Incentives Behind Clearinghouse Default Waterfalls

Agostino Capponi* W. Allen Cheng† Jay Sethuraman‡

Abstract

We study the incentives provided by the conventional clearinghouse default loss allocation mechanism. Heterogenous members participate in clearing to obtain default protection via loss-mutualization, but incur the cost of financing their default fund contributions. The clearinghouse commits equity to attract members and increase her revenue, but this exposes her to default losses. The equilibrium default fund and equity levels imply an equilibrium level of expected funding shortfall, our proposed measure of systemic risk. We show that both default funds and equity increase with the amount of risk-sharing and decrease with funding cost. Systemic risk decreases with risk-sharing but increases with members' revenues. High risk members' equilibrium surplus is decreasing in their default risks, showing that the current loss allocation mechanism potentially mitigates moral hazard problems. A minimum equity requirement policy can increase default fund contributions, which mitigates systemic risk but decreases economic surplus because of the increased funding costs.

1 Introduction

Financial institutions mutualize counterparty risk by becoming clearing members of a clearinghouse. As the effective central counterparty (CCP) to all members, the clearinghouse's arrangement of various layers of loss-absorbing capital has significant implications for the allocation and management of counterparty risk.

The losses originated from defaults of members within a clearing network are allocated among the surviving members according to a "default waterfall" (Pirrong (2011), ISDA (2013a)). The first line of defense against such losses is the initial margin posted by each member to the clearinghouse, capital used only to absorb losses generated by

*Department of Industrial Engineering and Operations Research, Columbia University, New York, NY 10027, USA, `ac3827@columbia.edu`. Research supported by a grant from the Global Risk Institute.

†Department of Industrial Engineering and Operations Research, Columbia University, New York, NY 10027, USA, `wc2232@columbia.edu`. Research supported by the National Science Foundation Graduate Research Fellowship under Grant No. DGE-11-44155. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

‡Department of Industrial Engineering and Operations Research, Columbia University, New York, NY 10027, USA.`jay@ieor.columbia.edu`.

the defaulting member's portfolio. The role and choice of initial margins within central clearing have been extensively investigated in the literature.[1]

In this paper, we analyze a clearinghouse's incentives behind the determination of the default fund requirement and her equity commitment, resources further down the default waterfall. Default funds are additional contributions, beyond initial margins, made by clearing members and used to absorb losses when their posted margins are insufficient. Different from the initial margin, however, the default fund can be used to absorb losses originating from another member's default, and hence is a bona fide resource of loss-mutualization. The equity commitment of the clearinghouse, also referred to as the clearinghouse's "skin in the game", is a layer of loss-absorbing capital utilized when a defaulting member's default fund contribution is exhausted but before other members' default fund contributions are deployed. The default loss borne by the clearinghouse is typically limited to the amount of equity that she commits. Established to align the interests of the clearinghouse with those of their members,[2] the appropriate level and rules behind a clearinghouse's skin in the game is still an active source of regulatory debate.[3]

In this paper, we assume that the typical default waterfall structure is in place (see Section 2 for a brief review), and construct a model where agents make decisions based on it. In our two-period model economy, there is a continuum of potential clearing members and a profit-maximizing clearinghouse. Each member can be of safe or risky type, and this is private information. At time one, the clearinghouse declares her default fund requirement and equity commitment rule. The default fund requirement is the amount of costly funds a member needs to post with her, should he choose to participate in the clearing process. The equity commitment is the amount of losses the clearinghouse commits to bear, should defaulting members' default funds be exhausted. In response to the clearinghouse's declaration, members individually choose whether or not to participate in clearing. Both the member and the clearinghouse derive income from the member's participation. At time two, members may default, generating losses to the surviving clearing members and the clearinghouse. To model market stress, we assume that default losses are random but perfectly correlated across defaulting members, so that they are not predictable ex-ante. Realized losses are then allocated among clearing members and the clearinghouse according to the default waterfall.

[1] Figlewski (1984) relates initial margin rules to pre-specified quantiles of the loss distribution. Telser (1981) describes the competing incentives behind margin setting due to market riskiness and funding costs. This is quantified and supported by the framework developed by Capponi and Cheng (2016). Hedegaard (2014) documents the empirical relation between futures margins and volatility. Duffie and Zhu (2011) study how central clearing affects collateral usage.

[2] This is analogous to the case where a creditor's interest is aligned with that of the lender's when sufficient creditor capital is at stake (Acharya et al. (2012)).

[3] Skin in the game was one of the major topics discussed during the Global Markets Advisory Committee Meeting held on May 14, 2015 by the U.S. Commodity Futures Trading Commission (CFTC). Clearing members have generally argued for more contributions from clearinghouses to align interests with those of their members. While major clearinghouses generally agree on the proposed incentive effects, they have argued against their skins in the game being a major source of loss absorption (LCH Clearnet (2014a), CME Group (2015)).

Our model is designed to capture the fundamental trade-offs faced by the clearinghouse and her potential clearing members. Large default fund requirements increase the funding costs borne by members and decrease the amount of (counterparty) risk they can offload to the clearing network; on the other hand, they increase protection from other members' defaults and reduce the clearinghouse's exposure to default losses. Large equity commitments increase members' expected profits by providing a further layer of protection, attracting more members to participate and increasing the clearinghouse's revenue; however, they also increase the clearinghouse's potential for losses.

We fully characterize and provide tractable expressions for the prevailing equilibrium default fund and equity levels. These are related to the amount of risk-sharing, default probabilities, clearing revenue, and funding cost. The prevailing equilibrium can be separating (only risky members participate) or pooling (both types participate). Further, the equilibrium equity commitment is positive only when a clearinghouse wants to attract both types of members to participate.

Our results highlight the incentives behind a clearinghouse's choices. For example, our model shows that, without clearinghouse equity or large incomes from clearing, safer members would not participate in clearing as the loss-mutualization mechanism creates a wealth transfer from them to the riskier members. The clearinghouse is thus incentivized to commit positive equity in order to increase clearing revenue. This serves to explain why clearinghouses commit positive equity in the absence of regulatory minimums,[4] a seeming anomaly as they bear no default losses when they do not.

The tractability of our framework allows for succinct comparative statics analyses both on the clearinghouse's and members' equilibrium choices, and on the distribution of economic surplus within the model economy. Consistent with intuition, we find that a higher funding cost generally decreases the equilibrium default fund, and induces a decrease in the equilibrium equity commitment, as the clearinghouse's exposure to default losses increases. Higher risk-sharing has the opposite effect, increasing both the default fund and equity commitment, as safe members demand more guarantees from risky members and the clearinghouse to participate. Higher default rates increase the overall riskiness of the market, and thus the clearinghouse's choice of default fund and equity. Interestingly, our results also show that in non-degenerate equilibria, the surplus that the risky members capture is decreasing in the amount of risk-sharing. In other words, the clearinghouse's optimal mix of default funds and equity resources can incentivize risky members to lower the risk that they impose onto others. This result supports the current default waterfall structure.[5]

We measure systemic risk generated by the clearing network with the expected funding shortfall that can arise when all pre-funded resources are exhausted, and perform a comparative statics analysis. We find that the levels of risk-sharing and funding cost impact differently the expected funding shortfall and the level of default resources. For

[4]Such requirements are part of the European Market Infrastructure Regulation (EMIR) but not imposed in the U.S.. Both ICE Clear Credit and CME's CDS Clearing segment boast of $50 million of corporate default resource contributions.

[5]Acharya et al. (2009) discuss the additional usefulness of clearinghouses to mitigate operational risks and increasing market transparency.

instance, while more risk-sharing increases the equilibrium default fund and hence the funding costs the model economy has to bear, the clearing network is safer since the expected funding shortfall is lower. In addition, increased clearing revenue for members may increase systemic risk. This is against the common argument that large corporate revenues serve as a capital buffer: the clearinghouse's equilibrium response is to reduce the layer of default resources. While some members may enjoy a larger economic surplus, the equilibrium result is actually a riskier clearing network.

We use our model to assess the impact of minimum equity requirements, a regulatory measure which can be used to mitigate systemic risk. We find that the equilibrium effect of such a requirement is to increase equilibrium default funds, which reinforces systemic risk mitigation. However, this also creates additional funding costs to the model economy. Our model provides guidance to a welfare-maximizing regulator who needs to take into account the equilibrium effects of such a requirement, balancing the additional funding costs with the reinforced systemic risk mitigation.

Our paper contributes to the very recent, but exponentially growing, literature on clearinghouses and in particular the branch analyzing the central clearing default waterfall structure. Closely related to our theoretical framework is the empirical work of Armakola and Laurent (2015), who analyze clearinghouse default resources data. They argue that the quality and heterogeneity of clearing members are key to the monitoring and supervision of CCPs and must be taken into account when sizing the default fund. Our results echo theirs in showing that the level of risk-sharing, a measure of member heterogeneity, is central to the determination of equilibria. This is further supported by Ruffini et al. (2015), who argues that risk-sharing is an important characteristic when considering centrally cleared systems. In our model, how much (default) risk is being shared greatly affects members' participation in clearing, similar to Castiglionesi and Navarro (2008) where risk-sharing incentives induces the formation of financial linkages.[6] Biais et al. (2016) consider the incentives in risk-sharing activities performed through derivative trades and show it may undermine risk-prevention incentives. In contrast, we find that the typical default waterfall potentially supports risk-prevention.

Murphy and Nahai-Williamson (2014) analyze the adequacy and usefulness of setting the default fund level using the "Cover 2" requirement.[7] Menkveld (2016) analyzes systemic liquidation within a crowded trades setting and considers the default fund as the minimum level of funds needed to cover default losses in "extreme but plausible" conditions. The incentives provided by default fund funding costs are featured in the default resource calibration framework of Ghamami and Glasserman (2016). Different from these studies, which follow risk-measure based rules to determine default funds or equity commitments, we solve for equilibrium default waterfall resources from first

[6]We do not model moral hazard as in Castiglionesi and Navarro (2008), but our analysis extends straightforwardly. We discuss the moral hazard problem in Section 5.

[7]The Cover 2 requirement states that the aggregate default fund should be sufficient to cover the default losses of the two largest clearing members, as advocated by the Principles for Financial Market Infrastructures (CPMI and IOSCO (2012)). However, since the clearinghouse has much discretion over the precise estimation and simulation approach, there is still much ambiguity as to how default funds are actually determined.

principles. Amini et al. (2015) consider how central clearing can be used to reduce systemic risk in a Eisenberg-Noe (Eisenberg and Noe (2001)) type clearing network, and propose an alternative structure to the default fund to reduce liquidation costs, taking into account incentive compatibility of the requirements. In contrast, our model focuses entirely on agents' incentives to pin down the equilibria.[8]

The rest of the paper is organized as follows. Section 2 provides a brief overview of the typical default waterfall. We present our model in Section 3. Section 4 develops the equilibrium solution and the comparative statics analysis. Section 5 analyzes the equilibrium welfare distribution and systemic risk. Section 6 presents empirical predictions of our model analysis. Section 7 discusses policy implications. Section 8 concludes. Technical proofs are delegated to the appendix.

2 Preliminaries

We give a brief overview of the typical clearinghouse default waterfall. We refer the reader to Pirrong (2011), ISDA (2013a), Domanski et al. (2015), and Armakola and Laurent (2015) for a more detailed review of current practices, trends, and default resource data.

When a clearing member defaults, the clearinghouse inherits the open positions and needs to offload them to return to a balanced book. Usually an auction is held to allocate the defaulting member's positions among those who are solvent. Since the portfolio value may deteriorate during this "liquidation period", the clearinghouse may incur default losses.

The first line of defense against default losses is the defaulting member's initial margins. When margins are insufficient, losses accrue to the member's contribution to the default fund. If losses exceed the defaulting member's contribution, they are absorbed by capital contributions (equity) committed by the clearinghouse first, and then by default funds contributed by other clearing members. In the event that all such pre-funded capital is exhausted, clearinghouses typically have the right to call for additional member contributions to the default fund (referred to as un-funded default funds or assessment rights), which are often capped at a multiple of the original default fund contribution. "End-of-the-waterfall" procedures are implemented when even this is insufficient.[9]

The end-of-the-waterfall procedure advocated by the International Swaps and Derivatives Association is variation margin gain haircutting (ISDA (2013a)), which subjects members who have accrued gains over the liquidation period to receive a pro rata reduction on their variation margin gains, allowing the clearinghouse to use the haircut to

[8] Our model does not take into account the presence of multiple competing clearinghouses. Glasserman et al. (2016) discuss the equilibrium effect of multiple central counterparties to collateral requirements, whereas Duffie et al. (2015) empirically document the increase in collateral demand arising from central counterparties segregation.

[9] In the event that the clearinghouse fails, resolution procedures are implemented. This is beyond the scope of our paper and we refer the interested reader to Duffie (2015).

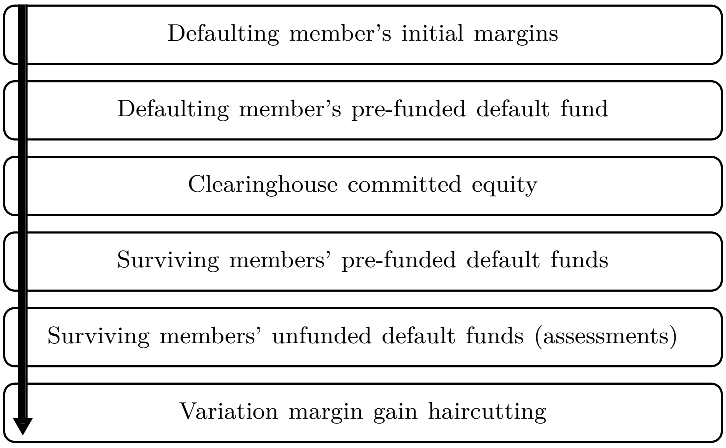

Figure 1: Graphical demonstration of a default waterfall.
When losses exceed the defaulting member's margins, they are sequentially allocated to the defaulting member's contribution to the default fund, clearinghouse equity, and other default fund resources. When these are all exhausted, end-of-the-waterfall procedures are enacted.

cover default losses. ISDA (2013a) argues that this "winner-pays" procedure is robust and almost always a sufficient resource. This default waterfall structure is graphically demonstrated in Figure 1.

We remark that clearinghouses may impose certain variations to their default waterfalls. For instance, LCH creates additional incentives for clearing members to participate in the auction process by allocating losses first to default funds belonging to clearing members who did not participate in bidding (LCH Clearnet (2014b)). In the interest of capturing the main economic forces, we do not model such exotic features to the default waterfall.

3 The model

We consider a two-period model economy consisting of one clearinghouse and a continuum of potential members. All agents maximize expected profits. Of the unit mass of potential members, $m \in [0,1]$ are risky (H) and $1-m$ are safe (L). Risky members default with probability d_H and safe ones default with probability d_L, where $1 > d_H > d_L > 0$. We also refer to d_L as the *baseline* default rate. Potential members' types are known only to themselves; however, d_H, d_L and m are common knowledge.

At time zero, the clearinghouse assigns a default fund requirement $G \in R^+$ and declares her equity rule, which specifies her committed equity, E, as a percentage, Θ, of aggregate expected default losses. We model the committed equity as such to reflect the general consensus that a clearinghouse's skin in the game should reflect the riskiness of the cleared market.

In response, members choose their propensities $\{p_H^i, p_L^j\}_{i\in[0,m),j\in[m,1]}$ of participating in the cleared market. We focus on symmetric rational expectations equilibria, i.e. $p^i_H = p_H$ and $p^j_L = p_L$. Given the participation propensities $\{p_H, p_L\}$, the law of large numbers (Judd (1985)) gives the mass of participating members who will default as $d := mp_H d_H + (1 - m)p_L d_L$, and the mass of participating members who will remain solvent as $s := mp_H(1 - d_H) + (1 - m)p_L(1 - d_L)$. Members incur a per unit funding cost α for posting the default fund.

For each participating member, B units of revenue are generated in the model economy. The parameter $B \geq 0$ is a reduced form specification for the preferences of members' clients to execute centrally cleared trades over bilateral ones, and quantifies the premium paid for such services. Revenue is split among the clearinghouse and the participating members, with the clearinghouse receiving $K := (1-\phi)B$ as fee and the member receiving $R := \phi B$, where $\phi \in [0, 1)$.

At time one, members default independently and exogenously. We model a stressed market, where default losses are perfectly correlated and distributed according to an exponentially distributed random variable with parameter λ, $Z \sim \text{Exponential}(\lambda)$. Realized aggregate default loss is hence Zd, and aggregate expected default loss is $\frac{d}{\lambda}$. This also yields the committed equity $E = \Theta\frac{d}{\lambda}$.

The objective of the paper is to analyze the economic role of default funds and clearinghouse equity, thus we do not model initial margins which usually serve as the first line of defense against default losses. This comes without loss of generality, however, as we can view Z as a random variable describing the losses that exceed initial margin requirements. In fact, the current market is moving towards a "defaulter-pays" model where initial margins are set at very conservative levels. This implies that a deeply stressed market is required for initial margins to be exhausted, and supports our assumption of strong loss correlation among parties, as losses tend to be more correlated under stressed market conditions (Hull (2012)).

Losses are covered first by using the defaulting member's default fund, second with the clearinghouse's equity, and third allocated to surviving clearing members' pre-funded default funds. When the allocated losses exceed the pre-funded amount, the clearinghouse utilizes her assessment rights to obtain addition capital from members. When even that capital is exhausted, losses are allocated to solvent members who are "in the money" (ITM). We assume that the clearinghouse's assessment rights is a multiple δ of the default fund, and that the probability of a solvent member being out of the money (OTM) is $1 - q$, $0 < q < 1$. The model timeline is provided in Figure 2.

3.1 Members

The payoff function of a participating member depends on his state at the end of time 1, and is given by:

$$X(\text{Default}) = R - \alpha G - G + (G - Z)^+, \tag{3.1}$$

$$X(\text{Solvent \& OTM}) = R - \alpha G - (1 + \delta)G + \left((1 + \delta)G - \frac{(d(Z - G) - E)^+}{s}\right)^+, \tag{3.2}$$

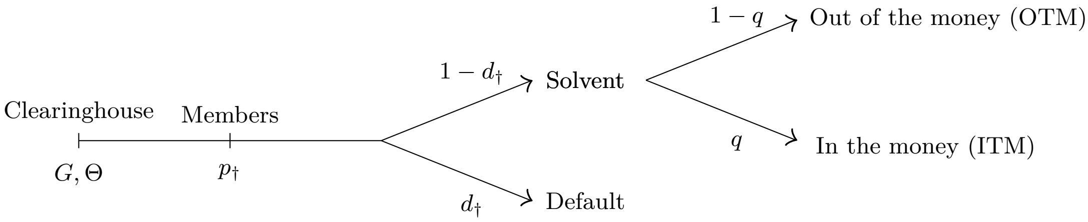

Figure 2: Model Timeline and Type † Member evolution.
Members choose their participation propensities $p_\dagger$ ($\dagger \in \{H, L\}$) in response to the clearinghouse's declaration of default fund requirement G and equity commitment Θ. Then members of type † default exogenously with probility $d_\dagger$. Conditioned on solvency, they are in the money with a fixed probability q.

$$X(\text{Solvent \& ITM}) = R - \alpha G - (1+\delta)G + \left((1+\delta)G - \frac{(d(Z-G)-E)^+}{s}\right)^+ - \left(\frac{(d(Z-G)-E-(1+\delta)Gs}{sq}\right)^+. \tag{3.3}$$

In all cases, the member receives income R for participating and pays upfront a funding cost αG because of the default fund requirement. In Eq. (3.1), $(G-Z)^+ - G$ reflects that the defaulting member's default fund is used to absorb the losses first, with the remaining losses absorbed by next tiers in the default waterfall. The term $\frac{(d(Z-G)-E)^+}{s}$ in Eq. (3.2) and Eq. (3.3) reflects that losses allocated to surviving members are those exceeding defaulting members' default fund and the clearinghouse's committed equity. Eq. (3.2) reflects that when members are out of the money, their contributions to absorbing losses are limited to a multiple $(1+\delta)$ of the default fund, taking into consideration the clearinghouse's assessment rights. Finally, the $\left(\frac{(d(Z-G)-E-(1+\delta)Gs}{sq}\right)^+$ term in Eq. (3.3) reflects that all losses not covered by pre-funded default fund, clearinghouse committed equity, and assessment rights, are borne by members who are in the money (say, via variation margin gain haircutting).

The expected payoff of a member, conditioned on the default losses Z and solvency, is then given by

$$\begin{aligned}\mathbb{E}[X(\text{Solvent})|Z] &= qX(\text{Solvent \& ITM}) + (1-q)X(\text{Solvent \& OTM}) \\ &= R - \alpha G - \frac{(d(Z-G)^+ - E)^+}{s}.\end{aligned} \tag{3.4}$$

Because agents are assumed to have linear utility, Eq. (3.4) shows that the payoff function is equivalent to the one where the clearinghouse has unlimited assessment rights, and that the parameters δ and q do not affect agents' decisions.

We can then compute the overall expected payoff for a member of type $\dagger \in \{H, L\}$ given by

$$\mathbb{E}[X_\dagger] = R - \alpha G - \frac{d_\dagger}{\lambda}(1 - e^{-\lambda G}) - \frac{(1-d_\dagger)d}{s\lambda} e^{-\lambda G - \Theta}. \tag{3.5}$$

If a member chooses to not participate in clearing, his expected payoff is the expected loss generated when he defaults, $-\frac{d_\dagger}{\lambda}$. Thus his economic profit for participating is $E[X_\dagger] + \frac{d_\dagger}{\lambda}$.

3.2 Clearinghouse

The clearinghouse's payoff function is given by:

$$Y = KM - E + (E - d(Z-G)^+)^+,$$

where $M := mp_H + (1-m)p_L$ is the mass of participating members. The KM term represents her revenue, whereas $-E+(E-d(Z-G)^+)^+$ indicates that the clearinghouse absorbs losses that exceed defaulting members' default funds, up to the amount of her committed equity. Her expected payoff is

$$\mathbb{E}[Y] = KM - \frac{d}{\lambda}e^{-\lambda G}(1-e^{-\Theta}). \tag{3.6}$$

4 Equilibrium Solutions

In this section we solve for symmetric rational expectations subgame perfect equilibria of our model and present associated equilibrium quantities.

Definition 1. *An equilibrium is a pair* $(p_H^*(G,\Theta), p_L^*(G,\Theta)) \in \{R_+^2 \to [0,1]^2\}$ *such that for all* $(G,\Theta)) \in R_+^2$

1. $p_H^*(G,\Theta) \in \operatorname*{argmax}_{p_H\in[0,1]}\ p_H\left(\mathbb{E}\left[X_H\left(p_H\middle|G,\Theta,\frac{d_L}{\lambda}, p_L^*(G,\Theta)\right)\right]\right) - (1-p_H)\frac{d_H}{\lambda}$
2. $p_L^*(G,\Theta) \in \operatorname*{argmax}_{p_L\in[0,1]}\ p_L\left(\mathbb{E}\left[X_L\left(p_L\middle|G^*,\Theta^*,\frac{d_L}{\lambda}, p_H^*(G,\Theta)\right)\right]\right) - (1-p_L)\frac{d_L}{\lambda}$

and a pair $(G^*,\Theta^*) \times R_+^2$ *such that*

$$(G^*,\Theta^*) \in \operatorname*{argmax}_{G,\Theta\geq 0}\ \mathbb{E}[Y(G,\Theta|p_H^*(G,\Theta), p_L^*(G,\Theta))]$$

By Eq. (3.5), we can normalize all monetary quantities $(G, E, B, \mathbb{E}[X], \mathbb{E}[Y])$ by using the expected default loss $\frac{1}{\lambda}$ as the unit of account. Hence, there is no loss of generality in assuming $\lambda = 1$, a convention we maintain throughout the rest of the paper.

It follows from Eq. (3.5) that

$$\mathbb{E}[X_H] + d_H > \mathbb{E}[X_L] + d_L, \tag{4.1}$$

which implies that $p_H^* \geq p_L^*$ holds in equilibrium. It is thus possible to observe two types of equilibria: separating equilibria ($p_L^* = 0$) where the clearinghouse's choice of default resources fail to incentivize safe members to participate, or pooling equilibria ($p_L^* > 0$)

where safe members have a positive propensity of participating in the loss-mutualization process. We individually consider the problems the clearinghouse faces when solving for separating equilibria and pooling equilibria. Their equilibrium default fund and equity rules are denoted by (G_s^*, Θ_s^*) and (G_p^*, Θ_p^*), respectively.

For separating equilibria, safe members do not participate in the clearing network. The clearinghouse's problem is

$$\max_{G,\Theta\geq 0} \quad \mathbb{E}[Y] \tag{P_s}$$

$$\text{subject to} \quad R - \alpha G + d_H e^{-G} - \frac{(1-d_H)d}{s} e^{-G-\Theta} \geq 0, \tag{IR$_H$}$$

$$R - \alpha G + d_L e^{-G} - \frac{(1-d_L)d}{s} e^{-G-\Theta} < 0. \tag{IR$_L^o$}$$

Here (IR$_H$) is the risky members' individual rationality (participation) constraint, and (IR$_L^o$) is the safe members' constraint for no participation.

For pooling equilibria, the clearinghouse's problem is:

$$\max_{G,\Theta\geq 0} \quad \mathbb{E}[Y] \tag{P_p}$$

$$\text{subject to} \quad R - \alpha G + d_L e^{-G} - \frac{(1-d_L)d}{s} e^{-G-\Theta} \geq 0. \tag{IR$_L$}$$

Here (IR$_L$) is the safe members' participation constraint. Notice that (IR$_H$) is automatically satisfied when (IR$_L$) is.

The interplay between the clearinghouse's equity rule and the IR constraints plays a central role in our analysis. The following key lemma allows us dichotomize the analysis of equilibrium equity rules.

Lemma 1. *For any separating equilibria with $\Theta_s^* > 0$, (IR$_H$) is binding. For any pooling equilibria with $\Theta_p^* > 0$, (IR$_L$) is binding.*

The economic intuition behind Lemma 1 is as follows. By Eq. (3.6), for a fixed level of positive equity, the clearinghouse would like to set default funds as high as possible, so to protect her committed equity. Since default funds introduce linear costs to her members, she sets default funds to the level where the marginal participating member is indifferent between participating or not. This implies that (IR$_H$) is binding for separating equilibria and (IR$_L$) is binding for pooling equilibria.

We observe that when $\Theta = 0$, there may be multiple levels of default funds among which the clearinghouse is indifferent, since she bears no default losses. We break the tie by assuming that the clearinghouse chooses the minimum possible default fund, as this minimizes funding costs and thus maximizes aggregate surplus captured by members.

4.1 Equilibrium decisions

In this section we solve for equilibrium default fund and equity contributions.

4.1.1 Separating Equilibria ($p_L^* = 0$)

When safe members do not participate, in equilibrium agents expect that the market consists only of risky members and compute their expected payoffs accordingly. The mass of agents who participate, default, and survive can then be expressed as $M = mp_H$, $d = mp_H d_H$, $s = mp_H(1-d_H)$, respectively. (P_s) can be rewritten as

$$\begin{aligned} \max_{G,\Theta\geq 0} \quad & mp_H \left(K - d_H e^{-G}(1-e^{-\Theta})\right) \\ \text{subject to} \quad & R - \alpha G + d_H e^{-G}\left(1 - e^{-\Theta}\right) \geq 0, && (4.2) \\ & R - \alpha G + d_L e^{-G} - \frac{(1-d_L)d_H}{1-d_H} e^{-G-\Theta} < 0. && (4.3) \end{aligned}$$

We now solve for separating equilibria:

Proposition 1. *For separating equilibria,* $(p_H^*, p_L^*, \Theta_s^*) = (1, 0, 0)$*. Moreover, suppose* $R \leq \frac{d_H - d_L}{1-d_L}$*, then* $G_s^* = 0$*; otherwise* G_s^* *is the unique solution to*

$$(R - \alpha G)e^G = \frac{d_H - d_L}{1 - d_L}. \tag{4.4}$$

The result in Proposition 1 is intuitive. When participating members are homogenous (only risky members participate) the risks they contribute to the clearing network are the same. Thus, when rewards for joining are low, $R \leq \frac{d_H - d_L}{1-d_L}$, all risky members would participate as long as there is no default fund requirement. This is because default funds introduce additional costs to members. When rewards from participating are high, however, the clearinghouse must set default funds high enough to disincentivize safe members from participating.[10] This also implies that the clearinghouse can attract all risky members to participate without committing equity, which minimizes her potential loss and thus maximizes her profit.

4.1.2 Pooling Equilibria ($p_L^* > 0, p_H^* = 1$)

We consider the case where safe members participate with positive propensity ($p_L^* > 0$). When safe members do participate, in equilibrium agents expect that all risky members participate. That is, by Eq. (4.1), $p_H^* = 1$ when $p_L^* > 0$. The mass of agents who participate, default, and survive can then be expressed as $M = m + (1-m)p_L$, $d = md_H + (1-m)d_L p_L$, and $s = m(1-d_H) + (1-m)(1-d_L)p_L$, respectively.

The clearinghouse's problem (P_p) can be rewritten as:

$$\begin{aligned} \max_{G,\Theta\geq 0} \quad & K(m + (1-m)p_L) - de^{-G}(1-e^{-\Theta}), && (P_p') \\ \text{subject to} \quad & R - \alpha G + d_L e^{-G} - \frac{(1-d_L)d}{s} e^{-G-\Theta} \geq 0. && (4.5) \end{aligned}$$

[10] As we show in Section 4.1.3, it may be more profitable for the clearinghouse to have only risky members participate rather than having both risky and safe members.

We can carry out the analysis of pooling equilibria by first showing that mixed strategies do not arise in equilibrium.

Lemma 2. *For pooling equilibria,* $(p_H^*, p_L^*) = (1, 1)$*.*

Lemma 2 can be understood as follows: because of linear utilities, the marginal profit safe traders contribute to the clearinghouse is constant. In addition, the clearinghouse wants safe traders to participate only when the marginal profit they contribute is positive. Thus, the fact that the equilibrium is pooling ($p_L^* > 0$) implies that the marginal profit safe traders contribute is a positive constant, in which case the clearinghouse incentivizes all of them to participate ($p_L^* = 1$).

Having solved for members' joining propensities, we next solve for the clearinghouse choices of G and Θ. We define the *risk-sharing coefficient* as

$$\gamma := \frac{m(d_H - d_L)}{m(1-d_H) + (1-m)(1-d_L)} = \frac{D - d_L}{S}.$$

Here $D = md_H + (1-m)d_L$ and $S = m(1-d_H) + (1-m)(1-d_L)$ are, respectively, the mass of defaulting and solvent members under full participation. The parameter γ is the aggregate additional default risk (above the "baseline" default rate of d_L) introduced by the risky members, which is allocated to surviving members. When γ is large, more risk is transferred to surviving members.

Our next result relates the arising pooling equilibria to funding cost α, the baseline default rate d_L, reward for participation R, and the amount of risk-sharing γ.

Proposition 2. *Define* $\bar{h}(x) := 1 + \log x$*, and* $\underline{h}(x) := -\frac{d_L}{x} + \log x$*. Then:*

(i) if $\gamma \geq \alpha$ *and* $\bar{h}(\gamma) > \frac{R}{\alpha} + \log\alpha > \underline{h}(\gamma)$ *then* $\Theta_p^* = -\log\left(\frac{\gamma}{\gamma + d_L}\left(\frac{R}{\alpha} - \log\frac{\gamma}{\alpha} + \frac{d_L}{\gamma}\right)\right)$ *and* $G_p^* = \log\frac{\gamma}{\alpha}$*.*

(ii) If $\alpha > \gamma > R$ *then* $\Theta_p^* = \log\frac{d_L+\gamma}{d_L+R}$ *and* $G_p^* = 0$*.*

(iii) If $\frac{R}{\alpha} + \log\alpha < \underline{h}(\gamma)$*, then* $\Theta_p^* = \infty$ *and* G_p^* *is the unique solution to*

$$R - \alpha G + d_L e^{-G} = 0. \tag{4.6}$$

(iv) If $R < \gamma$*,* $\frac{R}{\alpha} + \log\alpha \geq \bar{h}(\gamma)$ *and* $\gamma \geq \alpha$*, then* $\Theta_p^* = 0$ *and* G_p^* *is the unique solution to*

$$R - \alpha G - \gamma e^{-G} = 0 \tag{4.7}$$

smaller than $\frac{R}{\alpha} - 1$*.*

(v) If $R \geq \gamma$ *then* $\Theta_p^* = G_p^* = 0$*.*

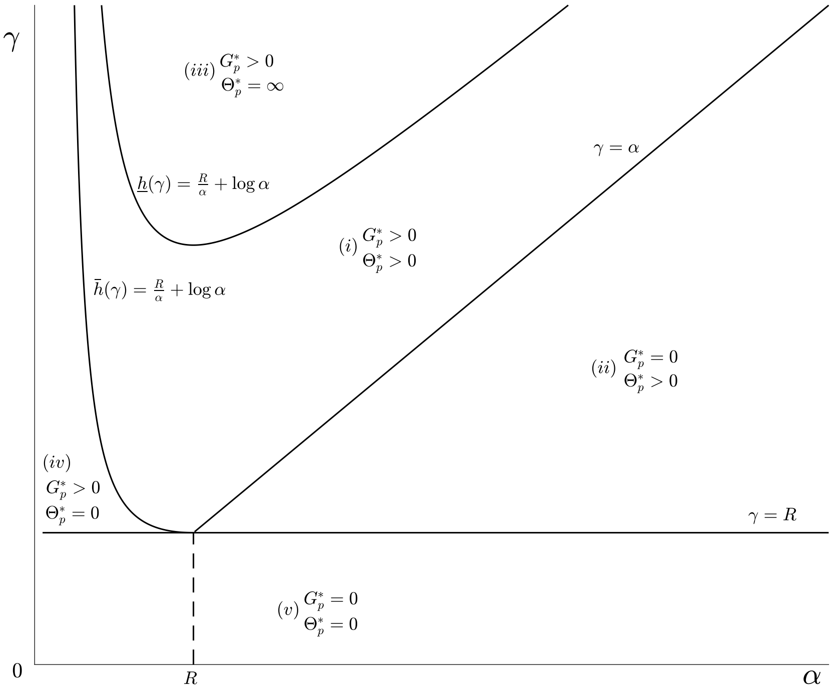

Figure 3: Sectioning of the $\alpha - \gamma$ plane into different pooling equilibria regimes.
Depending on the level of risk-sharing (γ) and funding cost (α), different types of pooling equilibria arise. Non-degenerate pooling equilibria (those with finite and positive default fund and equity) arise when both risk-sharing and funding cost are moderate.

Remark 1. *The risk-sharing coefficient γ plays a prominent role in the determination and analysis of pooling equilibria. The use of γ allows us to reduce the parameter space $(m, d_H, d_L, \alpha, K) \in [0,1]^2 \times [0, d_H] \times R_+^2$ to $(d_L, \gamma, \alpha, K) \in [0,1] \times R_+^2$ for the analysis of pooling equilibria. Importantly, given d_L, γ can take on any value in the interval $[0, \infty)$, allowing us to perform comparative statics analyses without constraints.*

Proposition 2 is graphically demonstrated in Figure 3. Depending on the levels of (γ, R, α, d_L), it shows that there are different regimes of pooling equilibria. When the clearinghouse can attract all members to participate without committing equity, she would do so (cases (iv) and (v)) since she then bears no losses. In these cases, she may use default funds to mitigate the wealth transfer from safe members to risky ones, so to incentivize them to participate (case (iv)). To see this, note that with zero equity, safe members' expected profits are $R - \alpha G - \gamma e^{-G}$ and risky members' expected profits are $R - \alpha G + \frac{1-m}{m} \gamma e^{-G}$. The term γe^{-G} term represents the wealth transfer resulting from loss-mutualization, and decreases with G.

Since equity directly increases members' profits, the profit-maximizing clearinghouse may need to commit equity to increase their participation (cases (i), (ii), and (iii)).

Notice that when $R = 0$, cases (iv) and (v) are precluded, and thus the clearinghouse's equity must be positive. This reflects the fact that, without additional incentives from clearing revenue, the clearinghouse must provide equity protection to attract safe members. This highlights the economic role of clearinghouse equity: it mitigates the losses generated by risky members, which safe members must bear due to loss-mutualization, and hence attracts safe members to participate.

When the clearinghouse commits equity, she has the additional incentive to require large default funds to protect her equity, which is reigned in by the funding costs that members would bear. For moderate levels of risk-sharing and funding cost, it is optimal for the clearinghouse to use a finite combination of equity and default fund (Case (i)), which we refer to as non-degenerate pooling equilibria. When the funding cost is too high relative to the amount of risk-sharing, the clearinghouse abandons the use of the default fund requirement altogether, so to help members save on paying the costs. This allows the clearinghouse to reduce the amount of committed equity, lowering her potential loss (Case (ii)). When there is a lot of risk-sharing and low funding cost (Case (iii)), the clearinghouse can impose a high default fund requirement while committing a large amount of equity. Because of the high default fund requirement, her expected loss from utilizing her equity to cover losses is low. In this extreme case, she essentially commits to absorbing all losses in excess of defaulting members' default funds, and members are completely insulated from each other.

We remark that the expressions given in Proposition 2 may result in default fund and equity levels that violate the clearinghouse's individual rationality constraint. For example, when the level of risk-sharing is high and B is low, the financing cost of default fund contributions may exceed the aggregate revenue K that the clearinghouse can capture. In this case, she is necessarily making negative expected profits and is better off not clearing trades at all. Recall, however, that the clearinghouse can capture a profit of mK in separating equilibria, so in the aforementioned situation the clearinghouse would set requirements to create a separating equilibrium rather than pooling ones.

4.1.3 Prevailing equilibria

This section studies the prevailing equilibrium based on the model parameters. This is achieved by using Proposition 1 to compute a separating equilibrium, Proposition 2 to compute a pooling equilibrium, and then selecting the equilibrium where the clearinghouse captures more profit.

Proposition 1 shows that the clearinghouse's profit in a separating equilibrium is mK. Comparing this with (P_p'), we see that the pooling equilibrium prevails if and only the clearinghouse's extra profit from choosing a pooling equilibrium $(1-m)K - De^{-G_p^*}(1-e^{-\Theta_p^*})$ is positive; otherwise the separating equilibrium prevails. Using Proposition 2, we can classify the type of prevailing equilibria by directly plugging the expressions given in Proposition 2.

Corollary 1. *The prevailing equilibrium is a pooling equilibrium if and only if one of the following conditions hold* *(i)* $\gamma \geq \alpha$, $\bar{h}(\gamma) > \frac{R}{\alpha} + \log\alpha > \underline{h}(\gamma)$ *and* $(1-m)K \geq \frac{\alpha}{1+\gamma}\left(1 - \frac{R}{\alpha} + \log\frac{\gamma}{\alpha}\right)$,

(ii) $\alpha > \gamma > R$ *and* $(1-m)K \geq \frac{\gamma - R}{1+\gamma}$,

(iii) $\frac{R}{\alpha} + \log\alpha < \underline{h}(\gamma)$ *and* $(1-m)K \geq \frac{\gamma + d_L}{1+\gamma}e^{-G_p^*}$, *where* G_p^* *is the unique solution to Eq.* (4.6),

(iv) $R \geq \alpha$ *and* $\bar{h}(\gamma) \leq \frac{R}{\alpha} + \log\alpha$.

(v) $R \geq \gamma$,

Otherwise the prevailing equilibrium is a separating equilibrium.

Corollary 1 also shows that separating equilibria are not vacuous. In the first three cases, there is always a large enough K so that pooling equilibria prevail, and a small enough K so that separating equilibria prevail, ceteris paribus.

4.2 Comparative Statics

We provide a comparative statics focusing on the major economic forces at play: (I) the cost and benefit tradeoff from loss-mutualization, (II) the clearinghouse's equity at risk and the benefit provided by equity to her members, and (III) the protection provided by default funds and the funding costs it imposes on her members. We assume $R = 0$ for most of this section as this allows to better highlight the key insights, but we also emphasize additional effects that can arise when $R > 0$.

When $R = 0$, the results of propositions 1 and 2 simplify. The second case of Proposition 1, and the fourth and fifth cases of Proposition 2 are precluded. The restrictions of the parameter set for the first three cases of Proposition 2 also simplify to $\left\{\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}\right\}$, $\{\alpha > \gamma\}$, and $\left\{\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}\right\}$, respectively.

Default fund and equity levels are always zero for the case of separating equilibria. The more interesting case is then when pooling equilibria arise. The sensitivity of the clearinghouse's equilibrium choices to changes in model parameters are tabulated in Table 1. The complete set of results for both cases presented in Proposition 1, and all five cases presented in Proposition 2 when $R > 0$, is delegated to the appendix.

While the precise relation depends on which case outlined in Proposition 2 takes place, we see from Table 1 that the relations remain consistent across cases (there is no change in sign). Hence, we will focus our discussion on the non-degenerate case (first column), and note that the same economic intuition follows for the degenerate ones.

When the funding cost (α) increases, the clearinghouse observes the lowered incentives of potential members to participate, and consequently lowers the default fund requirement. This increases her potential losses, however, as the default fund acts as a cushion to absorb losses before her equity is used. Thus she also lowers her equity contribution. On the other hand, increased risk-sharing (γ) increases both the default fund requirement and equity contribution. When there is more risk-sharing, the safe traders are less willing to participate since they need to bear more of the risky members'

$G_p^* \sim$ param.	$\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$	$\alpha > \gamma$	$\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$
α	decrease	no effect	decrease
γ	increase	no effect	no affect
d_L	no effect	no effect	increase
$\Theta_p^* \sim$ param.	$\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$	$\alpha > \gamma$	$\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$
α	decrease	no effect	no effect
γ	increase	increase	no effect
d_L	decrease	decrease	no effect

Table 1: Comparative statics for the equilibrium default fund and equity rule.
The relation between default resources and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. The sign of the relation is generally consistent with that of the non-degenerate case (first column).

losses. In response, the clearinghouse increases their default protection by increasing her equity contribution, but also increases her default fund requirement to protect her equity. Finally, an increase in the baseline default rate (d_L) means that there is a higher probability that the clearinghouse has to suffer default losses, hence she lowers her equity contribution in response. In the extreme case where she commits to absorbing all losses in excess of defaulting members' default funds (third column), she asks for more default funds to protect her equity.

We next present interesting phenomena that arise when $R > 0$:

Proposition 3. *The following statements hold:*

1. $\frac{\partial \Theta_p^*}{\partial R} \leq 0$.
2. *For non-degenerate pooling equilibria,* $\frac{\partial \Theta_p^*}{\partial \alpha} < 0$ *if* $R < \alpha$*, and* $\frac{\partial \Theta_p^*}{\partial \alpha} \geq 0$ *otherwise.*
3. *If* $R < \gamma$*,* $\frac{R}{\alpha} + \log \alpha \geq \bar{h}(\gamma)$ *and* $\gamma \geq \alpha$*,* $\frac{\partial G_p^*}{\partial \alpha} > 0$.

Proposition 3 highlights the fact that members' clearing revenue can alter their incentives and produces interesting phenomena. First, increased member revenue means that participation is less costly for safe members, lowering the amount of equity the clearinghouse needs to commit to attract them. Second, when member revenue is high ($R > \alpha$), the increase in funding cost creates a decrease in default funds, which greatly increases the wealth transfer from safe members to risky ones. In this case, the clearinghouse responds by increasing her equity commitment to keep the safe members from exiting. Third, a similar situation can arise when there is a lot of risk-sharing $\gamma \geq \max(\alpha, R)$. While the clearinghouse can attract all members to participate without committing equity, an increase in funding cost greatly lowers the safe members' surplus, which must be compensated for by decreasing the wealth transfer from safe members to risky ones. This is achieved by *increasing* the default fund requirement.

5 Welfare and systemic risk

In this section we investigate the distribution of equilibrium economic profits and a measure of systemic risk. We also provide a comparative statics analysis, and apply our results to analyze the incentives provided to agents by the default waterfall.

5.1 Equilibrium welfare distribution

We study the distribution of economic profits for the different types of equilibria. We denote the equilibrium economic profits of the clearinghouse, risky members, and safe members by K_C, $K_H := E[X_H^*] + d_H$, and $K_L := E[X_L^*] + d_L$, respectively.

For separating equilibria, it follows from Proposition 1 that $K_{C,s} = Km$, $K_{L,s} = 0$ and $K_{H,s} = R$ when $R \leq \frac{d_H - d_L}{1-d_L}$. When $R \geq \frac{d_H-d_L}{1-d_L}$, however, $K_{H,s} = \frac{d_H-d_L}{1-d_L} e^{-G_s^*}$ where G_s^* is the solution to Eq. (4.4).

For pooling equilibria, we know that $p_L^* = 1$. In addition, safe members make zero economic profit when $\Theta_p^* > 0$. We can then compute risky members' economic profit as

$$K_{H,p} = (d_H - d_L)e^{-G_p^*} \left(1 + \frac{D}{S} e^{-\Theta_p^*}\right), \tag{5.1}$$

and the clearinghouse's economic profit as

$$K_{C,p} = K - De^{-G_p^*}(1 - e^{-\Theta_p^*}). \tag{5.2}$$

Using Proposition 2, we can compute explicitly these equilibrium quantities. The corresponding expressions are given in Corollary 2, and come directly from plugging the clearinghouses's equilibrium choices into Eqs. (5.1) and (5.2).

Corollary 2. *For pooling equilibria, agents' economic profits admit the following expressions (the numbering of cases correspond to that provided in Proposition 2.):*

(i) $mK_{H,p} = \frac{\gamma}{1+\gamma}R + \frac{\alpha}{1+\gamma}\left(1 - \gamma \log \frac{\gamma}{\alpha}\right)$; $K_{C,p} = K - \frac{\alpha}{1+\gamma}\left(1 + \log\frac{\gamma}{\alpha} - \frac{R}{\alpha}\right)$; $K_{L,p} = 0$.

(ii) $mK_{H,p} = \frac{\gamma}{1+\gamma}R + \frac{\gamma}{1+\gamma}$; $K_{C,p} = K - \frac{\gamma - R}{1+\gamma}$; $K_{L,p} = 0$.

(iii) $mK_{H,p} = \frac{\gamma(1-d_L)}{1+\gamma} e^{-G_p^*}$; $K_{C,p} = K - \frac{\gamma + d_L}{1+\gamma} e^{-G_p^*}$; $K_{L,p} = 0$. *Here G_p^* is the unique solution to Eq. (4.6).*

(iv) $mK_{H,p} = mR - m\alpha G_p^* + (1-m)\gamma e^{-G_p^*}$; $K_{C,p} = K$; $K_{L,p} = R - \alpha G_p^* - \gamma e^{-G_p^*}$. *Here G_p^* is the unique solution to Eq. (4.7) smaller than $\frac{R}{\alpha} - 1$.*

(v) $mK_{H,p} = mR + (1-m)\gamma$; $K_{C,p} = K$; $K_{L,p} = R - \gamma$.

Next, we provide a comparative statics analysis on the welfare distribution. For ease of exposition we focus on the case $R = 0$.[11] All members' surpluses are held at zero

[11] When $R > 0$ safe members require less protection to participate. This reduces the need for default funds and equity, and is expected to increase the profit of both the risky members and the clearinghouse.

for the case of separating equilibria. Again, the more interesting case is when pooling equilibria arise. In these equilibria safe members' profits are still zero, but both the clearinghouse and her risky members make positive economic profit. The aggregate welfare $(B - \alpha G_p^*)M$ consists of the aggregate economic profit of risky members and the clearinghouse. From Table 1 and Proposition 2 we see that increasing funding cost and risk-sharing lead to higher αG_p^*, and thus lower aggregate welfare.

The effects of an increase in model parameters on the equilibrium welfare distribution are tabulated in Table 2. The relations remain mostly consistent across cases.

$mK_{H,p}^* \sim$ param.	$\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$	$\alpha > \gamma$	$\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$
α	increase	no effect	increase
γ	decrease	increase	increase
d_L	no effect	no effect	decrease
$K_{C,p}^* \sim$ param.	$\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$	$\alpha > \gamma$	$\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$
α	decrease	no effect	decrease
γ	decrease	decrease	decrease
d_L	no effect	no effect	increase

Table 2: Comparative statics for the equilibrium welfare distributions.

The relation between the equilibrium welfare distributions and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. Except for the dependency of risky members' profits on the level of risk-sharing, the sign of the relation is generally consistent with that of the non-degenerate case (first column). Since risky members' profits decrease with risk-sharing only in non-degenerate pooling equilibria, a positive combination of default fund and equity can align their incentives with that of the clearinghouse.

When the funding cost increases, the clearinghouse lowers the default fund requirement so as to incentivize safe members to participate (Table 1). There is more loss-mutualization among members which benefits the risky members. This comes at the detriment of the clearinghouse, whose equity contribution is at a higher stake. On the other hand, Table 1 indicates that when there is more risk-sharing, the clearinghouse's choices of default fund and equity commitment both increase; however, Table 2 shows that the additional protection coming from the increased default fund is overpowered by the higher potential losses due to the higher equity commitment, leading to a lower overall profit. The baseline default rate (d_L) mostly has no effect on economic profits except for the extreme case where members are essentially insulated from each other (third column). In this case, an increase in d_L leads the clearinghouse to increase her protection against default losses through increasing G_p^*, which lowers risky members' profits.

An interesting phenomenon is that an increase in risk-sharing *increases* aggregate economic profit of risky members in degenerate pooling equilibria, but *decreases* it in non-degenerate pooling equilibria. In a non-degenerate equilibrium, the clearinghouse's choices actually incentivize risky members to "be safer" and lower the amount of risk that is shared.[12] In general, risk-sharing means that the clearinghouse and safe members have to subsidize more of the risky members' default losses. In models with moral hazard, this can lead to incentives for members to "shirk" – to increase their default probabilities and capture economic surplus from default protection, which is indeed the case in our degenerate pooling equilibria. However, Table 2 shows that the optimal combination of default funds and a positive equity commitment can effectively align members' incentives with that of the clearinghouse.

5.2 Systemic risk

A clearing network may impose systemic risk and negative externalities on the real economy when its pre-funded capital stock is exhausted and external capital to absorb losses needs to be provided. Our measure of systemic risk is the "funding shortfall", the amount of external capital the clearing network requires, which we denote by Q. Using the funding shortfall as a measure for systemic risk is quite intuitive, as larger funding needs tend to be more disruptive (Greenwood et al. (2015)). As we are considering periods of market stress, this funding shortfall measure is similar in spirit to the SRISK measure proposed by Brownlees and Engle (2016), where they consider the expected capital shortfall of an entity conditioned on a systemic event.[13] Acharya et al. (2016) consider Systemic Expected Shortfall (SES), the expected funding shortfall conditioned on the event that the entire financial system is under-capitalized, again a systemic risk measure based on funding shortfall.

The (ex-ante) expected funding shortfall, $\mathbb{E}[Q]$,[14] can be straightforwardly computed as

$$\mathbb{E}[Q] := \mathbb{E}[(dL - MG - E)^+] = de^{-\frac{MG}{d} - \Theta}. \tag{5.3}$$

Having solved for the equilibrium decisions in Section 4.1, we can calculate the equilibrium expected funding shortfall $\mathbb{E}[Q^*]$.

Corollary 3. *For separating equilibria, if* $R \leq \frac{1}{m}\frac{\gamma}{1+\gamma}$, $\mathbb{E}[Q_s^*] = md_L$, *otherwise* $\mathbb{E}[Q_s^*] = md_L e^{-G_s^*}$ *where* G_s^* *solves Eq. (4.4). For pooling equilibria, we have:*

(i) $\mathbb{E}[Q_p^*] = \frac{\gamma}{1+\gamma}\left(\frac{\alpha}{\gamma}\right)^{\frac{1+\gamma}{\gamma+d_L}}\left(\frac{R}{\alpha} - \log\frac{\gamma}{\alpha} + \frac{d_L}{\gamma}\right)$,

(ii) $\mathbb{E}[Q_p^*] = \frac{R+d_L}{1+\gamma}$,

[12] In our model, members' default probabilities are exogenous to the model. A straightforward extension, analogous to the moral hazard model of Holmström and Tirole (1997), can introduce the possibility that members can choose to increase their default probabilities by a small amount. The clearinghouse can detect whether such actions are taken, but cannot ensure who the risk members are. In equilibrium, such an action would lower risky members' profits.

[13] Brownlees and Engle (2016) show that such a shortfall measure can effectively rank financial institutions in terms of systemic risk and predict declines in real economic activity.

[14] Another measure of systemic risk is the probability that pre-funded capital stock is exhausted $\mathbb{P}(dL \geq MG + E)$. One can easily show that using this measure gives analogous insights to the funding shortfall.

(iii) $\mathbb{E}[Q_p^*] = 0$,

(iv) $\mathbb{E}[Q_p^*] = \frac{\gamma+d_L}{1+\gamma} e^{-\frac{1+\gamma}{\gamma+d_L} G_p^*}$, *where* G_p^* *solves Eq.* (4.7),

(v) $\mathbb{E}[Q_p^*] = \frac{\gamma+d_L}{1+\gamma}$.

Above, the numbering corresponds to conditions on parameters provided in Proposition 2.

The results follow from plugging the expressions given by Propositions 1 and 2 into Eq. (5.3) and using the identity $D = \frac{\gamma+d_L}{1+\gamma}$.

We next provide a comparative statics analysis on equilibrium systemic risk. For ease of exposition we first consider the case where $R = 0$.[15] In the case of separating equilibria, the expected funding shortfall is md_H. The effects of an increase in model parameters for pooling equilibria are tabulated in Table 3. The relations remain consistent across cases.

$\mathbb{E}[Q_p^*] \sim$ param.	$\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$	$\alpha > \gamma$	$\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$
α	increase	no effect	no effect
γ	decrease	decrease	no effect
d_L	increase	increase	no effect

Table 3: Comparative statics for the systemic risk measure.
The relation between the equilibrium systemic risk and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. The sign of the relation is generally consistent with that of the non-degenerate case (first column). Increases in the funding cost and decreases in risk-sharing both lead to less pre-funded default resources and hence a higher level of systemic risk.

Since both equity and default funds decrease (for non-degenerate pooling equilibria) when the funding cost increases (Table 1), a higher funding cost leads to a higher expected funding shortfall. In addition, an increase in the baseline default risk leads to an overall riskier system.

Importantly, when there is more risk-sharing, increased default fund and equity commitment both contribute to mitigating expected funding shortfall. This implies that an increase in the default probabilities d_H of the risky members actually reduces systemic risk through the clearinghouse's equilibrium response of increasing default funds and equity. Recall, however, that it reduces the economic profits of clearing members (Section 5.1).

Another interesting phenomenon arises when we consider $R > 0$:

Corollary 4. $\frac{\partial \mathbb{E}[Q_p^*]}{\partial R} \geq 0$ *and* $\frac{\partial \mathbb{E}[Q_s^*]}{\partial R} \leq 0$.

[15] The full analysis for $R > 0$ is delegated to the appendix.

Corollary 4 considers the opposing effects that increasing member revenue can have on funding shortfall, depending on the type of equilibrium that arises. Interestingly, as increased revenues allow the clearinghouse to set lower equity and default fund levels in pooling equilibria, expected funding shortfall increase. This is against the common argument that large corporate revenues can increase the capital buffer that members can draw on during market stress: the clearinghouse's equilibrium response to members having higher revenue is to reduce the layer of default resources. While some members may enjoy a larger economic surplus (e.g. the risky members in our model), the equilibrium result is actually a riskier clearing network. On the other hand, if the prevailing equilibrium is separating, the clearinghouse increases default fund requirements to increase aggregate funding costs borne by members, and drive out safe members who may be attracted by the larger revenue. In this case expected funding shortfall decreases with member revenue.

6 Empirical predictions

Our model shows that the primary role of equity is to attract safer members to participate in clearing, and that default funds mitigate the wealth transfer from safe members to risky ones. A primary determinant of the equilibrium default waterfall is the level of risk-sharing, γ, which can also be interpreted as how diverse members' risk profiles are. For instance, γ is larger when risky members' default probabilities are significantly higher than those of safe members ($d_H - d_L$ is large).

Empirical Prediction 1. *Clearing networks where participants have more diverse risk profiles are (on a risk and size adjusted basis) (i) capitalized with more clearinghouse equity and default funds, (ii) less profitable, and (iii) safer from a systemic perspective.*[16]

Asymmetric information problems are more pronounced when risk profiles are diverse (large γ), since the clearinghouse's knowledge of the average member's default risk is less useful. Uncertainty in the risks imposed onto the clearing network by her members reduces clearing profit. The clearinghouse then requires a larger default fund and contributes more equity to attract safe members, which in equilibrium reduces systemic risk, even when risk profiles diversity stems from increased individual riskiness (d_H is larger).

Our results are supported by clearinghouse data in practice. For the quarter ended in September 2015, ICE Clear Credit, the leading US CDS clearinghouse, held \$19.5 billion of initial margins, \$1.9 billion of default funds, and \$50 million of equity contributions to capitalize their clearing network. In comparison, the CDS clearing branch of ICE Clear Europe, held \$7.7 billion of initial margins, \$1.1 billion of default funds, and \$31 million of equity contributions.[17] Using initial margins as a gauge for default risk and size, this

[16]Recall, as our model normalizes the mass of potential members to 1 and uses the expected default loss as the unit of account, it is important to compare on a risk and size adjusted basis.

[17]This data is retrieved from the International Organization of Securities Commissions (IOSCO) disclosures, which are available on the Intercontinental Exchange website `www.theice.com`.

implies that ICE Clear Europe requires more default funds and equity per unit of risk, which can be attributed to the more diverse profile of European market participants compared to those in the U.S..[18]

Empirical Prediction 2. *Diverse risk profiles and low funding costs lead to a "defaulter-pays" clearing network. When the funding cost is high compared to the level of risk-sharing, more loss-mutualization arises.*

As previously discussed, diverse risk profiles intensify asymmetric information, which then incentivizes the clearinghouse to have large member contributions to the default fund (Table 1). This is commonly referred to as a "defaulter-pays" business model, where the clearinghouse sets conservative capitalization requirements to reduce the need for loss-mutualization. At the same time, in low funding cost environments, a clearinghouse uses large default funds to protect her business and equity (Table 1 and Proposition 2); this is because in this case large default funds do not severely impact her members' profits but can provide a lot of default protection, also leading to a "defaulter-pays" default waterfall. In view of the current low interest rate environment, default fund levels are predicted to be relatively high. Indeed, ISDA (2013b) show that for their sample, clearinghouse default fund levels are quite conservative: on average less than 20% of the pre-funded default funds are used when defaults occur under stressed scenarios. On the other hand, when the funding cost is high compared to the level of risk-sharing, it becomes very costly to utilize default funds (Proposition 2). Since members' risk profiles are relatively similar in this case, the clearinghouse can use equity to attract many members to participate, and use the increased revenue to offset lost default protection from requiring lower default funds.

Empirical Prediction 3. *The clearinghouse may choose to not attract safer members to participate when her clearing revenue is low or when she is capital constrained.*

Our model shows that, while the participation of safer members lowers the average default risk, the clearinghouse can only attract them by subjecting her equity to default losses, especially when members gain little revenue from clearing. It is then beneficial for the clearinghouse to attract safe members if (I) the extra revenue she can capture from their participation covers the default losses that she must bear and (II) she has the equity capital to do so. In the event that her revenue is low or she is capital constrained, she may choose to not attract them at all (Corollary 1). This resembles the behavior of CC&G, whose member base is mostly of low quality, as compared to the larger ICE Clear Credit. The latter typically requires members to have a credit rating of A or higher, and has much stricter membership requirements (Armakola and Laurent (2015)).

7 Policy Implications

Our analysis informs policy making along three important dimensions (I) the impact of minimum equity requirements, (II) how the economics of risk-sharing impact systemic

[18] Armakola and Laurent (2015) show that ICE Clear Credit's member base consists only of good quality clearing members, whereas ICE Clear US has a significant portion of low quality ones.

risk and (III) the importance of relating default resource levels to varying funding cost.

7.1 Minimum equity requirements

Minimum equity requirements can induce increases in the default fund. We first discuss how the model proposed in this paper can be used by a regulator to assess holistically the impact of a minimum equity requirement. Such requirements are part of the European Market Infrastructure Regulation (EMIR) but not imposed in the U.S.. There has been much regulatory debate over the proper level of equity clearinghouses should commit. It was one of the major topics discussed during the Global Markets Advisory Committee Meeting held on May 14, 2015 by the U.S. Commodity Futures Trading Commission (CFTC). Clearing members have generally argued for minimum equity requirements and more contributions from clearinghouses to align interests with those of their members (JPMorgan Chase & CO. (2014)). While major clearinghouses generally agree on the proposed incentive effects, they have argued against their skins in the game being a major source of loss absorption (LCH Clearnet (2014a), CME Group (2015)).

In the context of our model, a minimum equity requirement imposes a constraint $\Theta > \theta$ on the clearinghouse's optimization problem. The regulator wants to balance the objectives of (I) maximizing welfare of the clearinghouse and clearing members, and (II) minimizing systemic risk. We can quantify his tradeoff via the following maximization problem:

$$\begin{aligned} \max_{\theta \geq 0} \quad & M(B - \alpha G^*) - \rho \mathbb{E}[Q] \\ \text{subject to} \quad & \Theta \geq \theta. \end{aligned}$$

The first term of the objective function represents the aggregate welfare in the model economy; the second term represents systemic risk generated by the clearing network, where ρ is a parameter quantifying the systemic risk generated per unit expected funding shortfall. We will denote the clearinghouse's choice of equity and default fund levels under the minimum equity requirement by G_I^* and Θ_I^*, respectively.

In line with our previous results, such a minimum equity requirement is binding only when it exceeds the unconstrained equilibrium equity levels.

Lemma 3. *For separating equilibria, the constraint introduced by the policy intervention is binding if and only if $\theta \geq \Theta_s^*$, in which case $\Theta_{s,I}^* = \theta$ and $G_{s,I}^*$ is the unique solution to*

$$R - \alpha G + d_H e^{-G}(1 - e^{-\theta}) = 0 \tag{7.1}$$

For pooling equilibria, the constraint introduced by the policy intervention is binding if and only if $\theta \geq \Theta_p^*$, in which case $\Theta_{p,I}^* = \theta$ and $G_{p,I}^*$ is the unique solution to

$$R - \alpha G + d_L e^{-G} - (1 - d_L)\frac{D}{S} e^{-G-\theta} = 0 \tag{7.2}$$

Quite intuitively, Lemma 3 shows that the minimum equity requirement has an impact only when it is higher than the prevailing equilibrium equity level. When the equity requirement is binding, G_I^* is given by the IR constraint of the marginal participating member. Using Lemma 3, we can assess the impact of the requirement:

Proposition 4. *Assume that all clearing revenue is captured by the clearinghouse, i.e. $R = 0$. Then $\frac{\partial G^*_{s,I}}{\partial \theta} > 0$ when $\theta \geq \Theta^*_s$. Similarly, $\frac{\partial G^*_{s,p}}{\partial \theta} > 0$ when $\theta \geq \Theta^*_p$.*

Proposition 4 has important implications with regards to minimum equity regulations. In particular, there are two effects from the clearinghouse's equilibrium response to such regulations, one that mitigates systemic risk and one that decreases welfare. Proposition 4 shows that the clearinghouse's response to such a regulation is generally to increase her default fund requirement so to protect her equity, which is still incentive compatible for members since they are better protected with the increased equity. The clearinghouse's response complements the regulator's goal of mitigating systemic risk through lowering $\mathbb{E}[Q]$. However, since increased default funds also increase funding costs, aggregate welfare $(B - \alpha G)M$ is reduced. A proper minimum equity policy should balance and internalize these two effects.

7.2 Clearing membership policies

Risk-sharing can mitigate systemic risk but reduces clearing participation. We show that the level of risk-sharing, a measure of member heterogeneity, is central to the determination of default resource equilibria and the associated welfare and systemic risk. This is pertinent to the discussion on how clearinghouses should screen for potential members, and whether it is beneficial to pool the risk of members with various credit qualities. Since $\frac{\partial \mathbb{E} Q^*_p}{\partial \gamma} < 0$, in equilibrium systemic risk decreases with member heterogeneity, and pooling members with different risk profiles may be beneficial in terms of mitigating systemic risk. However, since the mitigation stems from increased default resources, this reduces the profitability of clearing and may increase funding costs, which may in turn reduce clearing participation.

7.3 Hidden risks from increasing funding costs

Low funding costs create the illusion that the clearing network is safe. Another dimension of regulatory policy is to endogenize the effect of varying funding costs. Recall that default funds generally decrease with funding cost, $\frac{\partial G^*}{\partial \alpha} < 0$. The current low interest rate environment indicates that funding costs are generally low, which implies that large amounts of capital can be tied up in the clearing network. This may reduce productive capital investments, and thus regulatory policies *limiting* the increase in default funds when funding costs decrease may be socially desirable. At the same time, the large stock of default resources may be taken as evidence that systemic risk stemming from the clearing network is low. However, when market stress is preceded by a period of rising funding costs, the clearinghouse has the incentive to reduce default fund requirements. She does so to increase profit before the stress event, reducing the resilience of the clearinghouse. Policies should thus take into account the sensitivity of the stock of default resources to variations in interest rates.

8 Conclusion

We analyze the incentives behind the determination of the different layers of default protection in the clearinghouse's default waterfall structure.

The main incentive driving the clearinghouse's choice of equity requirements is the profitability of her clearing business, subject to regulatory constraints. While the precise dependence of the equity levels on market fundamentals is complex, the equity commitment serves to insulate members from each other's default losses, and makes clearing profitable for members who would not participate otherwise. The clearinghouse then has an incentive to use default funds to protect her equity. This also lowers the wealth transfer from safer members to risky ones due to loss-mutualization. Our model provides a framework to quantify the interplay of these incentives, and delivers tractable expressions for equilibrium default fund and equity levels.

Our model focuses mainly on the typical default waterfall, and does not consider more exotic types of loss allocation mechanisms (e.g. additional layers of equity commitments), which can be accommodated by an extension of our framework. In a future continuation of this research, it would be useful to account for the extra funding costs borne by members when there is a funding shortfall, so to reflect their exposure to fire sale losses generated from raising capital under market stress. Members would then need to internalize part of the systemic risk generated, and would most likely prefer larger contributions to default fund resources.

A Comparative Statics Computations

In this section, we provide a full comparative statics analysis of equilibrium default funds and equity commitments for $\phi \in [0, 1)$. It will be useful to define the function

$$h(G) := (R - \alpha G)e^G.$$

We note the following properties of $h(G)$:

1. It is uniquely maximized at $G_0 := \frac{R}{\alpha} - 1$. $h'(G) > 0$ for $G < G_0$ and $h'(G) < 0$ for $G > G_0$.
2. $h(G) \geq 0$ for $G \leq \frac{R}{\alpha}$ and $h(G) < 0$ for $G < \frac{R}{\alpha}$.
3. $h(0) = R$.

We start with default funds, and report the results in Table 4

Computations for Table 4. For separating equilibria with $R \geq \frac{d_H - d_L}{1 - d_L} = \frac{1}{m}\frac{\gamma}{1+\gamma}$, G_s^* solves

$$h(G) = \frac{1}{m}\frac{\gamma}{1+\gamma}.$$

Differentiating the expression, we obtain:

$$\frac{\partial G_s^*}{\partial \alpha} = \frac{G_s^*}{R - \alpha - \alpha G_s^*} \leq 0, \quad \frac{\partial G_s^*}{\partial \gamma} = \frac{1}{(R - \alpha - \alpha G_s^*)e^{G_s^*}}\frac{1}{m}\frac{\gamma}{(1+\gamma)^2} \leq 0,$$

$G_s^* \sim$ param.	$R \leq \frac{\gamma}{m(1+\gamma)}$	$R \geq \frac{\gamma}{m(1+\gamma)}$
α	0	$-$
γ	0	$-$
d_L	0	0
R	0	$+$

$G_p^* \sim$ param.	(i)	(ii)	(iii)	(iv)	(v)
α	$-$	0	$-$	$+$	0
γ	$+$	0	0	$+$	0
d_L	0	0	$+$	0	0
R	0	0	$+$	$-$	0

Table 4: Comparative statics for the equilibrium default fund. The relation between default funds and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. For pooling equilibria, the sign of the relation is generally consistent with that of the non-degenerate case (first column).

$$\frac{\partial G_s^*}{\partial R} = \frac{-1}{R - \alpha - \alpha G_s^*} \geq 0,$$

since $G_s^* \geq \frac{R}{\alpha} - 1$. We also note that the second type of separating equilibria is not vacuous. Indeed, one can consider $m = 0.95, \gamma = e^{-1} > \alpha = R = \frac{1}{0.9(1+e)}$, and $K > 100\alpha$. Then the resulting equilibrium is separating, and yields a positive default fund.

For pooling equilibria, cases $(i), (ii)$ and (v) are trivial. For case (iii), we have $h(G) = -d_L$ and hence $G_p^* \geq \frac{R}{\alpha} - 1$. This implies

$$\frac{\partial G_p^*}{\partial \alpha} = \frac{G_p^*}{R - \alpha - \alpha G_p^*} \leq 0, \quad \frac{\partial G_p^*}{\partial d_L} = \frac{-1}{(R - \alpha - \alpha G_p^*)e^{G_p^*}} \leq 0, \quad \frac{\partial G_p^*}{\partial R} = \frac{-1}{R - \alpha - \alpha G_p^*} \geq 0.$$

For case (v), we have $h(G) = \gamma$ and $G_p^* \leq \frac{R}{\alpha} - 1$, thus

$$\frac{\partial G_p^*}{\partial \alpha} = \frac{G_p^*}{R - \alpha - \alpha G_p^*} \geq 0, \quad \frac{\partial G_p^*}{\partial \gamma} = \frac{1}{(R - \alpha - \alpha G_p^*)e^{G_p^*}} \geq 0, \quad \frac{\partial G_p^*}{\partial R} = \frac{-1}{R - \alpha - \alpha G_p^*} \leq 0.$$

□

Next, we consider the equity commitment:

$\Theta_p^* \sim$ param.	(i)	(ii)	(iii)	(iv)	(v)
α	$\pm$	0	0	0	0
γ	$+$	$+$	0	0	0
d_L	$-$	$-$	0	0	0
R	$-$	$-$	0	0	0

Table 5: Comparative statics for the equilibrium equity rule. Here, $\frac{\partial \Theta_p^*}{\partial \alpha} > 0$ when $R \geq \alpha$ and $\frac{\partial \Theta_p^*}{\partial \alpha} < 0$ otherwise. The relation between equity and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. The sign of the relation is generally consistent with that of the non-degenerate case (first column).

Computations for Table 5. For pooling equilibria, cases $(ii), (iii), (iv)$ and (v) are trivial. We thus focus on the non-degenerate case:

$$-\frac{\partial e^{-\Theta_p^*}}{\partial \gamma} = \frac{(2d_L + \gamma) - d_L\left(\frac{R}{\alpha} - \log\frac{\gamma}{\alpha}\right)}{(\gamma + d_L)^2} \geq \frac{(2d_L + \gamma) - d_L}{(\gamma + d_L)^2} > 0, \quad -\frac{\partial e^{-\Theta_p^*}}{\partial R} = -\frac{\gamma}{\alpha(\gamma + d_L)} < 0,$$

$$-\frac{\partial e^{-\Theta_p^*}}{\partial d_L} = -\frac{\gamma}{(\gamma+d_L)^2} < 0.$$

Moreover,

$$-\frac{\partial e^{-\Theta_p^*}}{\partial \alpha} = \frac{\gamma(R-\alpha)}{\alpha^2(\gamma+d_L)},$$

so $\frac{\partial \Theta_p^*}{\partial \alpha} \geq 0$ when $R \geq \alpha$ and $\frac{\partial \Theta_p^*}{\partial \alpha} < 0$ otherwise. □

We next develop the computations of the sensitivities of the welfare distribution, displayed in Table 2.

Computations for Table 2. The results follow from a straightforward differentiation of the expressions presented in Proposition 5.1. The case that requires extra attention is when $\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$. We arrive at

$$\begin{aligned}
\frac{\partial mK_{H,p}^*}{\partial \gamma} &= -\frac{\alpha}{(1+\gamma)^2}\left(2+\gamma+\log\frac{\gamma}{\alpha}\right) < 0,\\
\frac{\partial mK_{H,p}^*}{\partial \alpha} &= 1-\frac{\gamma\log\frac{\gamma}{\alpha}}{1+\gamma} > 0,\\
\frac{\partial K_{C,p}^*}{\partial \gamma} &= -\frac{\alpha}{(1+\gamma)^2}\left(1-\gamma\log\frac{\gamma}{\alpha}\right) < 0.\\
\frac{\partial K_{C,p}^*}{\partial \alpha} &= -\frac{\log\frac{\gamma}{\alpha}}{1+\gamma} < 0,
\end{aligned}$$

where we have used the fact that $\log\frac{\gamma}{\alpha} \geq 0$ and $\gamma\log\frac{\gamma}{\alpha} \leq d_L < 1$. In addition, when $\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$, G_p^* solves Eq. (4.6), and through implicit differentiation we have

$$\frac{\partial G_p^*}{\partial d_L} = \frac{1}{\alpha e^G + d_L} > 0.$$

Thus we can compute:

$$\frac{\partial mK_{H,p}^*}{\partial d_L} = -\frac{\gamma e^{-G_p^*}}{1+\gamma}\left(1+(1-d_L)\frac{\partial G_p^*}{\partial d_L}\right) < 0.$$

Furthermore

$$\frac{\partial K_{C,p}^*}{\partial d_L} = \frac{e^{-G_p^*}}{1+\gamma}\left((\gamma+d_L)\frac{\partial G_p^*}{\partial d_L}-1\right) = \frac{e^{-G_p^*}}{1+\gamma}\frac{\gamma-\alpha e^{G_p^*}}{\alpha e^{G_p^*}+d_L}.$$

Since (I) $\frac{\partial \alpha e^{G_p^*}}{\partial \alpha} = \frac{e^{G_p^*}}{1+G_p^*} > 0$, and (II) when $\alpha = \gamma e^{-\frac{d_L}{\gamma}}$ we must have $G_p^* = \frac{d_L}{\gamma}$, we conclude that $\gamma - \alpha e^{G_p^*} \geq \gamma - \gamma e^{-\frac{d_L}{\gamma}} e^{\frac{d_L}{\gamma}} = 0$. Together, this means that $\frac{\partial K_{C,p}^*}{\partial d_L} \geq 0$. □

Last, we present the computations for the sensitivities of the expected funding shortfall, tabulated in Table 6.

Computations for Table 6. In view of tables 4 and 5, the only non-trivial case is related to $\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha}$ for non-degenerate pooling equilibria. We have

$$\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha} = \frac{\gamma}{(1+\gamma)\alpha^2}\left(\frac{\alpha}{\gamma}\right)^{\frac{1}{D}+1}\left(\frac{d_L}{D}+\gamma-\gamma\frac{R}{\alpha}+\frac{\gamma}{D}\left(\frac{R}{\alpha}-\log\frac{\gamma}{\alpha}\right)\right).$$

Using the parameters restrictions for non-degenerate pooling equilibria, we obtain

$$1+2\gamma-\gamma\frac{R}{\alpha} \geq \frac{d_L}{D}+\gamma-\gamma\frac{R}{\alpha}+\frac{\gamma}{D}\left(\frac{R}{\alpha}-\log\frac{\gamma}{\alpha}\right) \geq \gamma-\gamma\frac{R}{\alpha}.$$

Thus if $R < \alpha$, $\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha} > 0$, whereas if $R > \frac{1+2\gamma}{\gamma}\alpha$, $\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha} < 0$. Notice that the second case is not vacuous since for case (i), $\alpha < \gamma$, and thus $R > 1+2\alpha$ suffices. □

$\mathbb{E}[Q_s^*] \sim$ param.	(i)	(ii)	$\mathbb{E}[Q_p^*] \sim$ param.	(i)	(ii)	(iii)	(iv)	(v)
α	0	$+$	α	$\pm$	0	0	$-$	0
γ	0	$+$	γ	$-$	$-$	0	$-$	0
d_L	0	0	d_L	$+$	$+$	0	0	0
R	0	$-$	R	$+$	$+$	0	$+$	0

Table 6: Comparative statics for equilibrium systemic risk. Here, $\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha} > 0$ when $R < \alpha$ and $\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha} < 0$ when $R > 1 + 2\alpha$. The relation between systemic risk and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. For pooling equilibria, the sign of the relation is generally consistent with that of the non-degenerate case (first column).

B Proofs

Proof of Proposition 1. Observe that, for a fixed p_H, if there exists $(G \geq 0, \Theta = 0)$ that satisfy Eqs. (4.2) and (4.3), then they necessarily maximize the expected profit of the clearinghouse. Moreover, the clearinghouse is indifferent between which level of G prevails. Recall that in this case, we assume that the clearinghouse choses the minimum feasible default fund requirement G.

Consider first the case $G_s^* = 0$. Eqs. (4.2) and (4.3) are both satisfied if and only if $R \leq \frac{d_H - d_L}{1 - d_L}$, in which case the equilibrium default fund level is zero. Otherwise, if $R > \frac{d_H - d_L}{1 - d_L}$, the minimum possible default fund level G is a solution to

$$(R - \alpha G)e^G - \frac{d_H - d_L}{1 - d_H} = 0.$$

Simple calculus shows that there is a unique solution $G_s^* \leq \frac{R}{\alpha}$. This characterizes the equilibrium default fund level. Finally, since the expected profit per member is $K > 0$ for $\Theta = 0$, the clearinghouse prefers full participation ($p_H^* = 1$). □

Proof of Lemma 2. We start with the case $\Theta_p^* = 0$. Then the clearinghouse's expected profit is monotonically increasing in p_L. In addition, since $\frac{d}{s}$ is decreasing in p_L, the feasible region given by Eq. (4.5) increases with p_L. Thus $p_L^* = 1$.

Next, consider the case $\Theta_p^* > 0$. From Lemma 1 we know that (IR_L) is binding. Plugging in the constraint, we see that the clearinghouse's objective can be rewritten as

$$J := K(1 - m)p_L - de^{-G} + \frac{s}{1 - d_L}\left(R - \alpha G + d_L e^{-G}\right)$$

Differentiating the transformed objective function J with respect to p_L leads to

$$\begin{aligned}\frac{\partial J}{\partial p_L} &= \phi(1 - m)B - (1 - m)d_L e^{-G} + \frac{(1 - m)(1 - d_L)}{1 - d_L}\left(\phi B - \alpha G + d_L e^{-G}\right) \\ &= (1 - m)(B - \alpha G).\end{aligned}$$

We now argue that the clearinghouse's expected profit is increasing in p_L. When $B < \alpha G$, the net outflows of the model economy exceed production, and the clearinghouse necessarily makes negative expected profit. Thus, we must have $G_p^* < \frac{B}{\alpha}$, which implies $\frac{\partial J}{\partial p_L} > 0$. Hence, $p_L^* = 1$ in equilibrium. □

Proof of Proposition 2. We first consider the case $\Theta_p^* = 0$. This is always the prevailing equilibrium should there exist $G_p^* \geq 0$ such that (G_p^*, Θ_p^*) is feasible. Using Lemma 2, the clearinghouse's problem becomes

$$\max_{G,\Theta \geq 0} \quad K,$$

$$\text{subject to} \quad R - \alpha G - \gamma e^{-G} \geq 0.$$

Recall that when the clearinghouse is indifferent among default fund levels, we assume that default funds are set to the minimum feasible level. Simple calculus shows that $G_p^* = 0$ if $R \geq \gamma$, and is the unique solution to $R - \alpha G - \gamma e^{-G} = 0$. This is smaller than $\frac{R}{\alpha} - 1$ if $R < \gamma$, $\frac{R}{\alpha} + \log \alpha \geq \bar{h}(\gamma)$, and $\gamma \geq \alpha$. This gives the cases (iv) and (v).

Next, we consider the remaining cases, in which we must have $\Theta_p^* > 0$. By Lemma 1, (IR$_L$) is binding. We can rewrite the clearinghouse's problem as

$$\begin{aligned} \max_{G,\Theta \geq 0} \quad & J_1 := -De^{-G} + \frac{S}{1-d_L}\left(R - \alpha G + d_L e^{-G}\right), \\ \text{subject to} \quad & R - \alpha G + d_L e^{-G} - (1-d_L)\frac{D}{S} e^{-G-\Theta} = 0, \end{aligned} \tag{B.1}$$

We observe that

$$\frac{\partial^2 J_1}{\partial G^2} = e^{-G}\left(\frac{S d_L}{1-d_L} - D\right) = -\frac{m(d_H - d_L)e^{-G}}{1-d_L} < 0.$$

Hence, J_1 is concave in G and there is a unique maximizer given by $\frac{\partial J_1}{\partial G} = 0$.

The solution to the first order condition is $\tilde{G} = \log \frac{\gamma}{\alpha}$. Plugging it into Eq. (B.1) gives

$$\tilde{\Theta} = -\log\left(\frac{\gamma}{\gamma + d_L}\left(\frac{R}{\alpha} - \log\frac{\gamma}{\alpha} + \frac{d_L}{\gamma}\right)\right).$$

These quantities are the clearinghouse's equilibrium choices if $(\tilde{G}, \tilde{\Theta}) \in R_+^2$. $\tilde{G} \in R_+$ implies $\gamma \geq \alpha$, whereas $\tilde{\Theta} \in R_+$ implies $\bar{h}(\gamma) > \frac{R}{\alpha} + \log \alpha > \underline{h}(\gamma)$.

We observe that (I) the relation implied by $\bar{h}(\gamma) = \frac{R}{\alpha} + \log \alpha$, $\bar{\gamma}(\alpha) = \alpha e^{\frac{R}{\alpha} - 1}$, is a convex function of α satisfying $\lim\limits_{\alpha \to 0^+} \bar{\gamma}(\alpha) = \lim\limits_{\alpha \to \infty} \bar{\gamma}(\alpha) = \infty$. Moreover, the minimum value $\bar{\gamma}(\alpha_0) = R$ occurs at $\alpha_0 = R$; (II) the relation implied by $\underline{h}(\gamma) = \frac{R}{\alpha} + \log \alpha$, $\underline{\gamma}(\alpha)$, is a convex function of α satisfying $\lim\limits_{\alpha \to 0^+} \underline{\gamma}(\alpha) = \lim\limits_{\alpha \to \infty} \underline{\gamma}(\alpha) = \infty$. The minimum value occurs at $\alpha_0 = R$ and $\underline{\gamma}(\alpha) \geq \alpha$; (III) $\underline{\gamma}(\alpha) \geq \bar{\gamma}(\alpha)$ since $\bar{h}(\gamma) \geq \underline{h}(\gamma)$. Given R and d_L, this implies that the $\alpha - \gamma$ plane can be sectioned as presented in Figure 4.

Continuity of the maximum (Berge's Maximum Theorem) implies that we can analyze which of the constraints $G \geq 0$, $\Theta \geq 0$, and $\Theta < \infty$ are binding by looking at Figure 4. Straightforwardly, we see that when the constraint $\underline{h}(\gamma) \leq \frac{R}{\alpha} + \log \alpha$ is violated, we have $\Theta_p^* = \infty$ and G_p^* solves Eq. (4.6). Next, when $\alpha > \gamma \geq R$, it is the constraint $G \geq 0$ that becomes binding. In this case, we must have $G_p^* = 0$, which when plugged into Eq. (B.1) gives $\Theta_p^* = \log \frac{d_L + \gamma}{d_L + R}$. Notice that $\Theta_p^* \in R_+$ since $\gamma \geq R$. In addition, for $\alpha \geq R$, $\gamma = R$ implies $\Theta_p^* = 0$. □

Proof of Proposition 3. See Tables 4 and 5. □

Proof of Corollary 4. See Table 6 □

Proof of Lemma 3. Using the same argument as that of Lemma 1, we see that one of the IR constraints must be binding in equilibrium.

For separating equilibria, the IR constraint of the risky members must then be binding. In this case, the clearinghouse would set $\Theta_{s,I}^* = \Theta_s^*$ if $\Theta \geq \theta$ were not binding. Thus, if $\theta > \Theta_s^*$, $\Theta_{s,I}^* = \theta$ and $G_{s,I}^*$ solves Eq. (7.1). The cases for pooling equilibria and Eq. (7.2) follow analogously. □

Proof of Proposition 4. For the case of separating equilibria, using implicit differentiation we can directly compute

$$\frac{\partial G_{s,I}^*}{\partial \theta} = \frac{d_H e^{-G_{s,I}^*}(1 - e^{-\theta})}{\alpha + d_H e^{-G_{s,I}^*}(1 - e^{-\theta})} > 0.$$

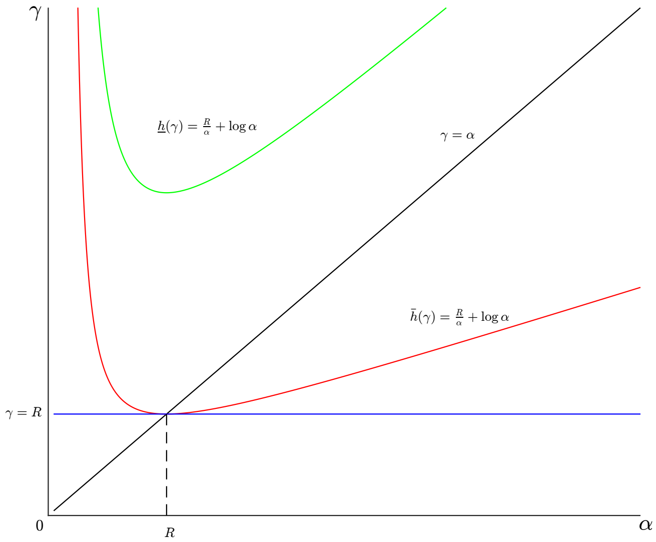

Figure 4: Sectioning of the $\alpha - \gamma$ plane. The various constraints in the optimization problem section the parameter space into several regions. Our analysis shows that the behavior of the optimal default resources depend critically on which region the parameters are in.

For the case of pooling equilibria, using implicit differentiation we obtain

$$\frac{\partial G^*_{p,I}}{\partial \theta} = \frac{(1-d_L)\frac{D}{S}e^{-G^*_{p,I}-\theta}}{\alpha + d_L e^{-G^*_{p,I}} - (1-d_L)\frac{D}{S}e^{-G^*_{p,I}-\theta}}.$$

Hence, the sign of the derivative depends on the sign of the denominator. We can compute:

$$\alpha + d_L e^{-G^*_{p,I}} - (1-d_L)\frac{D}{S}e^{-G^*_{p,I}-\theta} = \alpha + d_L e^{-G^*_{p,I}} - (\gamma + d_L)e^{-G^*_{p,I}-\theta} = \alpha + \alpha G^*_{p,I} > 0.$$

Thus $\frac{\partial G^*_{p,I}}{\partial \theta} > 0$. $\square$

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