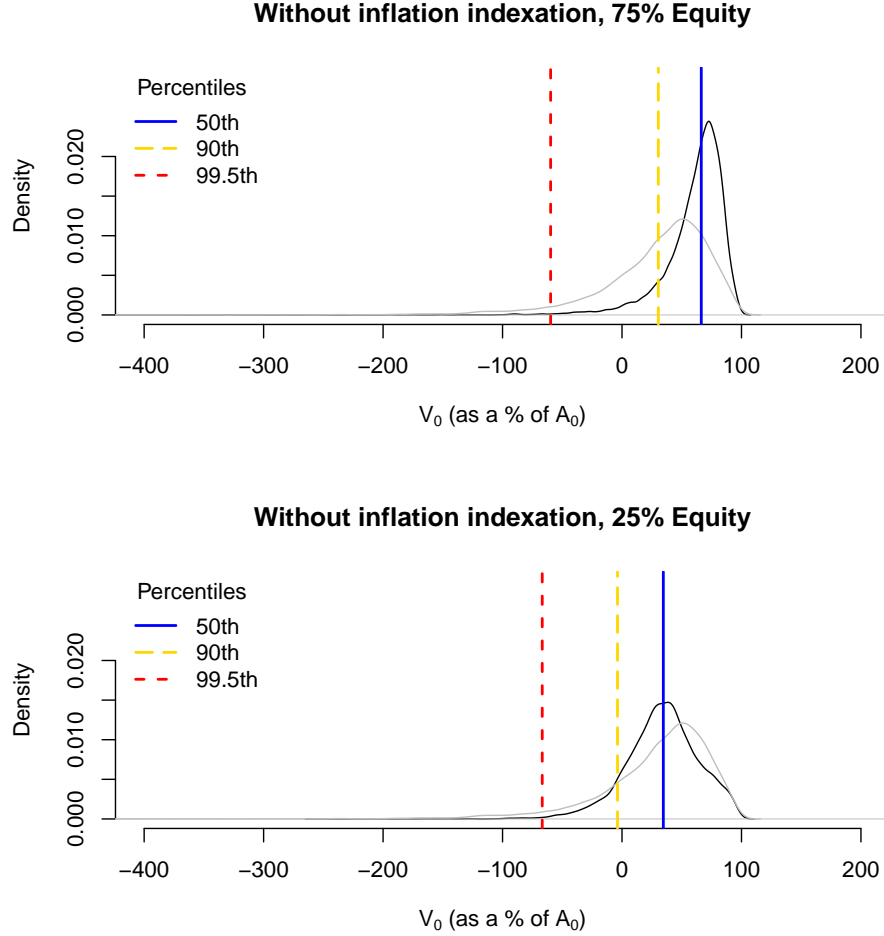
Table 5: Economic capital (as a percentage of A_0)

Equity Allocation	75%		55%		25%	
Percentile	VaR	ES	VaR	ES	VaR	ES
50	66	43	62	42	35	10
10	30	2	31	10	-4	-26
0.5	-58	-116	-31	-64	-66	-99

The relative results are similar to the case where the benefits are “indexed” – little impact of moving from 55% equities to 75% equities, but a material impact to reduce to 25% equities. In this case, where a significant proportion of the benefits are nominal, there is a worsening of the financial position of the plan by increasing the “matching” characteristics of the asset allocation. Unlike the previous section, the sole reason for this is the significant reduction in expected return when looking at a very long time horizon.

The full distribution of outcomes is shown in Figure 6. The description is similar to that in the previous section. Visually, the changes do not look quite so large because the spread of the distribution is smaller, but the horizontal scale on the graphs is kept the same to facilitate visual comparison with Figure 5.

Figure 6: Comparison of Equity Allocation under Faster Aging



4 Conclusion

In broad summary, as the demographic variable we have chosen (the inverse of an old age dependency ratio) moves from its 75th percentile to its 25th percentile value (i.e., an older population), expected annual equity returns are lower by over .75% and expected annual long bond returns are lower by over 0.25%. This has a moderate effect on the financial status of pension

plans in the centre of the distribution of results, but quite a stark impact in the tails of the distribution of results.

We start with the plan provisions and membership data of the Ontario Teachers' Pension Plan (OTPP). To make the results more generalizable to plans in Canada, we present results both with and without inflation protection after termination or retirement.

The base case projections (using the median path of the demographic variable) show that the spread of financial results for the OTPP is very large. In 10% of the economic scenarios, plan assets would need to be augmented by 29% or more in order to provide all of the promised benefits. On the other hand, the median results shows that we could remove over a third of plan assets and still be able to pay all of the benefits.

The risk measure that we analyze is based on the run-off of liabilities for current plan members. This is a very long term measure. The results show that shifting the asset allocation of the plan into long bonds has an adverse effect on the financial status of the plan, both for the OTPP, which is fully indexed, and for a plan that does not grant inflationary increases after termination of employment or retirement. This is a much different finding from most asset allocation analyses that typically focus on a much shorter time horizon. The reason for this is that the reduction in expected return by moving part of the asset allocation from equities to bonds has a much longer time period over which to operate.

In conclusion, the impact of population aging represents at least a moderate risk to the finances of pension plans due to its impact on investment returns. This is a risk that plan managers should be considering.

A Graphical Model

A.1 Introduction

This Appendix provides an overview of an Economic Scenario Generator (ESG) for Canada using a graphical modelling approach developed by Oberoi et al. (2019). The interested reader can refer to that paper for full details of this approach. To build our ESG, we require Canadian data for consumer price inflation (I), salary inflation (J), dividend yield (Y), dividend growth (K) and long term bond yield (C) over a long time horizon. In Section A.2, we provide an overview of the data we use. In Section A.3, we show the parameter estimates and the graphical structures using simultaneous p-values. Finally, in Section A.4, we show simulations from the graphical structures.

A.2 Data

As mentioned in the body of this paper, the demographic variables come from the Canadian census provided by Statistics Canada. For price inflation and salary inflation, we use data from two sources. Data between 1950 and 2000 comes from Emmanuel Saez who provides data for retail price index and average wages. From 2001 onwards, we use inflation data and salary inflation data from the Federal Reserve Economic Data (FRED) database. For dividend yield, dividend growth and long term bond yield, we use data from Statistics Canada which provide data for the Toronto Stock Exchange index, Toronto Stock Exchange dividend yield and 10-year government bond yield for the whole period.

A.3 Modelling

A graph, $G = (V, E)$, is a structure consisting of a finite set of variables V (or vertices or nodes) and a finite set of edges E between these variables. The existence of an edge between two variables represents a connection or some

form of dependence. The absence of this connection represents conditional independence.

For instance, if we have a set of three variables, $V = (A, B, C)$, where A is connected to B and not to C , but B is connected to C , then A is connected to C via B . A is then conditionally independent of C , given B . Such a structure can be graphically represented by drawing circles or solid dots representing variables with lines between them representing edges. The graphs we consider here are called undirected graphs because the edges do not have a direction (which would otherwise be represented by an arrow). Such graphs model association rather than causation. The graphical model described here with three variables, A , B and C , is shown in Figure 7.

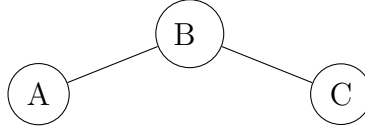


Figure 7: Example of a Graphical Model

Graphical models enable us to represent the covariance structure with dimension reduction, by effectively capturing conditional independence between pairs of variables and shrinking the relevant bivariate links to zero while allowing for weak correlations to exist in the simulated data.

The aim of a graphical model ESG is to give importance to long-run stable relationships and to generate a distribution of joint scenarios. This takes the approach of estimating the joint distribution of the residuals of individual time series regressions and focuses on the dependence between the residuals. For each variable other than the demographic variable, a time series model is fitted independently, and then a graphical model is fitted to the time series

residuals across variables. For each time series, X_t , the following AR(1) time series model formulation is used.

$$\begin{aligned}\mu_x &= E[X_t] \\ Z_t &= X_t - \mu_x \\ Z_t &= \beta Z_{t-1} + e_t, \text{ where } e_t \sim N(0, \sigma^2)\end{aligned}$$

From Figure 1, we note the downward trend in the population demographic data¹¹. Given that the data is not mean-reverting, we apply a first order differencing to the data and then model the differenced series as an AR(1). In other words, we model the demographic ratio as an ARIMA(1,1,0).

$$\begin{aligned}\mu_x &= E[X_t] \\ Z_t &= X_t - \mu_x \\ Z_t &= Z_{t-1} + \beta_x (Z_{t-1} - Z_{t-2}) + e_{x,t} \quad \text{where } e_{x,t} \sim N(0, \sigma_x^2)\end{aligned}$$

The parameter estimates from the ARIMA(1,1,0) and AR(1) regressions are provided in Table 6.

Table 6: Time series parameter estimates Canada.

	μ	β	σ
I_t	0.0372	0.7836	0.0192
J_t	0.0554	0.6091	0.0326
Y_t	0.0328	0.9157	0.0050
K_t	0.0808	0.1310	0.1767
C_t	0.0687	0.9669	0.0084
D_t	5.0735	0.9524	0.0207

The resulting partial correlation matrix is given in Table 7. Clearly, some of the partial correlations in the matrix are small. Our goal is to identify the

¹¹The projections prepared by Statistics Canada from the most recent census follow the same general time trend as our projections. The “high fertility”, “medium fertility”, and “low fertility” projections lie between the 50th and 75th percentiles of our distribution.

graph(s), with the minimum number of edges, which describe the underlying data adequately. From Table 7, we note a positive correlation between population demographic and the other five variables. The correlation between population demographic and dividend growth is relatively high compared to the remaining four variables.

Table 7: Partial correlations of residuals

	I_t	J_t	Y_t	K_t	C_t	D_t
I_t	1					
J_t	0.71	1				
Y_t	0.14	-0.20	1			
K_t	-0.24	0.42	0.27	1		
C_t	0.38	-0.34	0.15	0.41	1	
D_t	0.13	-0.25	0.02	0.35	-0.14	1

Selecting only those pairs of correlations where the p-values are 0.6 or less results in the selected model shown in Figure 2.

A.4 Simulations

Figure 8 shows the fanplots from 10,000 simulations for each of the six variables. For the case of price inflation, salary inflation, dividend yield, dividend growth and bond yield, we note the projections tend towards their long-term mean. This is expected given that these variables are modelled as an AR(1) process calibrated to historic averages and standard deviations. For the demographic factor, we see a downward trend in the projections. This is consistent with the both historic data and future projections prepared by Statistics Canada. The demographic factor as at 2012 (our last data point) is around 3.7. Based on our central projection, this is expected to fall to around 1.3 by 2060. As discussed before, we use an ARIMA(1,1,0) to model the demographic factor. When projecting the demographic factor forward, we add

a restriction such that the demographic ratio does not fall below zero. In the projections prepared by Statistics Canada, their “low”, “medium”, and “high” fertility projections end up with factors in 2060 of 1.84, 2.00, and 2.10, respectively. These all lie in the 3rd quartile of our projection results.

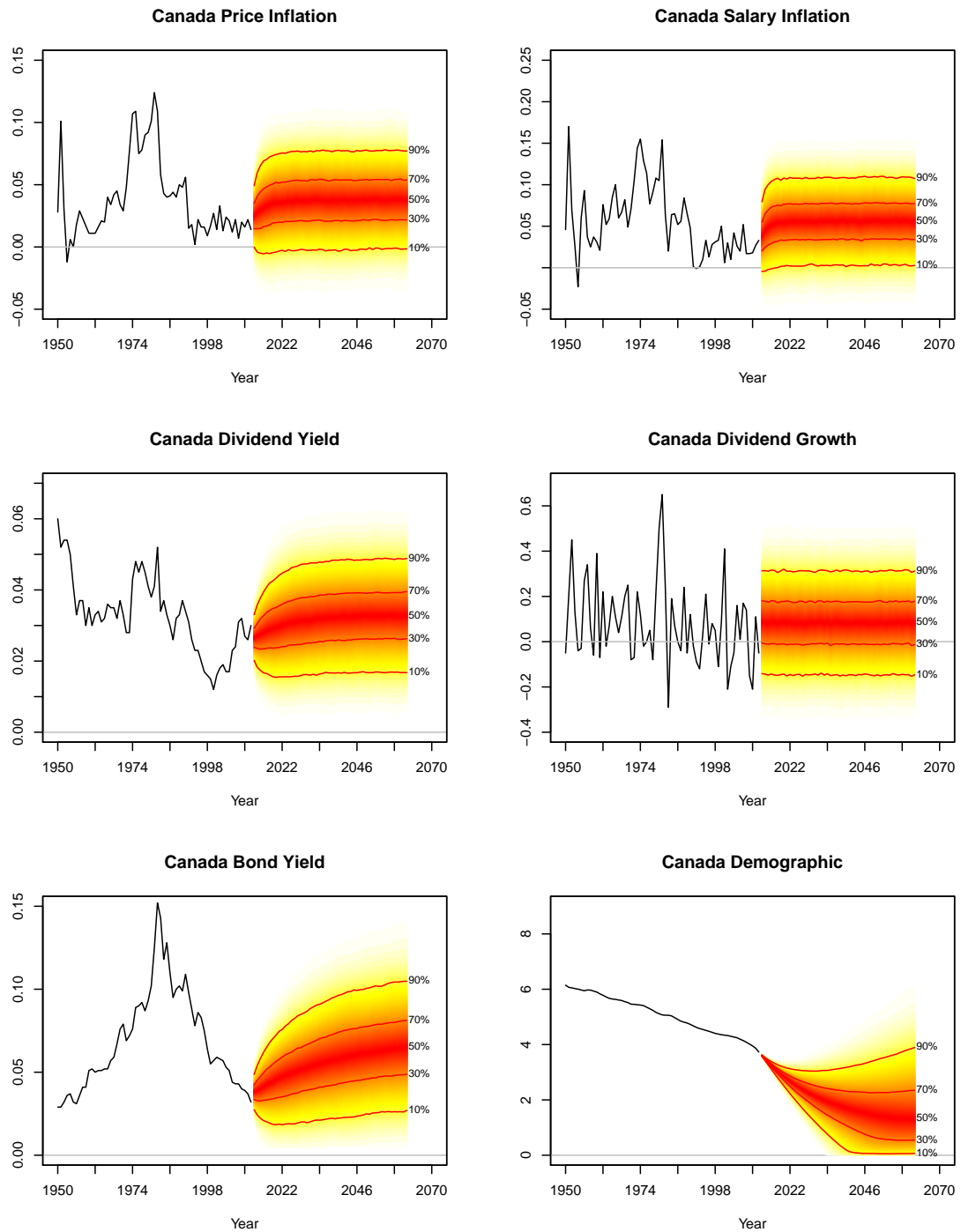


Figure 8: Fanplots of simulations.

B OTPP Description

B.1 Introduction

In this Appendix, we describe the membership profile, benefit structure, and plan assets of the Ontario Teachers' Pension Plan (OTPP), which we use as the base case in presenting our results. The OTPP is a large open defined benefit pension plan operating in Canada with over 300,000 plan members. The numbers presented here are based on the latest available valuation carried out for the plan as at January 1, 2018.

B.2 Membership profile

Table 8: Membership profile

Active	Number	144,325
	Average pensionable earnings	\$90,468
	Average age	44.4
	Average past service	14.6
Deferred Members	Number	71,205
	Average deferred pension	\$1,965
	Average age	45.1
Pensioners	Number	129,785
	Average lifetime pension	\$41,154
	Average age	71.1

Table 8 shows the membership profile as presented in the 2018 valuation report. As can be seen from the table, only a single average age is provided for the active members, which is not sufficient to capture the overall risk characteristics of the plan. We need a range of model points to capture the inter-generational risk dynamics. The valuation report also provides information on the proportion of active members in different age bands, based on

which, we use an age distribution of active members shown in Table 9.

Table 9 also shows the past service and salary assumptions for active members for each model point. These have been set so that the average past service and average salary of active members broadly match the figures from Table 8.

Table 9: Model points, past service and earnings of active OTPP members

Age	Proportion	Number	Past service	Earnings
30	15%	21,649	5	\$75,000
40	35%	50,514	12	\$85,000
50	35%	50,514	17	\$95,000
60	15%	21,649	25	\$105,000
Total	100%	144,326		
Average			14.7	\$90,000

For deferred members and pensioners, we use single model points to represent each of these membership categories. We also assume a 50:50 gender split and no salary differential between genders.

B.3 Benefit structure

B.3.1 Pension benefits

The annual pension is equal to:

- 2% of the member's highest 5-year average salary (i.e., the average salary in the 5 (not necessarily consecutive) school years with the greatest annualized pensionable salary) multiplied by the number of years of credited service

LESS

- 0.45% of the lesser of:
 - the member’s highest 5-year average salary, or
 - the average of the maximum pensionable earnings under the Canada Pension Plan in the year of cessation of employment and the 4 preceding years.

For simplicity, we assume that the annual pension is 1.7% of the member’s final salary multiplied by the number of years of credited service.

All pensions are assumed to increase in line with CPI¹². Members’ salaries increase in line with projected wage increases plus experience-related salary increases. The OTPP valuation allows for these experience-related salary increases (shown in Table 10).

Table 10: Experience-related increases assumptions for OTPP members.

Years of credited service	Experience-related increase
1	7.00%
2	6.60%
3	6.10%
4	5.70%
5	5.30%
6	5.30%
7	4.30%
8	3.90%
9	3.00%
10	2.00%
11	1.10%
12-35	0.40%
36+	0.00%

¹²Pension increases are not fully guaranteed but we have assumed that they will be granted in any event.

B.3.2 Withdrawal benefits

Members who withdraw from the plan are entitled to a deferred pension. The deferred pension is fully indexed between the time of withdrawal and the time it becomes payable. Table 11 shows the withdrawal assumptions for the OTPP.

Table 11: Withdrawal assumptions for OTPP.

Age	< 5 years of service		5-10 years of service		10+ years of service	
	Males	Females	Males	Females	Males	Females
20-29	4.5%	4.5%	1.0%	1.5%	0.5%	0.5%
30-39	4.0%	6.0%	1.0%	1.5%	0.5%	0.5%
40-49	5.5%	5.5%	1.0%	1.5%	0.5%	0.5%
50+	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

B.3.3 Death benefits

When a member dies prior to receiving a pension, a benefit equal to the value of the deferred pension entitlement with respect to post-1986 service is payable. In addition, a refund equal to the excess of the member's contributions (made after 1986) plus interest over one-half of the commuted value of the post-1986 pension is payable.

On the death of a pensioner, a spouse's pension of half the amount of member's pension is payable. It is assumed in the valuation report that 85% of male members and 75% of female are married.

B.3.4 Contributions

Both employers and employees contribute to the OTPP. Each party contributes 10.4% of pensionable earnings up to the Year's Maximum Pensionable Earnings (YMPE) and 12.0% of pensionable earnings in excess of the

YMPE. For 2018, the YMPE is \$55,900. The YMPE is projected to increase in line with wage inflation.

B.4 Plan Assets

As at January 1, 2018, plan assets amounted to \$227.5 Bn. The OTPP invests approximately 54% in real assets and 46% in fixed assets. Table 12 shows the asset allocation of the OTPP as given in the 2017 Accounts and Reports. For purposes of our calculation, we assume an asset allocation of 55% equity and 45% bonds for the base case.

Table 12: OTPP investment mix.

Assets	Allocation (%)
Equities	36
Inflation sensitive assets	14
Real estate and infrastructure	25
Money market instruments	(21)
Total real	54
Fixed interest	33
Credit and absolute return strategies	13
Total fixed	46

C Mortality Model

We need to project future mortality rates forward as well. The OTPP uses the 2014 OTPP Generational Mortality Table and Projection Scale TT (two dimensional tables), which were developed by the University of Waterloo based on a study of the plan's experience to the end of 2013. These tables are deterministic tables and provide a single projection path. To capture the mortality risk, we use Model M7 from Cairns et al. (2009) calibrated to Canadian data from the UN's Human Mortality Database. We then adjust the projected mortality rates such that the central projection from M7 matches the projection from the 2014 OTPP Generational Mortality Table and Projection Scale TT.

Model M7 provides the best overall fit to UK and US mortality data. As such, we use it for Canada as well. The actual model is as follows:

$$\text{logit } q(t, x) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \kappa_t^{(3)}[(x - \bar{x})^2 - \sigma^2] + \gamma_{t-x}^{(4)},$$

where the κ 's and the γ are fit to the data, and $\sigma^2 = n^{-1} \sum (x - \bar{x})^2$.

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